Study of natural oscillations to ensure seismic resistance of underground structures

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Abstract. The scientific research illustrates that the eigenfrequencies of vibrations of an elastic spherical layer located in an infinite elastic medium are investigated. The problem of natural and forced oscillations is solved by reduction to a plane problem of elasticity theory. The spectrum of complex natural frequencies for spherical mechanical systems in an elastic medium is obtained in this work depending on various geometric and mechanical parameters of the system.

1 Introduction

The damping of elastic waves in a solid is caused by spatial differences in elastic properties. Differences in elastic properties and density are caused by various reasons, which are of interest in the study of the properties of a solid body, as well as in the field of seismic vibrations of inclusions (vibration technique). These differences can be associated with defects or groups of defects: such as elastic phase inclusions, composition inhomogeneities, magnetic domain walls, and up to atomic scale defects. In general, any inhomogeneity will be a center of attenuation (or scattering), and with an increase in the frequency of elastic waves, it becomes possible to detect defects of smaller and smaller sizes and in smaller numbers [1].

Vibration technology brings with it to geophysics qualitatively new research methods associated with the possibility of controlling the frequency and amplitude of the seismic signal sent. Therefore, having a controlled probing signal in our hands, we can come close to studying the resonance properties of various inhomogeneities in the depths Earth, and thereby obtain information about such geophysical parameters characterizing this heterogeneity as seismic wave velocities, density, transverse dimensions [2].

2 Problem statement

Let's consider natural vibrations of piecewise homogeneous spherical bodies (Figure 1) or a spherical cavity located in an infinitely elastic medium. Equation of motion for displacement $\vec{U}$ in an isotropic elastic medium has the form [1]:

$$(\lambda + \mu)\nabla\nabla\vec{u} + \mu\nabla^2\vec{u} = \rho\frac{\partial^2\vec{u}}{\partial t^2} \quad (1)$$

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where λ and μ - Lame coefficients, ρ-medium density. For natural vibrations

$$\vec{u} = \vec{u}_0 e^{i\omega t}$$

where $\overrightarrow{u_0}$ - a function of only spatial coordinates, which can be represented as

$$\overrightarrow{S_0} = -\nabla\varphi + \nabla_x[\nabla(\vec{r}\Psi)] \tag{2}$$

The potentials ψ and φ satisfy the equations

$$(\nabla^2 + K^2)\varphi = 0; \;\; (\nabla^2 + X^2)\psi = 0$$

where

$$K = \omega\left(\frac{\rho}{\lambda+2\mu}\right)^{\frac{1}{2}} = \frac{2\pi}{\lambda_e}; \quad x = \omega\left(\frac{\rho}{\mu}\right)^{\frac{1}{2}} = \frac{2\pi}{\lambda_t} \tag{3}$$

λe and λt -respectively, the lengths of longitudinal and transverse waves. The potentials φ and ψ represent the longitudinal and transverse parts of the wave.

3 Solution method

The waves in the medium are the sum of the incident and scattered waves:

$$\overrightarrow{u_0} = \vec{u}_p + \vec{u}_S$$

where indexes p and s - correspond to the incident and scattered waves. Equations (1) and (2) are linear, so for the region outside the sphere

$$\varphi_1 = \varphi_p + \varphi_S; \qquad \psi_1 = \psi_p + \psi_S = \psi_S;$$

$$(\nabla^2 + K_1^2)\varphi_p = 0; \quad (\nabla^2 + K_1^2)\varphi_S = 0; \quad (\nabla^2 + X_1^2)\psi_S = 0;$$

where indexes 1 - refers to the environment [3].

Inside the sphere

$$\varphi_2 = \varphi_q; \quad \psi_2 = \psi_q; \quad (\nabla^2 + K_2^2)\varphi_q = 0; \quad (\nabla^2 + X_2^2)\psi_q = 0;$$

Index 2 - refers to the material of which the sphere is composed, and index q refers to the wave inside the sphere. The solutions of the wave equations must be symmetrical with respect to the axis passing through the spheres and parallel to the direction of propagation. Choosing a coordinate system (r, θ, φ) with origin at the center of the sphere and with angles θ and φ, which respectively determine the angles of rotation from the axis around the symmetry, we find that the potentials φ and ψ will be functions of only r and θ. The general solution has the form [4]

$$\sum_{m=0}^{\infty} C_m Z_m(e\vec{r}) P_m(\cos\theta) \tag{4}$$

where Cm - permanent, Zm- linear combination of spherical Bessel and Neumann functions m - order, l - wave number (k and x), Pm - Legendre polynomial of degree m

