# Pressure calculation in a roller pair

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**Abstract.** As a result of studies on force interaction in a roller pair, a calculation formula was obtained for the specific pressure in a roller pair, for the case when the material being processed is elastically viscous and the rollers are drive. Analysis of the calculation formula showed that the specific pressure in the roller pair depends on the geometric parameters of the material being processed and the roller coating, the viscosity coefficient of the material being processed, and the strain rates of contacting bodies. A formula was obtained to determine the value of the strain coefficient of the processed material under recovery, providing conditions for conjugating strains in the compression and recovery zones.

## 1 Introduction

The pressure on the contact surface determines the technological efficiency of a roller pair. Under the influence of pressure, local strains occur in the contact zone of the roller pair, causing the occurrence of a contact area of a certain width. The width of the contact area depends on the load intensity and the compliance of the roller coatings and the material being processed. The strain of the material in the contact zone makes it possible to estimate the unit force at a specific point, and the non-uniformity of contact stresses [1-2]. At the ideal elasticity of materials, the normal contact stress along the width of the contact area is distributed by an elliptical law. For real materials with the properties of elasticity, viscosity, and plasticity, the pattern of distribution of normal stresses differs from elliptical one [3-10].

The pressure of roller pairs determines several indices of the technological process. In addition, the mechanical characteristics of the roller pair depend on it, especially the quality of the finished product [1, 11-12].

Methods for force research of roller pairs are known from [13-20], however, these studies do not fully examine the strain properties of the material being processed and their variations during the technological process, in particular, the features related to the viscosity of the material being processed [21-26].

In roller technological machines, the rollers have elastic coatings. The strain, geometric, and filtration properties of coatings are determinant in the force analysis of a roller pair [27-30]. The force interaction in roller pairs depends on the kinematic connection between the links since they determine the patterns of external forces acting on the roller. Therefore, the pressure assessment in a roller pair must be conducted considering the kinematic connection between the working rolls [7, 31-32].

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Thus, force analysis of a roller pair is critical and presents an area that still requires more research.

## 2 Materials and methods

Consider the action of forces on an elementary section of the roll contact curve of the roller pair with the drive rollers.

Let us select an element parallel to the $y$-axis in the compression area of the processed strip (Figure 1). Normal $dN_1$ and shear $dT_1$ elementary forces act on this element from the side of the drive roll; these forces are balanced by the vertical component of the strip reaction force $\sigma_{y1}dx_1$ [33]:

$$\sigma_{y1}dx_1 = dN_1\cos(\theta_1 - \psi_1) + dT_1\sin(\theta_1 - \psi_1), \tag{1}$$

where $\psi_1 -$ is the angle between the normal force and the polar radius.

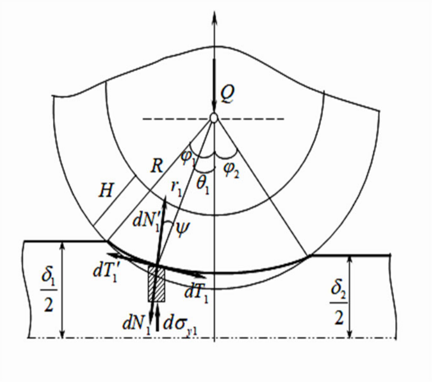

**Fig. 1.** Scheme of force interaction in a roller pair.

By Newton's law, this section of the roll is subject to elementary forces $dN_1'$ and $dT_1'$, equal in magnitude and oppositely directed to forces $dN_1$ and $dT_1$ (Figure 1).

From the condition of equilibrium of the roll in the compression zone, we have [7]:

$$Q_1 - T_{1y}' - N_{1y}' = 0$$

or

$$dQ_1 = dT_{1y}' + dN_{1y}'.$$

Considering the scheme of forces (Fig. 1), we obtain:

$$dQ_1 = dT_1'\sin(\theta_1 - \psi_1) + dN_1'\cos(\theta_1 - \psi_1),$$

or considering the equality of elementary forces:

$$dQ_1 = dN_1\cos(\theta_1 - \psi_1) + dT_1\sin(\theta_1 - \psi). \tag{2}$$

From equality (1) and (2), it follows that:

$$dQ_1 = \sigma_{y1}dx_1. \tag{3}$$

From Figure 1, it follows

$$x_1 = r_1\sin\theta_1.$$

Hence, we obtain:

$$dx_1 = (r_1'\sin\theta_1 + r_1\cos\theta_1)d\theta_1. \tag{4}$$

According to [21], the roll contact curves of the considered roller pair are described by the following equation:

$$r_1 = \frac{R}{1+k_1}\left(\cos\theta_1 + k_1\frac{\cos\varphi_1}{\cos\theta_1}\right), \quad -\varphi_1 \le \theta_1 \le 0, \tag{5}$$

$$r_2 = \frac{R}{1+k_2}\left(1 + k_2\frac{\cos\varphi_2}{\cos\theta_2}\right), \quad 0 \le \theta_2 \le \varphi_2, \tag{6}$$

where

$$k_1 = \frac{2H}{\delta_1}m_1, \quad k_2 = \frac{2H}{\delta_2}m_2, \tag{7}$$

here $m_1, m_2$ is the ratio of the strain rates of the strip of material and the roller coating under compression and recovery, $\varphi_1, \varphi_2$ – are the contact angles, $\delta_1, \delta_2$ – are the thicknesses of the strip of material, $H$ – is the thickness of the roll coating, $R$ – is the roll radius (Figure 1)

From equation (5), we obtain:

$$r_1' = \frac{R}{1+k_1}k_1\cos\varphi_1\frac{\sin\theta_1}{\cos^2\theta_1}. \tag{8}$$

With equalities (5) and (8), from equality (4), we find

$$dx_1 = \frac{R}{1+k_1}\left(\cos\theta_1 + k_1\cos\varphi_1\frac{1}{\cos^2\theta_1}\right)d\theta_1 . \tag{9}$$

By analogy with (9), for the recovery zone, we have

$$dx_2 = \frac{R}{1+k_2}\left(\cos\theta_2 + k_2\cos\varphi_2\frac{1}{\cos^2\theta_2}\right)d\theta_2 . \tag{10}$$

The reaction force $\sigma_{y1}$ is determined primarily by the strain properties of the material being processed, characterized by the stress-strain relationships.

An analysis of the strain properties of materials processed in roller machines has shown that descriptions of the compressive strain of a material can be taken as a model consisting of two parts, defining the elastic and viscous properties [1]:

$$\sigma_{y1} = A\varepsilon_1^{a_1} + \mu\varepsilon_1\frac{d\varepsilon_1}{dt}, \qquad -\varphi_1 \le \theta_1 \le 0, \tag{11}$$

where $A, a-$ are the compression strain coefficients of the material and $\mu-$ is the viscosity coefficient.

When the material is recovered, there is no viscous component, and the strain of the material is determined only by the elastic component, described by the power law [1]:

$$\sigma_{y2} = B\varepsilon_2^{b_2}, \qquad 0 \le \theta_2 \le \varphi_2, \tag{12}$$

where $B, b-$ are the strain coefficients of material recovery.

## 3 Results and discussion

From Figure 1, it follows that at the points of the compression zone, the thickness of the material changes as

$$h_1 = r_1\cos\theta_1 - R\cos\varphi_1$$

or considering expression (5):

$$h_1 = \frac{R}{1+k_1}(\cos\theta_1 - \cos\varphi_1)\,. \tag{13}$$

Then, we have:

$$\varepsilon_1 = \frac{2R}{\delta_1(1+k_1)}(\cos\theta_1 - \cos\varphi_1), \quad \frac{d\varepsilon_1}{dt_1} = \omega\frac{2R}{\delta_1(1+k_1)}\sin\theta_1, \tag{14}$$

here, the minus sign after differentiation is absent due to the different direction of the velocity and angle [1].

From equality (11) considering equalities (14), we obtain:

$$\sigma_{y1} = A\left(\frac{2R(\cos\theta_1 - \cos\varphi_1}{\delta_1(1+k_1)}\right)^a + \mu\omega\left(\frac{2R}{\delta_1(1+k_1)}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1$$

Let us transform this equality

$$\sigma_{y1} = A\left(1 - \frac{\delta_1(1+k_1) - 2R(\cos\theta_1 - \cos\varphi_1}{\delta_1(1+k_1)}\right)^a + \mu\omega\left(\frac{2R}{\delta_1(1+k_1)}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1$$

or in the first approximation

$$\sigma_{y1} = A\left(1 - a\frac{\delta_1(1+k_1) - 2R(\cos\theta_1 - \cos\varphi_1}{\delta_1(1+k_1)}\right) + \mu\omega\left(\frac{2R}{\delta_1(1+k_1)}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1\,. \tag{15}$$

By analogy with (9), for the recovery zone we have

$$\sigma_{y2} = B\left(1 - b\frac{\delta_1(1+k_2) - 2R(\cos\theta_2 - \cos\varphi_{21}}{\delta_2(1+k_2)}\right). \tag{16}$$

To match the strain curves in the compression and recovery zones, expressions (15) and (16) obtained, must be fulfilled under the following boundary conditions $\varphi_1 = \varphi_2 = 0$, $\sigma_{y1} = \sigma_{y2}$.

