Stress distributions along the roll contact arc

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Abstract. The article provides the results of a mathematical description of the distribution of contact stresses of a roll module with smooth rollers. Analysis of the developed models shows that when the rollers rotate, the maximum of the normal stress distribution diagram is displaced in the direction opposite to the movement of the material being processed; the displacement depends on the properties of the material being processed and the forces acting on the rollers. Under static contact, shear stresses are distributed symmetrically relative to the line of centers. In the same way, they are distributed along the arc of contact of the rotating roller close to rotational resistance equal to zero. As the rotational resistance increases, the distribution changes and becomes asymmetrical. In the drive roller, the neutral angle shifts in the direction opposite to the movement of the material, and the free roller, it shifts along the direction of movement.

1 Introduction

Prospective expansion of the use of roller machines requires the study of individual components of these machines [1, 2].

The main element of these machines is a roll module, which is a mechanism that consists of a pair of rollers and materials to be processed and pressed against each other to realize the technological process.

General patterns of contact interaction in a roll module are considered in [3-7]. However, these publications do not describe the stress state along the contact arc for specific conditions of material processing, velocity modes, and properties of contacting bodies [7-12].

The pressure distribution along the contact arc determines several technological indices of the process, such as the maximum pressure on the material, the tractive capacity of a pair of rolls, and the value of the contact arc. In addition, it affects the energy indicators of the process (mechanical characteristics of the operation of the pair of rolls, the resistance to rolling, the rolling velocity, etc.) [13-18].

Methods for studying the stressed state of roller mechanisms are discussed in [19-24], however, approximate models of friction stress used in these publications do not provide sufficient accuracy in determining the force parameters [14-25].

Interaction phenomena, including the stress state in the roll module, depend on the kinematic connection between the links since they determine the patterns of external forces acting on the roll and stresses distributed along the contact arc. The rollers are called driven or free depending on the kinematic connection of the links in the roll mechanism [14].

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This study is devoted to describing the patterns of distribution of contact stresses along the contact arc of the drive rolls of a symmetrical roll module.

2 Materials and methods

In a steady-state process, each roll of the module under consideration is acted upon by the pressure force of the gripping device Q, the reaction of the roll supports F, the moment of resistance M, and elementary normal dN and shear dT forces (Figure 1) [14].

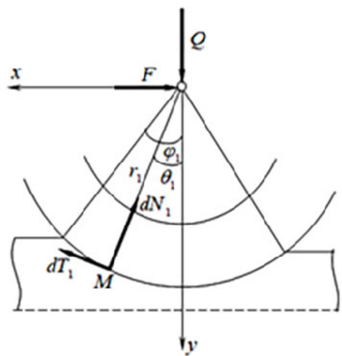

Fig. 1. Scheme of forces acting on the roller.

Let us present the elementary forces in the compression and recovery zones separately. Then, from the equilibrium condition of the roll in the compression zone, we have

$$-F_1 + T_{1x} - N_{1x} = 0, \qquad Q_1 - T_{1y} - N_{1y} = 0$$

or

$$dF_1 = dT_{1x} - dN_{1x}, \qquad dQ_1 = dT_{1y} + dN_{1y}.$$

Considering the scheme of forces (Figure 1), we obtain:

$$dF_1 = dT_1 \cos\theta_1 - dN_1 \sin\theta_1, \tag{1}$$

$$dQ_1 = dT_1 \sin\theta_1 + dN_1 \cos\theta_1. \tag{2}$$

3 Results and discussion

The pressure and reaction forces in the roll gripping device are related to the following dependence [21,26]

$$F = fQ, \tag{3}$$

where f – is the coefficient of friction in the roll support.

Let us assume that

$$F_1 = f_1 Q_1 \tag{4}$$

or

$$dF_1 = f_1 dQ_1.$$

Then from equality (1), we obtain

$$dQ_1 = \frac{1}{f_1}(dT_1 \cos\theta_1 - dN_1 \sin\theta_1). \qquad (5)$$

Equating the right-hand sides of Eqs. (2) and (5), we determine:

$$f_1 = \frac{dT_1 - dN_1 tg\theta_1}{dT_1 tg\theta_1 + dN_1}. \qquad (6)$$

The modules of elementary normal and shear forces are expressed as:

$$dN = ndl, \qquad dT = tdl, \qquad (7)$$

where $n, t -$ are the normal and shear stresses distributed along the contact arc, $dl -$ is the elementary contact arc.

Substituting (7) into equality (6), after transformations, we obtain:

$$t_1 = \frac{f_1 + tg\theta_1}{1 - t_1 tg\theta_1} n_1 \quad ,$$

or assuming that $f_1 = tg\nu_1$

$$t_1 = tg(\theta_1 + \nu_1), \qquad -\varphi_1 \le \theta_1 \le 0, \qquad (8)$$

where $\varphi_1 -$ is the angle of the beginning of the contact arc (the nip angle).

