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Mathematical description of the interaction process in a roll module

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Abstract. An analysis of the shape of the contact arc in the roll module was performed for the case when changes in the thickness of the material and the roller coating at each point of the contact zone occur in the radial direction to the roller axis. Mathematical models of the shape of the contact arc of the roll module were developed. Particular mathematical models of the contact arc shape were obtained for the cases when the material being processed is absolutely elastic, elastic-viscous, and plastic. It was determined that the shape of the contact arc in the roll module depends primarily on the geometric and deformation properties of the roller material and the roller coating.

1 Introduction

The interaction process in the roll module occurs between the rolled material and the elastic coatings of the rollers along the roll contact arcs.

The shape of each contact arc of the roller determines a number of technological indices of the process, such as the magnitude and distribution of contact stresses, the maximum pressure on the material, the tractive capacity of a roller pair, and the length of the contact arc. In addition, the shape of each contact arc affects the mechanical characteristics of the roll module operation, primarily the rolling resistance, and its dependence on the rolling velocity, that is, the energy indices of the process.

A mathematical description of the shape of the roll contact arc is one of the complex problems of contact interaction since when solving it, one has to consider deformation, geometric, kinematic, and force parameters of the rolled material and the roller coating.

Contact interaction in the roll module can be considered by analogy with the rolling of a wheel on the ground [1-5]. At present, the form of wheel contact with the ground is expressed by various lines, and accordingly, by various formulas, which do not indicate the scope of application [6-10]. For example, in [11], the shape of the contact line is immediately displayed by equations in three forms: a truncated circle, an ellipse, and a polynomial of the third degree, but the scope of application of any formula is not indicated.

The analysis showed that there are many different mathematical models of the form of wheel contact with the ground, which do not have a scope of application.

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In [1,12-17], the analytical description of the contact shape of the roll module is associated with a power-law empirical stress-strain relationship of the materials of contacting bodies, obtained experimentally.

The shape of the roll contact arc of a roll module is determined by the direction of change in the thickness of the contacting bodies in the contact zone [6,14,15]. The opinions of the authors of the study fell into two groups. In the first group, it is believed that changes in the thickness of the contacting bodies at each point of the contact zone occur in the vertical direction of the strip of rolled material [18-23], and in the second group - in the radial direction to the roller axis [24-26].

This study is devoted to the mathematical description of the shape of the roll contact arc in a symmetrical roll module, where changes in the thickness of the contacting bodies at each point of the contact zone occur in the radial direction to the roller axis.

2 Materials and methods

In the compression zone, point K is determined by the polar coordinates ρ_1 and φ_1, where $-\alpha_1 \le \varphi_1 \le 0$, α_1 – is the nip angle (Figure 1).

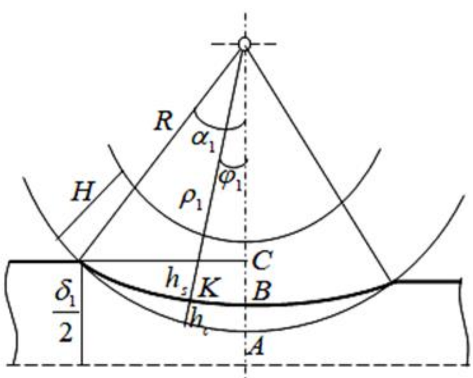

Fig. 1. Scheme of interaction in the roll module.

At point K, the thickness of the rolled material and the roller coating change radially to the roller axis:

$$h_{1s} = \rho_1 - R\frac{\cos\alpha_1}{\cos\varphi_1}, \qquad h_{1c} = R - \rho_1, \tag{1}$$

where $R-$ is the radius of the roller.

From equalities (1), it follows that

$$\frac{dh_{1s}}{dt} = \frac{d}{dt}\left(\rho_1 - R\frac{\cos\alpha_1}{\cos\varphi_1}\right), \qquad \frac{dh_{1c}}{dt} = \frac{d}{dt}(R-\rho_1)$$

or

$$\frac{d\varepsilon_{1s}}{dt} = \frac{2}{\delta_1}\frac{d}{dt}\left(\rho_1 - R\frac{\cos\alpha_1}{\cos\varphi_1}\right), \qquad \frac{d\varepsilon_{1c}}{dt} = \frac{1}{H}\frac{d}{dt}(R-\rho_1) \tag{2}$$

where $\delta_1 -$ is the initial thickness of the rolled material, $H-$ is the thickness of the roller coating, $\frac{d\varepsilon_{1s}}{dt}$, $\frac{d\varepsilon_{1c}}{dt}$ – are the deformation rates of the rolled material and the roller coating.