Hence, with expressions (15) and (16), we obtain:

$$A\left(1 - a\frac{\delta_1(1+k_1) - 2R(1 - \cos\varphi_1}{\delta_1(1+k_1)}\right) = B\left(1 - b\frac{\delta_1(1+k_2) - 2R(\cos\theta_2 - \cos\varphi_{21}}{\delta_2(1+k_2)}\right),$$

which allows us to determine the values of coefficient $b$:

$$b = \frac{A(\delta_1(1+k_1) + a(R\varphi_1^2 - \delta_1(1+k_1))\delta_2(1+k_2)}{B\delta_1(1+k_1)} - \frac{\delta_2(1+k_2)}{R\varphi_2^2 - \delta_2(1+k_2)}. \tag{17}$$

Let us substitute expressions $\sigma_{y1}$ and $dx_1$ from equalities (15) and (9) into equalities (3):

$$dQ_1 = \left(A\left(1 - a\frac{\delta_1(1+k_1) - 2R(\cos\theta_1 - \cos\varphi_1}{\delta_1(1+k_1)}\right) + \mu\omega\left(\frac{2R}{\delta_1(1+k_1)}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1\right)\times$$

$$\times\frac{R}{1+k_1}\left(\cos\theta_1 + k_1\cos\varphi_1\frac{1}{\cos^2\theta_1}\right)d\theta_1\,. \tag{18}$$

After integration of (18) on segment $[-\varphi_1;0]$ and transforms, we obtain

$$Q_1 = \frac{R}{1+k_1}\left(A\cdot\left((1-a)(1+k_1) + \frac{2Ra}{\delta_1(1+k_1)}\sin^2\frac{\varphi_1}{2}\right)\sin\varphi_1 + \right.$$

$$+\frac{4\mu\omega k_1}{\delta_1^2}\left(\frac{R}{1+k_1}\right)^2\left(1-4\frac{\sin^2\varphi_1}{2}\right)\Bigg)\Bigg)$$

or considering expression (7)

$$Q_1=\frac{R}{\delta_1(\delta_1+2Hm_1)^2}\left(A\cdot\left((1-a)(\delta_1+2Hm_1)^2+2R\delta_1 a\sin^2\frac{\varphi_1}{2}\right)\sin\varphi_1+\right.$$

$$\left.+8RH\mu\omega m_1\left(1-4\frac{\sin^2\varphi_1}{2}\right)\right). \tag{19}$$

By analogy with (19) and considering equalities (10) and (16), for the recovery zone we have

$$Q_2=\frac{R}{\delta_2(\delta_2+2Hm_2)^2}\left(B\cdot\left((1-b)(2_1+2Hm_2)^2+2R\delta_2 a\sin^2\frac{\varphi_2}{2}\right)\sin\varphi_2\right. . \tag{20}$$

Then from equalities (19) and (20), we obtain:

$$Q=\frac{R}{\delta_1(\delta_1+2Hm_1)^2}\left(A\cdot\left((1-a)(\delta_1+2Hm_1)^2+2R\delta_1 a\sin^2\frac{\varphi_1}{2}\right)\sin\varphi_1+\right.$$

$$\left.+8RH\mu\omega m_1\left(1-4\frac{\sin^2\varphi_1}{2}\right)\right)+$$

$$+\frac{R}{\delta_2(\delta_2+2Hm_2)^2}\left(B\cdot\left((1-b)(2_1+2Hm_2)^2+2R\delta_2 a\sin^2\frac{\varphi_2}{2}\right)\sin\varphi_2\right. . \tag{21}$$

## 4 Conclusion

The technological efficiency of a roller pair is determined by the pressure on the contact surface since it determines a number of indices of the technological process. In addition, the mechanical characteristics of the roller pair, primarily the quality of the finished product, depend on it.

When designing roller machines, the strain properties of the processed material and the roller coating are not taken into account to the full extent; their variation during the technological process leads to an artificial increase in the characteristics of the roller pair - mass, stiffness, and load.

A formula was derived that determined the value of the strain coefficient of the processed material under recovery, providing conditions for conjugating strains in the compression and recovery zones.

A calculation formula was obtained for the specific pressure in a roller pair, where the material being processed is elastically viscous and the rollers are driven.

Analysis of the calculation formula showed that the specific pressure in the roller pair depends on the geometric parameters of the material being processed and the roller coating, the viscosity coefficient of the material being processed, and the strain rates of contacting bodies.

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