By analogy with (8), for the recovery zone, we obtain:

$$t_2 = tg(\theta_2 + \nu_2), \qquad 0 \le \theta_2 \le \varphi_2, \qquad (9)$$

where $\varphi_2 -$is the angle of the end of the contact arc.

Note that for $\varphi_1 = \varphi_2 = 0$, then $t_1(0) = t_2(0),\ n_1(0) = n_2(0).$ Considering this condition and equalities (3) and (4), we have $tg\nu_1 = tg\nu_2 = tg\nu = \frac{F}{Q}$ or $\nu = arctg\frac{F}{Q}$, where $\nu -$is the friction angle in the roll support.

Then generalizing equalities (8) and (9), we obtain

$$t = tg(\theta + \nu)n, \qquad -\varphi_1 \le \theta \le \varphi_2. \qquad (10)$$

Equalities (10) determine the model of friction stress in a symmetrical roll module, where the rollers are driven.

If the roll module has free rollers, then forces T and F acting on the roller change their direction [14]. Then from equalities (10), it follows that

$$t = -tg(\theta - \nu)n, \qquad -\varphi_1 \le \theta \le \varphi_2. \qquad (11)$$

The distribution of normal stresses along the contact arc depends on the strain properties of the material being processed.

The properties of the processed material are characterized by the compression curve under interaction with the roller and the curve that determines the recovery of its shape [4]. These curves are determined by the dependences of compression and recovery strains on load (stress).

When the material is compressed, the strain from the load is nonlinear. For most materials processed in roller machines, the relationship between compressive strain and stress is described by dependencies of the following form [27-31]:

$$\sigma_1 = A_1 \varepsilon_1^{m_1}, \qquad (12)$$

where $A_1, m_1 -$ are the compression strain coefficients.

The shape of the curve during recovery depends on the nature of the material: it can be absolutely elastic, elastic-viscous, and plastic. In the first case, when leaving the roll bite, the material will completely restore its shape, in the second case - partially, in the third case - there will be no reverse strain [4]. The relation between recovery strain and stress has the following form [32-34]:

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$$\sigma_2 = A_2 \varepsilon_2^{m_2} , \tag{13}$$

where A_2, m_2 – are the recovery strain coefficients.

In the compression area of the strain zone, we select an element of material with length dx_1. The selected element of the material is affected by elementary normal dN_1 and shear dT_1 forces from the side of the roll ; these forces are balanced by the reaction force of the material $\sigma_{1y} dx_1$ [14]

$$-\sigma_{1y} dx_1 + dN_1 \cos\theta_1 + dT_1 \sin\theta_1 = 0$$

or considering equality (7):

$$\sigma_{1y} dx_1 = (n_1 \cos\theta_1 + t_1 \sin\theta_1) dl_1. \tag{14}$$

Let us equate the length of the elementary arc to the length of the chord

$$dl_1 \approx \frac{dx_1}{\cos\theta_1}.$$

Substituting this expression, we get

$$\sigma_{1y} = \frac{(n_1 \cos\theta_1 + t_1 \sin\theta_1)}{\cos\theta_1}$$

or wiht equality (10)

$$\sigma_{1y} = \frac{\cos\theta_1 + \sin\theta_1 tg(\theta_1 + \nu)}{\cos\theta_1} n_1.$$

Hence, we obtain:

$$n_1 = \frac{\cos\theta_1 \cos(\theta_1 + \nu)}{\cos\nu} \sigma_{1y}. \tag{15}$$

From Figure 1, we have

$$\varepsilon_1 = \frac{2R}{\delta_1} (\cos\theta_1 - \cos\varphi_1).$$

Then from equality (12), it follows

$$\sigma_{1y} = A_1 \left(\frac{2R}{\delta_1} (\cos\theta_1 - \cos\varphi_1 \right)^{m_1} .$$

After replacement of σ_{1y} in equality (15) and some transforms, we have

$$n_1 = B_1 (\cos\theta_1 - \cos\varphi_1)^{m_1} (1 + \cos 2\theta_1 - \sin 2\theta_1 tg\nu), \quad -\varphi_1 \le \theta_1 \le 0, \tag{16}$$

where $B_1 = \frac{A_1}{2} \left(\frac{2R}{\delta_1} \right)^{m_1}$, $tg\nu = \frac{F}{Q}$.

By analogy with (17), for the recovery zone, we obtain:

$$n_2 = B_2 (\cos\theta_2 - \cos\varphi_2)^{m_1} (1 + \cos 2\theta_2 - \sin 2\theta_2 tg\nu), \quad 0 \le \theta_2 \le \varphi_2, \tag{17}$$

where $B_2 = \frac{A_2}{2} \left(\frac{2R}{\delta_2} \right)^{m_2}$, δ_2 – is the thickness of the material when it leaves the roll contact zone.