As in the theory of the wheel [14], we assume that:

$$a_1 = \frac{\dfrac{d\varepsilon_{1c}}{dt}}{\dfrac{d\varepsilon_{1s}}{dt}}. \tag{3}$$

3 Results and discussion

From equality (3) considering expression (2), we obtain:

$$\frac{1}{H}\frac{d}{dt}(R-\rho_1) = a_1\frac{2}{\delta_1}\frac{d}{dt}\left(\rho_1 - R\frac{\cos\alpha_1}{\cos\varphi_1}\right)$$

or

$$\frac{d}{d\varphi_1}(R-\rho_1)\frac{2}{\delta_1}a_1\frac{d}{d\varphi_1}\left(\rho_1 - R\frac{\cos\alpha_1}{\cos\varphi_1}\right).$$

Hence

$$d\rho_1 = R\cos\alpha_1\frac{a_1b_1}{1+a_1b_1}\frac{\sin\varphi_1}{\cos^2\varphi_1}d\varphi_1, \tag{4}$$

or after integration

$$\rho_1 = R\cos\alpha_1\int\frac{a_1b_1}{1+a_1b_1}\frac{\sin\varphi_1}{\cos^2\varphi_1}d\varphi_1 + C, \tag{5}$$

where $b_1 = \frac{2H}{\delta_1}$, $C-$is the constant of integration.

If the roller in the roll module has a non-deformable coating, then $\frac{d\varepsilon_{1c}}{dt} = 0$, and $a_1 = 0$

From equality (5), it follows that

$$\rho_1 = C.$$

Here, using the boundary condition $\rho_1 = R$, for $\varphi_1 = -\alpha_1$, we obtain:

$$\rho_1 = R. \tag{6}$$

If the material rolled in the roll module is non-deformable, then $\frac{d\varepsilon_{1s}}{dt} = 0$, and $a_1 \to \infty$.

From equality (5), considering the above boundary condition, it follows that

$$\rho_1 = \frac{R\cos\alpha_1}{\cos\varphi_1}. \tag{7}$$

In a general case, a_1 is a function of time, therefore, of angle φ_1.

Consider an example where we assume that $a_1 = \cos\varphi_1$.

Then from equality (5), we have

$$\rho_1 = R\cos\alpha_1\int\frac{b_1}{1+b_1\cos\varphi_1}\frac{\sin\varphi_1}{\cos\varphi_1}d\varphi_1 + C,$$

after integration

$$\rho_1 = -b_1\cos\alpha_1\ln\frac{\cos\varphi_1}{1+b_1\cos\varphi_1} + C.$$

Having determined the constant of integration, considering the above boundary condition, we obtain:

$$\rho_1 = R\left(1 - b_1 \cos\alpha_1 \ln \frac{\cos\alpha_1 (1 + b_1 \cos\varphi_1)}{\cos\varphi_1 (1 + b_1 \cos\alpha_1)}\right)$$

or

$$\rho_1 = R\left(1 - b_1 \cos\alpha_1 \ln\left(1 - \frac{\cos\alpha_1 - \cos\varphi_1}{\cos\varphi_1 (1 + b_1 \cos\alpha_1)}\right)\right).$$

After expanding the logarithmic function in a series and restricting ourselves to the first approximation with the first term of expansion, we find

$$\rho_1 = R\frac{1 + b_1 \cos\varphi_1}{1 + b_1 \cos\theta_1}. \tag{8}$$

In the compression zone, the contact of the rolled material with the roller coatings occurs at a short time interval. Therefore, a_1 may be considered a constant. In this case, when $a_1 = const$, it follows from equality (5):

$$\rho_1 = \frac{Ra_1 b_1}{1 + a_1 b_1}\frac{\cos\alpha_1}{\cos\varphi_1} + C.$$

From here, using the boundary condition $\rho_1 = R$, for $\varphi_1 = -\alpha_1$, we determine:

$$\rho_1 = \frac{R}{1 + a_1 b_1}\left(1 + a_1 b_1 \frac{\cos\alpha_1}{\cos\kappa_1}\right), \quad -\alpha_1 \le \varphi_1 \le 0. \tag{9}$$

Let us make the notation $c_1 = \dfrac{AB}{BC}$ (Figure 1), where $AB = R - \rho_1(0)$, $BC = \rho_1(0) - R\cos\alpha_1$.

With equality (9) we have $c_1 = a_1 b_1$.

Then equality (9) has the following form:

$$\rho_1 = \frac{R}{1 + c_1}\left(1 + c_1 \frac{\cos\alpha_1}{\cos\varphi_1}\right), \quad -\alpha_1 \le \varphi_1 \le 0, \tag{10}$$

where $c_1 = \dfrac{2H}{\delta_1} a_1$.

By analogy with (10), for the recovery zone, we obtain:

$$\rho_2 = \frac{R}{1 + c_2}\left(1 + c_2 \frac{\cos\alpha_2}{\cos\varphi_2}\right), \quad 0 \le \varphi_2 \le \alpha_2, \tag{11}$$

where $c_2 = \dfrac{2H}{\delta_2} a_2$, a_2 –is the ratio of the strip and coating deformation rates under recovery.

Thus, formulas (10) and (11) describe the shape of the contact arc of each roller of the roll module, where the rollers have elastic coatings, in the case when changes in the thickness of the contacting bodies at each point of the contact zone occur in the radial direction to the roller axis.

