

Mathematical model of the running system of combine harvesters

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Abstract. Various factors affect the performance of mobile agricultural machinery, including the impact of the operator and external loads. The analysis of these factors is an urgent task, since the identification of patterns of their occurrence allows, if not to eliminate the cause, then to predict the malfunction and adjust the equipment of service centers. Various methods and criteria are used to assess the resource of machine units: an approximate assessment of reliability characteristics was carried out on the basis of statistical material on the submitted failures of 180 machines ($N_{\Sigma}=180$), which was the result of incomplete resource tests; differential equations were compiled using the Lagrange method. During the resource tests and according to the results of operation, material was collected on the failures of the undercarriage units of the combine and breakdowns of their parts. The developed model of the combine allowed to perform an amplitude and frequency analysis of the chassis of the chassis system when driving on country roads and to assess the smoothness of the machine and the working conditions of the operator. In addition, it allows you to rationally choose the speed at which over-resonant or pre-resonant ranges of operation will be provided.

1 Introduction

The performance of mobile agricultural machinery is influenced by various factors, including the impact of the operator and external loads [1, 2, 3]. The combination of these factors and the impact on the parts of the units are random [3-6]. The most characteristic failures leading to the failure of the running system of the combine harvester are: failure of the gearbox as a result of the destruction of the secondary shaft bearing; destruction of the gear teeth of the second and third gears; destruction of the satellites of the central gear, crown gear, half-axis, fracture of the axis of the on-board gearbox; destruction of the satellites of the bearings of the central and crown gear of the on-board gearbox; fracture of the primary shaft of the gearbox; destruction of the bearing of the primary shaft of the gearbox; breakage of the lugs of the gearbox attachment to the drive axle; cracks in the differential housing, breakage by welding of the cross beam bracket of the running bridge.

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The analysis of these failures is an urgent task, since the identification of patterns of their occurrence allows, if not to eliminate the cause, then to predict the malfunction and adjust the equipment of service centers.

2 Materials and methods

During the resource tests and according to the results of operation, material was collected on the failures of the undercarriage units of the combine and breakdowns of their parts. In Figure 1 shows the characteristic types of destruction of the parts of the on-board gearbox and gearbox of combines, and in Figure 2 - destruction of the elements of the glider gears of the on-board gearbox.

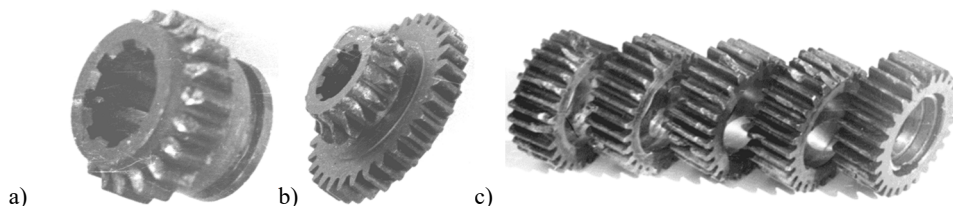


Fig. 1. Destruction of gear teeth of the gearbox: a) reverse gear stages; b) II gears; c) satellites.

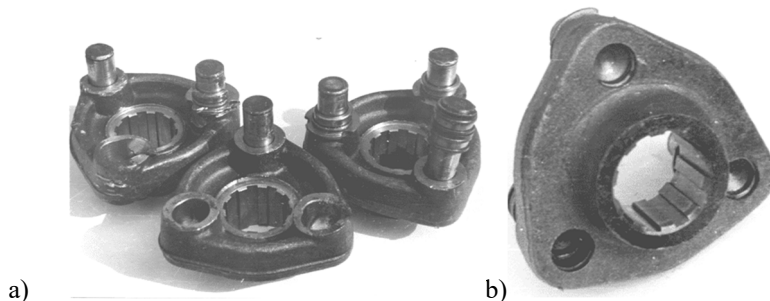


Fig. 2. Failure of the elements of the on-board gearbox: a) Destruction of the on-board gearbox driver; b) self-compression of the fingers of the driver.