$$\varphi_s = \sum_{m=0}^{\infty} (-1)^{m+1} a(2m+1) A_m h_m(K_1 r) P_m(\cos\theta);$$

$$\psi_s = \sum_{m=0}^{\infty} (-1)^{m+1} a(2m+1) B_m h_m(X_1 r) P_m(\cos\theta); \tag{5}$$

$$\varphi_q = \sum_{m=0}^{\infty} (-1)^{m+1} a(2m+1) C_m j_m(K_2 r) P_m(\cos\theta);$$

$$\psi_q = \sum_{m=0}^{\infty} (-1)^{m+1} a(2m+1) D_m j_m(X_2 r) P_m(\cos\theta);$$

where a- sphere radius, Am, Bm, Cm, Dm - constants determined by the boundary conditions, and

$$h_m(\xi) = j_m(\xi) - in(\xi \tag{6}$$

spherical Hankel functions of the Meansh order of the second kind, jm(ξ) and nm(ξ) - spherical Bessel and Neumann functions of means order [5,6].

Below are the eigenfrequencies for some interesting cases. Spherical cavity. In the case of a spherical cavity, there is no wave inside the sphere, therefore, it is necessary to determine the coefficients {An} and {Bn}, it is also required to set two boundary conditions.

Provided that the stress components were continuous at the boundary at r = a, we have

$$\sigma_{rr} = \sigma_{r\theta} = 0 \tag{7}$$

This condition leads to the following equations for ω.

$$\varepsilon_{11}^{(0)}\varepsilon_{12}^{(0)} - \varepsilon_{13}^{(0)}\varepsilon_{14}^{(0)} = 0 \tag{8}$$

where

$$\begin{gathered}
\varepsilon_{11}^{(0)} = [\beta_1^2 - 2n(n-1)]h_n(\alpha_1) - 4\alpha_1 h_{n+1}(\alpha_1) \\
\varepsilon_{12}^{(0)} = \left[\frac{1}{2}\beta_1^2 - (n^2-1)\right]h_n(\psi_1) - \beta_1 h_{n+1}(\beta_1) \\
\varepsilon_{13}^{(0)} = [2n(n+1)(n-1)]h_n(\beta_1) - \beta_1 h_{n+1}(\beta_1) \\
\varepsilon_{14}^{(0)} = [(n-1)]h_n(\alpha_1) - \alpha_1 h_{n+1}(\alpha_1) \\
\alpha_1 = k_1 a = \frac{\omega^2}{c_{p_1}^2}\alpha; \qquad \beta_r = \frac{\omega^2}{c_{s_1}^2}\alpha
\end{gathered} \tag{9}$$

The frequency equation (9) is solved numerically using the approximate Muller method on a computer.

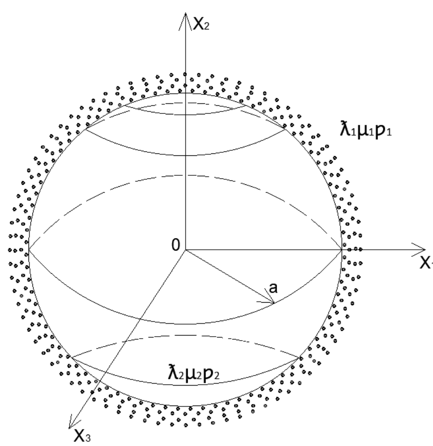

Fig. 1. Calculation scheme for spherical bodies located in an infinite environment.

Torsional vibrations. They are characterized by the vanishing of the radial component of the displacement vector u, as well as div u. Obviously, only the part that includes the coefficients Cmn corresponds to the general solution (4). Substituting this part into the boundary conditions leads to the following system of equations for determining the coefficients cmn(i) and cmn(e):

$$\begin{gathered}
c_{mn}^{(i)} j_n\left(k_s^{(i)}, R\right) = c_{mn}^{(t)} h_n\left(k_s^{(e)}, R\right), \\
\mu_i c_{mn}^{(i)}\left[(n-1)j_n\left(k_s^{(1)}, R\right) - (k_s^{(i)}, R)j_{n+1}(k_s^{(i)}, R)\right] = \\
= \mu_i c_{mn}^{(i)}\left[(n-1)h_n\left(k_s^{(e)}, R\right) - (k_s^{(e)}, R)h_{n+1}(k_s^{(e)}, R)\right]
\end{gathered} \tag{10}$$

Equating the determinants of the system to zero, we obtain an equation for the natural frequencies of torsional vibrations of a spherical inclusion:

$$[n - 1 - g_i(\beta x)] = p[n - 1 - g_e(x)] \tag{11}$$

where

$$g_i(z) = \frac{z j_{n+1}(z)}{j_n(z)}, \quad g_e(z) = \frac{z h_{n+1}(z)}{h_n(z)}, \qquad x = \frac{\omega R}{c_{sc}},$$

reduced to the transverse velocity in the medium dimensionless frequency, β=cse/csi lateral speed ratio outside (cse) and inside (csi) spherical inclusion, η=ρe/ρi - ratio of densities and ρ=μe/μi=β2η the ratio of the shear moduli of the host medium and the inclusion [7].