Comparative analysis of formulas (6), (7), (8), (10), and (11) with previously derived formulas, for the case when changes in the thickness of the contacting bodies at each point of the contact zone occur in the vertical direction of the strip of rolled material [1], showed that they have the same form. Based on this, we can say that mathematical models of the shape of the roll contact arc do not depend on the direction of change in the thickness of the contacting bodies in the contact zone.

The resulting mathematical models (10) and (11) show that the shape of the roll contact arc depends primarily on the deformation properties of the rolled material and the roller coating, and the rate of their deformation.

The deformation properties of the processed material in a roll module are characterized by a compression curve during interaction with the roller coating and a line determining the recovery of its deformation (shape) [6].

When the material is compressed, the dependence of deformation on load is nonlinear. Therefore, the roll contact arc under compression is described by formula (10).

The shape of the curve during deformation recovery depends on the nature of the material: it can be absolutely elastic, elastic-viscous and plastic.

When the material being processed in the roll module is absolutely elastic, then $c_2 = c_1$ and $\alpha_2 = \alpha_1$, therefore, from formula (11), it follows that the roll contact arc under recovery is described by the following formula:

$$\rho_2 = \frac{R}{1+c_1}\left(1+c_1\frac{\cos\alpha_1}{\cos\varphi_1}\right), \quad 0 \le \varphi_1 \le \alpha_1 , \tag{12}$$

When the material being processed in the roll module is elastic-viscous, then $c_2 \ne c_1$ and $c_2 \ne c_1$, therefore, the contact arc is described by formula (11).

When the material being processed in the roll module is plastic, then $c_2 \to \infty$, therefore, from formula (11) it follows that

$$\rho_2 = \frac{R\cos\alpha_2}{\cos\varphi_2}, \quad 0 \le \varphi_2 \le \alpha_2 , \tag{13}$$

Thus, the shape of the roll contact arc consists of two parts: the first is the curve described by formula (10), and the second is the curve described by one of formulas (11) - (13). Moreover, these formulas depend on the ratio of the deformation rates of contacting bodies, which, in turn, are determined by deformation models that determine the stress-strain relationships under compression and recovery.

For most materials processed in roller machines and materials of roller coating, deformation models have the following form [14,24-26]:

$$\sigma_s = A_1\varepsilon_s^{m_1}, \qquad \sigma_c = B_1\varepsilon_c^{k_1}, \tag{14}$$

where $A_1, m_1, B_1, k_1 -$ are the compression deformation coefficients of the rolled material and the roller coating, respectively.

According to Newton's law, at each point of the compression zone of the contact arc the following equality of stresses holds:

$$A_1\varepsilon_s^{m_1} = B_1\varepsilon_c^{k_1}. \tag{15}$$

Differentiating, we have

$$A_1 m_1\varepsilon_s^{m_1-1}\frac{d\varepsilon_s}{dt} = B_1 k_1\varepsilon_c^{k_1-1}\frac{d\varepsilon_c}{dt}. \tag{16}$$

From equality (15) and (16), considering equality (1), we obtain

$$a_1 = \frac{k_1}{m_1}\frac{\varepsilon_{1c}}{\varepsilon_{1s}}. \tag{17}$$

For $a_1 = const$, the following equality holds:

$$\frac{\frac{d\varepsilon_{1c}}{dt}}{\frac{d\varepsilon_{1s}}{dt}} = \frac{\varepsilon_{1c}}{\varepsilon_{1s}}. \tag{18}$$

From equalities (17) and (18), it follows that $m_1 = k_1$. Considering this, from equality (15), we obtain:

$$\frac{\varepsilon_{1c}}{\varepsilon_{1s}} = \sqrt[m_1]{\frac{B_1}{A_1}}$$

or with equalities (1) and (18):

$$a_1 = \sqrt[m_1]{\frac{B_1}{A_1}}.$$

Then we have

$$c_1 = \frac{2H}{\delta_1}\sqrt[m_1]{\frac{B_1}{A_1}}. \tag{19}$$

Similarly, we find

$$c_2 = \frac{2H}{\delta_2}\sqrt[m_2]{\frac{B_2}{A_2}}. \tag{20}$$

Thus, the ratio of the deformation rates of the contacting bodies and, accordingly, the shape of the contact arc of the roll module depends on the thickness and deformation coefficients of the contacting bodies under compression and recovery.

4 Conclusion

A mathematical model of the shape of the contact arc of a roll module was developed for the case when changes in the thickness of the material and the roller coating at each point of the contact zone occur in the radial direction to the roller axis.

It was revealed that mathematical models of the shape of the roll contact arc do not depend on the direction of change in the thickness of the contacting bodies in the contact zone.

Analysis of the mathematical models obtained showed that the shape of the roll contact arc consists of two parts: the first is the curve described by formula (10), and the second is the curve described by one of formulas (11) - (13). All graphs lie between the graphs of curves $\rho = \frac{R\cos\alpha}{\cos\varphi}$ and $\rho = R$.

Particular types of the mathematical model of the contact arc shape were obtained for the cases when the rolled material is absolutely elastic, viscoelastic, and plastic.

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