Various methods and criteria are used to assess the resource of the machine units [4, 5, 7-9], an approximate assessment of the reliability characteristics (average (50%) and γ % of the resource (95%)) was carried out based on statistical material on the submitted failures of 180 machines ($N_{\Sigma}=180$), which was the result of incomplete resource tests.

In the distribution of failure rates, they were grouped with an interval of 20 hours. A cumulative was constructed from the distribution polygon [9] and then the experimental materials were approximated by the theoretical laws of distributions (Figure 3). Studies have shown that the nature of the initial section of the resource distribution function of the running system is closest to the logarithmically normal distribution [9]. In this case, the logarithm of the resource TR corresponding to the probability P is determined by the formula:

$$\lg T_p = \lg \bar{T} + u \sigma,$$

where \bar{T} is the average resource; and u is the quantile of the normal distribution; σ is the mean square deviation of the logarithm of the resource from the average value.

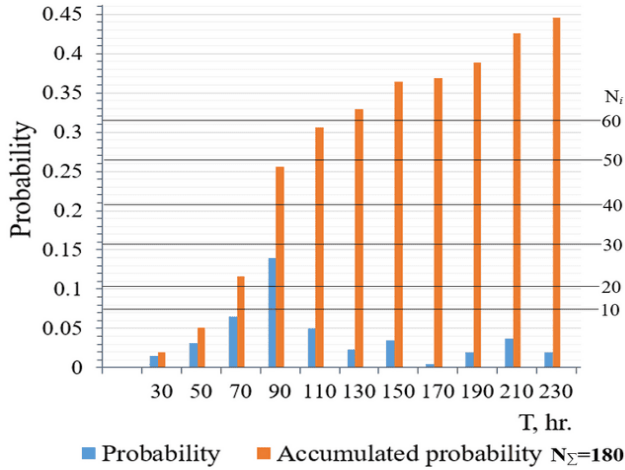


Fig. 3. Polygons of resource allocation of the driving bridge.

In Figure 4 the distribution function of the running system resource is constructed in logarithmic (horizontal) and probabilistic (vertical) scales, in this case the curve of the logarithmic-normal distribution law is aligned in a straight line, which allows us to determine the operating time for a given probability of trouble-free operation [4, 5]. The dots on the graph show experimental data. The analysis of the obtained graphical dependence allowed us to establish: the approximate value of the average resource is 280 hours, we accept $T_m = 250 \dots 300$ hours; a resource with a probability of 0.95 - 40 hours, we accept $T_{0.95} = 35 - 40$ hours. The average square deviation (σ) of the resource from the mathematical expectation (T_m) was 78 hours, $\sigma = 78$ hours.

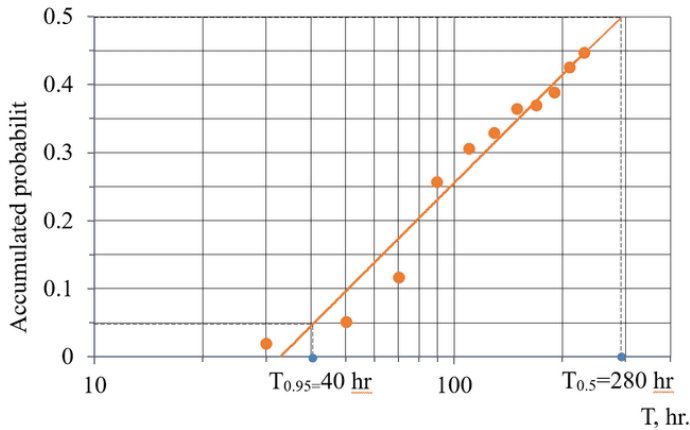


Fig. 4. The resource allocation function of the bridge of the running wheels.

The coefficient of variation of the resource allocation function was determined by the formula:

$$V_T = \frac{\sigma}{T_m} = \frac{78}{280} = 0.28 \tag{1}$$

The ratio of $T_{0.95}$ and T_m was:

$$K = \frac{T_{95}}{T_m} = \frac{40}{280} = 0.143 \tag{2}$$

The indicators determined by formulas (1) and (2) are the result of resource dispersion caused by different operating and manufacturing conditions. Similar parameters of resource dispersion for tractor and automobile running axles have values [2, 3]: for the coefficient of variation from 0.3...0.5, for the ratio T_{95} to T_m from 0.13...0.45.

