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Modeling contact stresses in roller mechanisms

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Abstract. The results of the interaction of elastic-viscous material with a roller mechanism are given in the article. Mathematical modeling of contact stress distribution in a roller mechanism interacting with the elastic-viscous material being processed, in cases with driven and free rollers, and under static contact, was conducted. It was revealed that the distribution of contact stresses, both under static contact and during movement, is directly related to the properties of the material being processed. The pattern of distribution of shear stresses is related to the type of kinematic link in the roller mechanism, and, accordingly, the nature of the external forces acting on the roller during the implementation of technological processes.

1 Introduction

For calculations related to the technological process and equipment, it is necessary to determine the magnitude and direction of the contact interaction forces in the base element. The roller mechanism (a module) is the basic element of roller machines. The main task when considering contact interaction forces in a roller mechanism is to model contact stresses in the contact zone [1, 2].

The general pattern of contact interaction in roller mechanisms is considered in [3-9]. However, these studies do not provide models of contact stresses during the processing of elastic-viscous material considering the kinematic links between the rollers.

Contact stresses determine several technological indicators of a pair of rolls and the value of the contact arc. In addition, the mechanical characteristics of the working rolls and the energy performance of the process depend on contact stresses [10].

Modeling of contact stresses was conducted in the theory of metal rolling [11-17] and in the theory of contact interaction in roller mechanisms in technological machines [18-26].

In studies related to metal rolling, approximate models of friction stress are used, which do not provide sufficient accuracy in determining force parameters [27, 28].

In research devoted to modeling contact stresses in roller mechanisms, models of strain in the processed material in the form of empirical functions were used, which did not consider the viscous properties of the processed material [29-33]. Such models do not correspond to the real stress-strain state in the roller mechanism of technological machines [34, 35].

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Interaction phenomena, including the stress state in the roller mechanism, depend on the kinematic connection between the links since they determine the patterns of external forces acting on the roller and, accordingly, the distribution of contact stresses. Therefore, models of contact stresses in the roller mechanism need to be developed considering the kinematic connection between the working rolls [36].

Thus, the modelling of contact stresses in the roll mechanism is crucial and is an area that still requires further research.

2 Materials and methods

Consider the problem of interaction of an elastic-viscous material with rollers of radius R, angular velocity ω and length L (Figure 1).

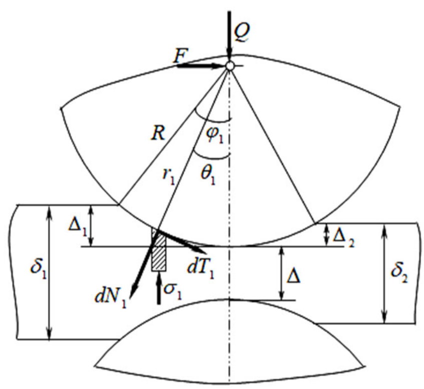

Fig. 1. Scheme of interaction in the roller mechanism

The distribution of contact stresses in the roller mechanism is determined primarily by the strain properties of the material being processed, characterized by compression and recovery curves when interacting with each roller of the mechanism [37].

The compression curve manifests itself in the form of a nonlinear dependence of strain on load, and the shape of the recovery curve depends on the nature of the material: it can be absolutely elastic, elastic-viscous, and plastic. In the first case, when leaving the roll tip, the material will completely restore its shape, in the second case - partially, in the third case - there will be no strain recovery. In a roller mechanism, these material properties determine the contact angles φ_1 and φ_2, which can be obtained from the stress-strain curves

$$\cos\varphi_1 = \frac{2R+\Delta-\delta_1}{2R}, \quad \cos\varphi_2 = \frac{2R+\Delta-\delta_2}{2R}, \tag{1}$$

where $\delta_1 -$ is the thickness of the material when it enters the roll tip, $\delta_2 -$ is the thickness of the material when it exits the roll tip, and $\Delta -$ is the distance (gap) between the rolls.

The compression and recovery curves of the material are determined by analytical equations relating stresses to strains. Such a relationship can manifest itself in a significant influence of elastic, viscous, and plastic properties.