Formulas (16) and (17) describe the patterns of distribution of normal stresses along the contact arc of the roll module under consideration.

Under static contact of the rollers with the material being processed $F = 0$ (or $\nu = 0$), therefore, from formulas (16) and (17), we obtain :

$$n_1 = 2B_1(\cos\theta_1 - \cos\varphi_1)^{m_1}\cos^2\theta_1, \quad -\varphi_1 \le \theta_1 \le 0, \tag{19}$$

$$n_2 = 2B_2(\cos\theta_2 - \cos\varphi_2)^{m_2}\cos^2\theta_2, \quad 0 \le \theta_2 \le \varphi_2. \tag{20}$$

The law of stress distribution along the contact arc of the rollers in the roll module under consideration is determined by substituting formulas (16) - (20) into equalities (10) and (11).

It has the following form

- for the drive roller

$$t_1 = B_1(\cos\theta_1 - \cos\varphi_1)^{m_1}(tg\nu + \sin 2\theta_1 + \cos 2\theta_1 tg\nu), \quad -\varphi_1 \le \theta_1 \le 0, \tag{21}$$

$$t_2 = B_2(\cos\theta_2 - \cos\varphi_2)^{m_1}(tg\nu + \sin 2\theta_2 + \cos 2\theta_2 tg\nu), \quad 0 \le \theta_2 \le \varphi_2; \tag{22}$$

- for the free roller

$$t_1 = B_1(\cos\theta_1 - \cos\varphi_1)^{m_1}(tg\nu - \sin 2\theta_1 + \cos 2\theta_1 tg\nu), \quad -\varphi_1 \le \theta_1 \le 0, \tag{23}$$

$$t_2 = B_2(\cos\theta_2 - \cos\varphi_2)^{m_1}(tg\nu - \sin 2\theta_2 + \cos 2\theta_2 tg\nu), \quad 0 \le \theta_2 \le \varphi_2; \tag{24}$$

- for static contact

$$t_1 = B_1(\cos\theta_1 - \cos\varphi_1)^{m_1}\sin 2\theta_1, \quad -\varphi_1 \le \theta_1 \le 0, \tag{25}$$

$$t_2 = B_2(\cos\theta_2 - \cos\varphi_2)^{m_1}\sin 2\theta_2, \quad 0 \le \theta_2 \le \varphi_2. \tag{26}$$

4 Conclusion

The pressure and reaction forces in the roll gripping device are related to the following dependence.

Mathematical models of friction stress (10) and (11) of a symmetrical roll module with smooth rollers were developed. Here, model (10) determines the relationship between the shear and normal stresses for the drive roller, and model (11) determines that for the free roller.

Formulas (16) – (20) were obtained to describe the laws of distribution of normal stresses along the contact arc between the rollers of a symmetrical roll module and smooth rollers. In this case, formulas (19) and 20 describe the law of distribution of normal stresses in a static state, and (17) and (18) during rotation of the rollers.

Analysis of the curves of these formulas showed that when the rollers rotate, the maximum of the normal stress distribution diagram shifts in the direction opposite to the movement of the processed material (Figure 2). The displacement depends on the properties of the material being processed and the forces acting on the rollers.

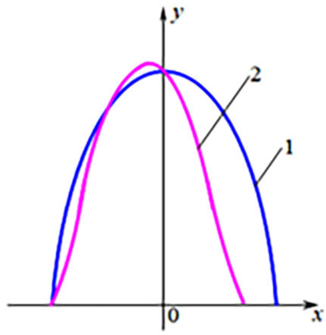

Fig. 2. Diagram of normal stress distribution: 1-in a static state; 2- when the roller rotates.

Formulas (21) – (26) were obtained to describe the laws of distribution of shear stresses along the contact arc between the rollers of a symmetrical roll module and smooth rollers. Here, formulas (25) and (6) describe the law of distribution of shear stresses in a static state, (21) and (22) for the drive roller, and (23) and (24) for the free roller.

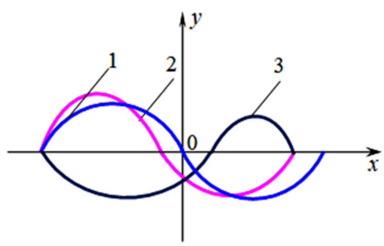

Fig. 3. Diagram of shear stress distribution: 1- in a static state; 2- in the drive roller; 3- in the free roller.

Analysis of the distribution graphs of shear stresses (Figure 3) showed that under static contact shear stresses have different signs and are distributed symmetrically relative to the line of centers. In the same way, they are distributed along the contact arc of the rotating roller with rotational resistance close to zero. As the horizontal reaction to the roll increases, the distribution changes and becomes asymmetrical. In the drive roller, the neutral angle shifts in the direction opposite to the movement of the material, and in the free roller, in the direction of movement.

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