The average and 95 % of the bridge resource are low. So, 95% of the life of the bridge of the running wheels, at the rate of 200 hours of annual loading, the machine should be about 2000 hours, and the average resource, based on the formula (2), respectively:

$$T_m = \frac{T_{95}}{K} = \frac{2000}{0.13} = 15400 \text{ hours} \tag{3}$$

In fact, 95% and the average resource is about 50 times less than the required one, which ensures failure of no more than 5% of the output of products over the 10-year service life of the machine. The analysis of the results shows that the increase in the life of the chassis of the combine is relevant and requires research.

3 Results and discussion

Equations should be centred and should be numbered with the number on the right-hand side.

The nature of breakdowns of parts of the aggregates of the running system of the combine shows the presence of overloads. Overloads are determined not only by dynamic external influences on the running system and the speed of movement of the combine, but also by the mass-geometric and elastic-dissipative characteristics of oscillatory systems. These characteristics are not constant and are determined by specific operational factors [3, 8, 9, 10].

In operating modes, the running system provides copying of the field relief and minimizes grain losses, in transport modes - comfortable working conditions of the machine operator [7, 11, 12]. In addition, when moving the combine, the running wheels should not come off the ground to ensure a minimum of energy spent on movement. These features determine the difference between dynamic models for different operating modes.

In transport modes, the smoothness of the course depends on the intensity of the vibrations of the combine when moving over the irregularities of the field. In this regard, the oscillation parameters of the combine are the initial data for the study of the running system [13-18].

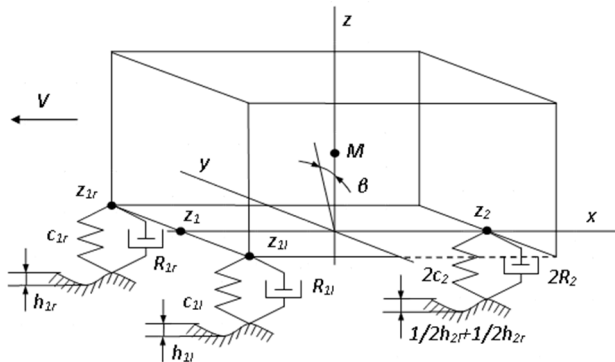


Fig. 5. An oscillatory system equivalent to a combine harvester.

When studying vibrations, the machine body was replaced by an equivalent oscillatory single-mass system, shown in Figure 5. The mass of the combine M is connected to the field by tires having stiffness c and attenuation k . Of all possible displacements, only vertical z oscillations create maximum accelerations.

Differential equations were compiled using the Lagrange method [4,6,18] in the form:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} = \frac{\partial P}{\partial q_i} - \frac{\partial R}{\partial q_i} \quad (\text{by } i = 1 \dots n), \tag{4}$$

The main movements of the combine were characterized by three movements: z_{1l} - movements of the left running wheel; z_{1r} - movements of the right running wheel; z_2 - movement of the bridge of the controlled wheels. In this case, the kinetic energy of the system:

$$T = \frac{1}{2} (M_1 z_1^2 + M_2 z_2^2 + 2M_3 z_1 z_2 + M_3 \rho_X^2 \beta^2) \tag{5}$$

where T is the kinetic energy of the system; P is the potential energy of the system; R is the dissipative function of the system; q_i is the generalized coordinate.

Potential energy of the system:

$$P = \frac{1}{2} \left\{ c_1 (z_{1l} - h_{1l})^2 + (z_{1r} - h_{1r})^2 + 2c_2 \left(z_2 - \frac{1}{2} h_{2r} - \frac{1}{2} h_{2l} \right) \right\} \tag{6}$$

where: c_1, c_2 - the stiffness of the running and steering wheels; $h_{1l}, h_{1r}, h_{2l}, h_{2r}$, - the height of the irregularities and the speed of movement, respectively, with which the wheels of the combine meet.

