Contact friction in roller mechanisms

Shavkat Khurramov¹, Shukhrat Hurramov², and Akmal Sultonov³

¹Tashkent University of Architecture and Civil Engineering, Tashkent, Uzbekistan
²University of Tashkent for Applied Sciences, Tashkent, Uzbekistan
³Jizzakh Polytechnic Institute, Jizzakh, Uzbekistan

Abstract. An analytical dependence was obtained that describes the patterns of friction stress distribution in the contact zone of roller mechanisms. The patterns of distribution of friction stresses from external forces acting on the roll supports, geometric and kinematic parameters of the roller mechanism, and strain properties of the processed material and elastic coatings of the rollers were determined. Mathematical models of friction stress of roller mechanisms were developed, considering the kinematics of the contact zone, the kinematic connection between the working rolls, and the compliance of the material being processed. It was established that the ratio of friction stress to normal stress at the points of the contact zone of the roller mechanism is not constant, but changes at each point of the contact zone.

1 Introduction

Contact friction in roller mechanisms is extremely important. Frictional forces arising in the contact zone between the processed material and the rollers have a great influence on the technological process. The friction force determines the allowable angle of nip of the strip of material being processed and the value of the advance slip of the strip [1-2]. In addition, it has a great influence on the energy-power and kinematic parameters of the process, at a decrease in the thickness of the material being processed, this influence increases [3-6].

A quantitative measure of friction force in roller mechanisms is function \( f(x) \) connecting contact friction \( t(x) \) and normal stress \( n(x) \) and constant \( f \) connecting friction force \( T \) and normal force \( N \).

Function \( f(x) \) is called the “Friction Stress Model”, and the proportionality coefficient \( f \) is the sliding friction coefficient (or friction coefficient) [7].

Many mathematical models describe the friction stress model and the friction coefficient; the main ones were proposed and used in rolling [8-14]. The following theories are used in rolling: dry friction (Amonton’s law), constant contact friction forces (E. Siebel’s law), liquid friction (A. Nadai’s law) and the ones allowing sticking (A. Tselikov’s law). Each of these theories is confirmed by practice under certain conditions.

Other models of friction stress are also known [1, 11, 15-20], including models with a variable friction coefficient [21-23]. However, these models are not widely used.

* Corresponding author: shavkat-xurramov59@mail.ru
From the above and the results of the study conducted by Ya.D. Vasilyev [24], it follows that the friction stress models used in the theory of rolling are very rough, have limited areas of application, do not predict the relationships between contact stresses, and do not provide high accuracy and reliability in predicting rolling parameters.

In the roller mechanisms of technological machines, the main and widely used model is the Amonton-Coulomb friction stress model [25-28]:

\[ f = f n , \]  

(1)

where \( f \) is the coefficient of friction, \( n, t \) – are the friction stress and normal contact stress.

Condition (1) is theoretically unsubstantiated and not confirmed experimentally, and naturally, it is neither a model of friction stresses, nor a law of friction [24]. This is simply one of many rough assumptions made in the theory of rolling [24] and in studies of contact interaction in roller mechanisms [7].

In the roller mechanisms of technological machines, the rollers have elastic coatings. The deformation and geometric properties of coatings also play a role in the distribution of contact friction [29-33].

The force interaction in roller mechanisms depends on the kinematic connection between the links since they determine the patterns of external forces acting on the roller. Therefore, the assessment of contact friction in roller mechanisms must be conducted considering the kinematic connection between the working rollers [7, 34].

From the above, it follows that a correct model for predicting contact friction stresses in roller mechanisms (including rolling processes) has not been created yet. The absence of such a model does not allow us to obtain patterns of contact friction distribution corresponding to experimental diagrams; it also reduces the accuracy and limits the range of performance of the obtained mathematical models of the distribution of contact stresses along the contact curves of the roller mechanism [7, 24].

Therefore, contact friction in roller mechanisms is critical and is an area that still requires more research.

2 Materials and methods

The problem aimed at developing a model of friction stress in roller mechanisms, considering the kinematics of the contact zone, the kinematic connection between the working rollers, the compliance of the processed material and the roll coating, which will improve the accuracy of determining contact friction.

The problem posed is solved for a roller mechanism consisting of a processed material of thickness \( h_1 \), two drive rolls with radii \( R \), and elastic coatings of thickness \( H \) (Figure 1).