To formalize the material in the form of models with a significant manifestation of certain types of these properties, there are analytical equations for the relationship between stresses and strains (strain rates). In roller mechanisms, the processed material is formalized as a continuous medium with elasticity and viscosity properties. Therefore, based on the analysis of models of the processed material, we accept the following analytical equations given in [37]:

- under compression:

$$\sigma_1 = A_1\varepsilon_1^{a_1} + \mu\varepsilon_1 \frac{d\varepsilon_1}{dt}, \qquad -\varphi_1 \le \theta_1 \le 0; \tag{2}$$

- under recovery:

$$\sigma_2 = A_2\varepsilon_2^{a_2}, \qquad 0 \le \theta_2 \le \varphi_2, \tag{3}$$

where $A_1, a_1,\ A_2, a_2 -$ are the strain coefficients of compression and recovery of the material, $\mu -$ is the viscosity coefficient.

From Figure 2, it follows that at the points of the compression zone the thickness of the material changes as:

$$h_1 = R(\cos\theta_1 - \cos\varphi_1). \tag{4}$$

From equality (4), we obtain :

$$\varepsilon_1 = \frac{2R}{\delta_1}(\cos\theta_1 - \cos\varphi_1), \quad \frac{d\varepsilon_1}{dt_1} = \omega\frac{2R}{\delta_1}\sin\theta_1, \tag{5}$$

here, there is no minus sign after differentiation due to different directions of speed and angle [37].

From equality (2) considering equalities (5), we obtain:

$$\sigma_1 = A_1\left(\frac{2R}{\delta_1}\right)^{a_1}(\cos\theta_1 - \cos\varphi_1)^{a_1} + \mu\omega\left(\frac{2R}{\delta_1}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1. \tag{6}$$

Similarly, from equality (3), we obtain :

$$\sigma_2 = A_2\left(\frac{2R}{\delta_2}\right)^{a_2}(\cos\theta_2 - \cos\varphi_2)^{a_2}. \tag{7}$$

3 Results and discussion

In the compression area of the strain zone, an element of material of a length dx_1 is selected (Figure 1). The selected element of the material is affected by elementary normal dN_1 and shear dT_1 forces from the side of the roll, which are balanced by the reaction force of the material $\sigma_1 dx_1$ [10]:

$$\sigma_1 dx_1 = dN_1\cos\theta_1 + dT_1\sin\theta_1. \tag{8}$$

The moduli of elementary normal and shear forces are expressed as:

$$dN = ndl, \qquad dT = tdl, \tag{9}$$

where $n, t -$ are the normal and shear stresses distributed along the contact arc, $dl -$ is the elementary contact arc.

With (9) from equality (8), we obtain:

$$\sigma_{1y} dx_1 = (n_1\cos\theta_1 + t_1\sin\theta_1)dl_1. \tag{10}$$

Let us equate the length of the elementary arc to the length of the chord

$$dl_1 \approx \frac{dx_1}{\cos\theta_1}.$$

Substituting these expressions into equalities (10), we have

$$\sigma_1 = \frac{(n_1\cos\theta_1 + t_1\sin\theta_1)}{\cos\theta_1}. \tag{11}$$

The pattern of distribution of shear stresses is related to the pattern of distribution of normal stresses. The stress relationship is determined by the friction stress model. We use the friction stress model developed in [10], which for the considered roller mechanism with drive rollers has the following form:

$$t_1 = tg(\theta_1 + \nu)n_1, \tag{12}$$

where

$$tg\,\nu = \frac{F}{Q}, \tag{13}$$

here $F, Q-$ are the forces acting on the roll supports.

Substituting t_1 into equalities (11) and after transforms, assuming that $\cos^2\theta_1 \approx 1$, we obtain:

$$n_1 = (1 - tg\,\nu\, tg\,\theta_1)\sigma_1$$

or with expression (13)

$$n_1 = \frac{Q - F tg\,\theta_1}{Q}\sigma_1.$$

Then, considering equality (6), we obtain:

$$n_1 = \frac{Q - F tg\,\theta_1}{Q}\left(A_1\left(\frac{2R}{\delta_1}\right)^{a_1}(\cos\theta_1 - \cos\varphi_1)^{a_1} + \mu\omega\left(\frac{2R}{\delta_1}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1\right). \tag{14}$$

Similarly, from equality (7), we have

$$n_2 = \frac{Q - F tg\,\theta_2}{Q}\left(A_2\left(\frac{2R}{\delta_2}\right)^{a_2}(\cos\theta_2 - \cos\varphi_2)^{a_2}\right). \tag{15}$$

Formulas (14) and (15) determine the pattern of distribution of normal stresses in the considered roller mechanism with drive rollers and elastic-viscous material being processed.