Dissipative function of the system:

$$R = \left(\frac{1}{2} k_1 \left[(\dot{z}_{1l} - \dot{h}_{1l})^2 + (\dot{z}_{1r} - \dot{h}_{1r})^2 \right] + 2k_2 \left(\dot{z}_2 - \frac{1}{2} \dot{h}_{2r} - \frac{1}{2} \dot{h}_{2l} \right) \right) \tag{7}$$

where: k_1, k_2 - damping coefficients of vibrations in the tires of the running and steering wheels.

After substituting the expressions for T, P and R into the Lagrange equation, the differential equations of the combine oscillations were obtained:

$$\left\{ \begin{array}{l} M_1 \ddot{z}_1 + 2k_1 \dot{z}_1 + M_3 \ddot{z}_2 + 2c_1 z_1 = (c_1 h_{1L} + k_1 \dot{h}_{1L}) + (c_1 h_{1n} + k_1 \dot{h}_{1n}); \\ M_2 \ddot{z}_2 + 2k_2 \dot{z}_2 + M_3 \ddot{z}_1 + 2c_2 z_2 = (c_2 h_{2L} + k_2 \dot{h}_{2L}) + (c_2 h_{2n} + k_2 \dot{h}_{2n}); \\ M \rho_X^2 \ddot{\beta} + \frac{k_1 K^2}{4} \dot{\beta} + \frac{c_1 K^2}{4} \beta = \frac{c_1 K}{2} (h_{1L} - h_{1L}) + \frac{k_1 K}{2} (\dot{h}_{1L} - \dot{h}_{1n}). \end{array} \right. \tag{8}$$

where: K is the track of the running wheels.

From this system, differential equations were obtained describing the vibrations of the body in the vertical plane, if the right and left wheels of the combine move along the road of the same profile:

$$\begin{aligned} \ddot{z}_1 + 2h_1 \dot{z}_1 + \omega_1^2 z_1 + \eta_1 \ddot{z}_2 &= \omega_1^2 h_{1K} + 2h_1 h_{1K} \\ \ddot{z}_2 + 2h_2 \dot{z}_2 + \omega_2^2 z_2 + \eta_2 \ddot{z}_1 &= \omega_2^2 h_{2K} + 2h_2 h_{2K} \end{aligned} \tag{9}$$

where $h_{1K} = h_{1l} = h_{1r}$; $h_{2K} = h_{2l} = h_{2r}$; ω_1, ω_2 - partial frequencies; η_1, η_2 - the coefficients of the relationship between the vibrations of the front and rear parts of the case.

For combines of the "Acros" - 1500, η_1 and η_2 are close to zero, so the vibrations of the front and rear parts can be considered independent and described by the equation:

$$\ddot{z} + 2h \dot{z} + \omega_0^2 z = \omega_0^2 h_k + 2h \dot{h}_k \tag{10}$$

In formula (10), the indices are omitted, because the equation has the same form for the front and back of the case.

Consider the forced vibrations of the combine occurring under the influence of external disturbing forces, having a periodic character [7]. With the sinusoidal shape of the obstacle, the differential equation (10) has the form:

$$\ddot{z} + 2h\dot{z} + \omega_0^2 z = \omega_0^2 h_0 (1 - \cos \omega t + 2 \frac{\psi_0 \omega}{\omega_0} \sin \omega t) \tag{11}$$

where: ψ_0 - relative attenuation coefficient; $h_k = h_0 (1 - \cos \omega t)$; $\dot{h}_k = \omega h_0 \sin \omega t$.

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Equation (11) has a solution:

$$z = Ah_0 (e^{-ht} B \sin t \sqrt{\omega_0^2 - h^2} - C \cos t \sqrt{\omega_0^2 - h^2} + [1 - \cos(\omega t - \varphi) + 2 \frac{\psi_0 \omega}{\omega_0} \sin(\omega t - \varphi)]) \tag{12}$$