We divide the contact zone into three sections - compression with lag slip 1, compression with advance slip 2, and recovery with advance slip 3.

The points of contact zone in polar coordinates are determined by the following equations:

\[ r = r(\theta_i) , \ i = 1,3 , \ -\varphi_1 \leq \theta_i \leq -\gamma , \ -\gamma \leq \theta_2 \leq 0 , \ 0 \leq \theta_2 \leq \varphi_2 , \]  

(2)

where \( i \) – is the index indicating the section number, \( \varphi_1, \varphi_2 \) – are the contact angles, \( \gamma \) – is the neutral angle.
Fig. 1. Scheme of interaction in the roll mechanism.

To solve this problem, the following models were used:

- friction stress models proposed in [24] for the lag zone:

\[
t_{0x} = -f n_{0x} \frac{h_0}{h_0 - h_y} \left( \frac{h_y}{h_x} - 1 \right),
\]

(3)

and for the advance zone:

\[
t_{1x} = -f n_{1x} \frac{h_1}{h_y - h_1} \left( \frac{h_y}{h_x} - 1 \right),
\]

(4)

where \( t_{0x}, n_{0x}, t_{1x}, n_{1x} \) – are the values of friction stress and normal contact stress in the considered section of the contact zone, respectively, in the lag and advance zones; \( h_x, h_y \) – are the values of strip thickness in the considered and neutral sections of the contact zone; \( h_0, h_1 \) – are the values of the strip thickness in the sections of the entrance and exit from the contact zone, respectively; \( f \) – is the friction coefficient.

- previously obtained models of roll contact curves [25] for the compression zone:

\[
r(\theta) = \frac{R}{1 + k_1} \left( \frac{1}{1} + k_1 \frac{\cos \varphi_1}{\cos \theta} \right), \quad -\varphi_1 \leq \theta \leq 0,
\]

(5)

for the advance zone:

\[
r(\theta) = \frac{R}{1 + k_2} \left( \frac{1}{1} + k_2 \frac{\cos \varphi_2}{\cos \theta} \right), \quad 0 \leq \theta \leq \varphi_2,
\]

(6)

where

\[
k_1 = \frac{2H}{h_0} m_1, \quad k_2 = \frac{2H}{h_1} m_2,
\]

(7)
$m_1, m_2$ are the indicators that determine the ratio of the deformation rates of the strip materials and the roller coating under compression and recovery, respectively.

3 Results and discussion

We rewrite equalities (2) in polar coordinates

$$t(\theta) = -fn(\theta) \frac{h(-\varphi_1)}{h(-\varphi_1) - h(-\gamma)} \left( \frac{h(-\gamma)}{h(\theta)} - 1 \right).$$

Hence, for section 1, we have:

$$t(\theta) = -fn(\theta) \frac{\delta_1}{\delta_1 - h(-\gamma)} \left( \frac{h(-\gamma) - h(\theta)}{h(\theta)} \right), \quad -\varphi_1 \leq \theta \leq -\gamma.$$  (8)

From Figure 1, it follows that

$$h(\theta) = h_0 - 2(r(\theta) \cos \theta - R \cos \varphi_1), \quad -\varphi_1 \leq \theta \leq -\gamma.$$

Then, taking into account equality (4), we obtain

$$h(\theta) = \frac{1}{1+k_1} (h_0(1+k_1) - 2R(\cos \theta - \cos \varphi_1))$$

or in the first approximation

$$h(\theta) = \frac{1}{1+k_1} (h_0(1+k_1) - R(\phi_1^2 - \theta^2)).$$  (9)

From here we find

$$h(-\gamma) = \frac{1}{1+k_1} (h_0(1+k_1) - R(\phi_1^2 - \gamma^2)).$$  (10)

Substituting equalities (8) and (9) into equalities (7) and after transform, we find

$$t(\theta) = fn(\theta) \frac{h_0(1+k_1)(\theta^2 - \gamma^2)}{(\phi_1^2 - \gamma^2)(\delta_0(1+k_1) - R(\phi_1^2 - \theta^2))}, \quad -\varphi_1 \leq \theta \leq -\gamma.$$  (11)

Formula (10) describes the law of friction stress distribution in section 1 of the contact zone.