From these formulas, with the friction stress model (12), we obtain a pattern of distribution of shear stresses in the considered roller mechanism with drive rollers and elastic-viscous material being processed:

$$t_1 = \frac{F + Q tg\,\theta_1}{Q}\left(A_1\left(\frac{2R}{\delta_1}\right)^{a_1}(\cos\theta_1 - \cos\varphi_1)^{a_1} + \mu\omega\left(\frac{2R}{\delta_1}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1\right), \tag{16}$$

$$t_2 = \frac{F + Q tg\,\theta_2}{Q}\left(A_2\left(\frac{2R}{\delta_2}\right)^{a_2}(\cos\theta_2 - \cos\varphi_2)^{a_2}\right). \tag{17}$$

In a free roller, the reaction force of the roll supports F and the friction forces change signs. Then from formulas (14) - (17) follow the pattern of distribution of contact stresses in a roller mechanism with free rollers:

$$n_1 = \frac{Q + F tg\,\theta_1}{Q}\left(A_1\left(\frac{2R}{\delta_1}\right)^{a_1}(\cos\theta_1 - \cos\varphi_1)^{a_1} + \mu\omega\left(\frac{2R}{\delta_1}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1\right),$$

$$n_2 = \frac{Q + F tg\,\theta_2}{Q}\left(A_2\left(\frac{2R}{\delta_2}\right)^{a_2}(\cos\theta_2 - \cos\varphi_2)^{a_2}\right),$$

$$t_1 = \frac{F - Q tg\,\theta_1}{Q}\left(A_1\left(\frac{2R}{\delta_1}\right)^{a_1}(\cos\theta_1 - \cos\varphi_1)^{a_1} + \mu\omega\left(\frac{2R}{\delta_1}\right)^2(\cos\theta_1 - \cos\varphi_1)\sin\theta_1\right),$$

$$t_2 = \frac{F - Qtg\theta_2}{Q}\left(A_2 \left(\frac{2R}{\delta_2} \right)^{a_2} (\cos\theta_2 - \cos\varphi_2)^{a_2} \right).$$

Under static contact $F = 0$. Then from equality (14) - (17), we have

$$n_1 = \left(A_1 \left(\frac{2R}{\delta_1} \right)^{a_1} (\cos\theta_1 - \cos\varphi_1)^{a_1} + \mu\omega \left(\frac{2R}{\delta_1} \right)^2 (\cos\theta_1 - \cos\varphi_1)\sin\theta_1 \right),$$

$$n_2 = \left(A_2 \left(\frac{2R}{\delta_2} \right)^{a_2} (\cos\theta_2 - \cos\varphi_2)^{a_2} \right),$$

$$t_1 = \left(A_1 \left(\frac{2R}{\delta_1} \right)^{a_1} (\cos\theta_1 - \cos\varphi_1)^{a_1} + \mu\omega \left(\frac{2R}{\delta_1} \right)^2 (\cos\theta_1 - \cos\varphi_1)\sin\theta_1 \right) tg\theta_1,$$

$$t_2 = \left(A_2 \left(\frac{2R}{\delta_2} \right)^{a_2} (\cos\theta_2 - \cos\varphi_2)^{a_2} \right) tg\theta_2.$$

4 Conclusion

Mathematical models of contact stress distribution in a roller mechanism interacting with the elastic-viscous material being processed were developed for the cases of driven and free rollers, and under static contact.

Based on the calculation of the obtained mathematical models, the following was revealed:

- the distribution of contact stresses under static contact and during movement, is directly related to the properties of the material being processed;
- with an elastic material being processed, the normal stress in the roller mechanism is distributed according to an elliptical law, and for an elastic-viscous material, the pattern of distribution of normal stresses differs from the elliptical law;
- the pattern of distribution of shear stresses is related to the type of kinematic link in the roller mechanism and, accordingly, the nature of the external forces acting on the roller during the implementation of technological processes; it also depends on the geometric parameters of the rollers and the material being processed.

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