Coefficients A, B, C and angle φ : $A = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2 \omega^2}}$;

$$B = \frac{\omega \sin \varphi \left(1 - 2 \frac{\psi_0 \omega h}{\omega_0} \right) - h(1 - \cos \varphi)}{\sqrt{\omega_0^2 - h^2}}$$

$$C = 1 - \cos \varphi + 2 \frac{\psi_0 \omega}{\omega_0} \sin \varphi$$

$$\text{tg} \varphi = \frac{2h\omega}{\omega_0^2 - \omega^2}$$

Differentiating the expression (12) twice for z , we determine the acceleration of the equivalent system:

$$\ddot{z} = Ah_0 \omega_0^2 e^{-ht} (C \cos t \sqrt{\omega_0^2 - h^2} + B \sin t \sqrt{\omega_0^2 - h^2} + Ah_0 \omega^2 \left[\cos(\omega t + \varphi) + 2 \frac{\psi_0 \omega}{\omega_0} \sin(\omega t - \varphi) \right]) \tag{13}$$

To have an idea of the steady-state fluctuations of the combine at different frequencies of external disturbances (various irregularities and speeds of movement of the combine), the amplitude-frequency characteristics of the combine were constructed (Figure 6).

In Figure 6 presents the amplitude-frequency characteristics of accelerations attributed to the unit of the height of the unevenness: accelerations of the bridge of the controlled wheels $\ddot{z}_2 / 2h_0$; acceleration of the left running wheel $\ddot{z}_{1l} / 2h_0$. On the graphs, a pre-resonant region ($\omega < 5 \text{ sec}^{-1}$) can be distinguished, corresponding to the low speed of the combine and the large length of the irregularities. This area is characterized by the fact that the combine wheels copy the profile, and the acting forces differ little from those statically applied [12, 19]. In the resonance region ($\omega = 5 \div 25 \text{ sec}^{-1}$), the accelerations of the combine increase due to the fact that the frequency of the disturbing force becomes close to the partial frequencies ω_l ,

ω_2 of the combine. In the resonant region ($\omega > 25 \text{ sec}^{-1}$) corresponding to the movement at the transport speed, the acceleration becomes somewhat smaller in magnitude, since the frequencies of external disturbances become higher than the natural frequencies of the combine.

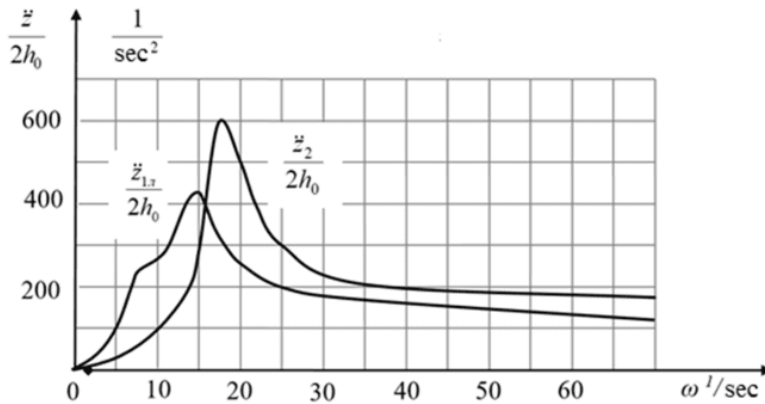


Fig. 6. Amplitude-frequency characteristics of the combine.

4 Conclusion

The analysis of the amplitude-frequency response makes it possible to rationally choose the speed at which over-resonant or pre-resonant ranges of operation will be provided, the load on the machine units is reduced. A characteristic feature of any mobile machine operated by the operator is his choice of such a maximum speed at which his comfortable condition will be ensured, and in this case sanitary standards must be observed.

The presented model of the combine allows you to perform an amplitude and frequency analysis of the chassis of the chassis system when driving on country roads and evaluate the smoothness of the machine and the working conditions of the operator. For a more detailed dynamic analysis, it is necessary to take into account the parameters of the attachments and consider the two-mass combine harvester and harvester.

Analysis of the results of statistical processing of the results of resource tests showed that the average and 95% life of the chassis, 40 hours, of the combine is lower than the standard indicator, 2000 hours, and the issue of increasing it remains relevant. The amplitude-frequency analysis allowed us to determine the resonant area of the combine, lying in the range from 5 to 25 sec^{-1} . For a more detailed dynamic analysis, it is necessary to take into account the parameters of the attachments and consider the two-mass combine harvester and harvester.

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