In section 2, the phenomena of compression with advance slip of the strip occur. Therefore, first, from dependence (7), we have

$$t(\theta) = -fn(\theta) \frac{h(0)}{h(-\gamma) - h(0)} \left( \frac{h(-\gamma) - h(\theta)}{h(\theta)} \right), \quad -\gamma \leq \theta \leq 0,$$  (12)

where $f_1 = -f(0)$ ($0 \leq f_1 \leq f$).

It was previously revealed [7] that at the point separating the compression and recovery zones of the contact zone of the roller mechanism, the value of the friction coefficient can be determined as

$$f_1 = \frac{F}{Q},$$

where $F$ is the horizontal reaction of the roll supports, $Q$ is the pressure force of the roll gripping device.
From equality (8), we obtain:
\[ h(\theta) = \frac{1}{1 + k_1} (h_0 (1 + k_1) - R(\varphi_1^2 - \theta^2)), \quad -\gamma \leq \theta \leq 0, \]  
\[ h(0) = \frac{1}{1 + k_1} (h_0 (1 + k_1) - R\varphi_1^2). \]  
(13)

Substituting equalities (9), (12) and (13) into equalities (11) and after transform, we find
\[ t(\theta) = -n(\theta) \frac{F}{Q} \frac{h_1 (1 + k_1) - R(\varphi_1^2 - \theta^2)}{(\delta_0 (1 + k_1) - R(\varphi_1^2 - \theta^2))}, \quad -\gamma \leq \theta \leq 0. \]  
(15)

Formula (14) describes the law of friction stress distribution in section 2 of the contact curve zone.

In section 3, the phenomenon of compression with advance slip of the strip occurs. Therefore, first, from dependence (6), we obtain:
\[ h(\theta) = \frac{1}{1 + k_2} (h_1 (1 + k_2) - R(\varphi_2^2 - \theta^2)), \quad 0 \leq \theta \leq \varphi_2, \]  
\[ h(0) = \frac{1}{1 + k_2} (h_1 (1 + k_2) - R\varphi_2^2). \]  
(17)

Substituting equalities (16) and (17) into equalities (15) and after transform, we find
\[ t(\theta) = -n(\theta) \left( \frac{F}{Q} + \left( \frac{f - F}{Q} \right) \frac{h_1 (1 + k_2) \theta^2}{\varphi_2^2 (h_1 (1 + k_2) - R(\varphi_2^2 - \theta^2))} \right), \quad 0 \leq \theta \leq \varphi_2. \]  
(19)

Formula (18) describes the law of friction stress distribution in section 3 of the contact zone.

Generalizing formulas (10), (15), and (18), we obtain:
\[ t(\theta) = n(\theta) \left\{ \frac{f}{Q} \frac{h_0 (1 + k_1) (\varphi_1^2 - \gamma^2)}{(\varphi_1^2 - \gamma^2) (h_0 (1 + k_1) - R(\varphi_1^2 - \theta^2))} - F \frac{(h_0 (1 + k_1) - R\varphi_1^2) (\gamma^2 - \theta^2)}{\gamma^2 (h_0 (1 + k_1) - R(\varphi_1^2 - \theta^2))}, \quad -\varphi_1 \leq \theta \leq -\gamma, \right. \]
\[ - \frac{F}{Q} \left( \frac{f - F}{Q} \right) \frac{h_1 (1 + k_2) \theta^2}{\varphi_2^2 (h_1 (1 + k_2) - R(\varphi_2^2 - \theta^2))}, \quad 0 \leq \theta \leq \varphi_2. \]  
(20)

From equality (19) considering equalities (9) and (20), we find
\[ t(\theta) = n(\theta) \left\{ \frac{f}{Q} \frac{h_0 (1 + k_1) (\varphi_1^2 - \gamma^2)}{(\varphi_1^2 - \gamma^2) (h_0 (1 + k_1) + 2Hm_1 - R(\varphi_1^2 - \theta^2))} - F \frac{(h_0 + 2Hm_1) - R\varphi_1^2) (\gamma^2 - \theta^2)}{\gamma^2 (h_0 + 2Hm_1) - R(\varphi_1^2 - \theta^2))}, \quad -\varphi_1 \leq \theta \leq -\gamma, \right. \]
\[ - \frac{F}{Q} \left( \frac{f - F}{Q} \right) \frac{h_1 (1 + k_2) \theta^2}{\varphi_2^2 (h_1 + 2Hm_2) - R(\varphi_2^2 - \theta^2))}, \quad 0 \leq \theta \leq \varphi_2. \]  
(21)
Thus, an analytical dependence was obtained that describes the patterns of friction stress distribution in the contact zone of the roller mechanisms; where the rollers are drive ones.

Hence, we determine the function of the ratio of friction stress to normal stress at the points of the roll contact zone:

\[ f(\theta) = \begin{cases} 
\frac{(h_0 + 2Hm_1)(\theta_1^2 - \gamma^2)}{(\phi_1^2 - \gamma^2)(h_0 + 2Hm_1) - R(\phi_1^2 - \theta^2)}, & -\varphi_1 \leq \theta \leq -\gamma, \\
-\frac{F}{Q} \frac{R}{\gamma^2} \frac{(h_0 + 2Hm_1) - R(\phi_1^2 - \theta^2)}{(h_0 + 2Hm_1) - R(\phi_1^2 - \theta^2)}, & -\gamma \leq \theta \leq 0, \\
-\frac{F}{Q} \left( f - \frac{F}{Q} \right) \frac{(h_1 + 2Hm_2)\theta_2^2}{\phi_2^2 (h_1 + 2Hm_2) - R(\phi_2^2 - \theta_2^2)}, & 0 \leq \theta \leq \varphi_2.
\end{cases} \] (22)

Function \( f(\theta) \) describes the mathematical model of friction stress. Thus, system (21) determines the model of friction stress in the contact zone of the roller mechanisms; where the rollers are drive ones.

In a free roll, contact friction and horizontal reaction of the roll supports change direction.

Then, from equality (21) follows the model of friction stresses in the contact zone of roller mechanisms, where the rollers are free

\[ f(\theta) = \begin{cases} 
-\frac{f}{(\phi_1^2 - \gamma^2)(h_0 + 2Hm_1) - R(\phi_1^2 - \theta^2)}, & -\varphi_1 \leq \theta \leq -\gamma, \\
-\frac{F}{Q} \frac{R}{\gamma^2} \frac{(h_0 + 2Hm_1) - R(\phi_1^2 - \theta^2)}{(h_0 + 2Hm_1) - R(\phi_1^2 - \theta^2)}, & -\gamma \leq \theta \leq 0, \\
-\frac{F}{Q} \left( f + \frac{F}{Q} \right) \frac{(h_1 + 2Hm_2)\theta_2^2}{\phi_2^2 (h_1 + 2Hm_2) - R(\phi_2^2 - \theta_2^2)}, & 0 \leq \theta \leq \varphi_2.
\end{cases} \] (23)

When the rolls are in static contact with the material being processed \( F = 0 \) (or \( \nu = 0 \)), therefore, from system (12) and (22), we obtain:

\[ f(\theta) = \begin{cases} 
\frac{(h_0 + 2Hm_1)\theta_1^2}{\phi_1^2 (h_0 + 2Hm_1) - R(\phi_1^2 - \theta^2)}, & -\varphi_1 \leq \theta \leq 0, \\
-\frac{(h_1 + 2Hm_2)\theta_2^2}{\phi_2^2 (h_1 + 2Hm_2) - R(\phi_2^2 - \theta_2^2)}, & 0 \leq \theta \leq \varphi_2.
\end{cases} \] (24)

### 4 Conclusion

The friction force in roller mechanisms determines the permissible angle of nip of the strip and the value of the advance slip of the strip and has a great influence on the energy-power and kinematic parameters of the process.

An analytical dependence was obtained that describes the patterns of friction stress distribution in the contact zone of roller mechanisms.

The patterns of distribution of friction stresses from external forces acting on the roll supports, geometric and kinematic parameters of the roller mechanism, and deformation properties of the processed material and elastic coatings of the rolls were determined.

Mathematical models of friction stress of roller mechanisms were developed that consider the kinematics of the contact zone, the kinematic connection between the working rolls, the
compliance of the processed material and the roller coating, which will improve the accuracy of determining contact friction.

It was established that the ratio of friction stress to normal stress at the points of the contact zone of the roller mechanism of technological machines is not constant, but changes at each point (angle) of the contact zone.

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