The solution to equation (11) is the set of complex frequencies x(k)=x0(k)+ix1(k). The natural frequency is the real part x0(k) complex number x(k), damping coefficient - imaginary part x1(k). In an ideal elastic medium, due to divergent spherical elastic waves, damping can be explained by the radiation of the energy of excited natural vibrations. If in (11) we pass to the limit corresponding to the case of the absence of an enclosing medium (an isolated elastic ball at p→0), then we obtain the frequency equation for the torsional vibrations of the ball [8]

$$n - 1 - g_i(x_{si}) = 0 \tag{12}$$

where xsi=ωR/csi. Since there is no radiation, this equation defines a discrete spectrum xsi(k) actual frequencies. In the opposite case, a spherical cavity in a continuous elastic medium, when p→∞, we arrive at a complex frequency equation

$$n - 1 - g_e(x) = 0, \tag{13}$$

describing the spectrum of complex values x(k) natural frequencies of torsional vibrations of the cavity [9,10].

Let's expand (13), for example, in the case of n=1 and n=2. We get for n=1

$$\beta x\ ctg\ \beta x = 1 - \frac{\beta^2 x^2 (1+ix)}{px^2 + 3(1-p)(1+ix)} \tag{14}$$

and at n=2

$$\beta x\ ctg\ \beta x = 1 - \frac{1}{3}\beta^2 x^2$$
$$\left[1 + \frac{\beta^2 x^2 \left[x^{-1}\left(1 - \frac{1}{3}x^2\right)\right]}{-\frac{i}{3}px^4 + [x^2(p-\beta^2) - 12(p-1)]\left[x^{-i}\left(1-\frac{1}{3}x^2\right)\right]}\right] \tag{15}$$

At p→0 the complex equations transform into transcendental equations for the natural frequencies of the ball vibrations, and as p→∞ they transform into complex equations for the natural frequencies and natural dampings of the torsional vibrations of the cavity. It is for n=1

$$x_{si} ctg x_{si} = 1 - \frac{x_{si}^2}{3}, \tag{16}$$
$$x^2 - 3ix - 3 = 0 \tag{17}$$

and at n=2

$$x_{si} ctg x_{si} = 1 + \frac{4x_{si}^2}{x_{si}^2 - 12}, \tag{18}$$
$$-5ix^2 - 12x + 12i = 0, \tag{19}$$

It is interesting to note that equations (17) and (19), as well as the more general transcendental equation, include both trigonometric and algebraic functions. Due to the periodicity property of trigonometric functions, for each number n we will have an infinite set of natural frequencies. An exception is the case of a cavity, when the eigenfrequencies are determined by algebraic equations of finite order, increasing with the number n.

4 Results and discussion

As noted in the case of torsional vibrations, the frequency equation (11) includes trigonometric and algebraic functions. It is the presence of trigonometric and algebraic functions that leads for each oscillation number n to a countable set of natural frequencies. The torsional vibration equation (11) can, in principle, be rewritten in the following general form:

$$\beta\chi ctg\beta\chi - f_{p\beta}(\chi) = 0 \tag{20}$$

where $f_{p\beta}(\chi)$- algebraic function X depending parametrically on p and β. The left side of (16) is a trigonometric function of the argument x that changes rapidly from -∞ to +∞, and the right side is a relatively smooth algebraic function. This fact leads to the presence of an infinite number of natural frequencies, each of which is contained in the interval (nπ/β,(n+1)π/β). In the case of spheroidal vibrations, as follows from the expressions for the elements of the determinant Δ_n, in particular equation (11) will be present $ctga\gamma\chi$ and $ctg\beta\gamma\chi$, each of which changes from -∞ to +∞ when χ changes in the interval or in the interval $(n\pi/\alpha\gamma,(n+1)\pi/\alpha\gamma)$.

This also leads to an infinite number of roots. In this case, the pattern of the distribution of roots over the indicated intervals will be much more complex than in the case of torsional vibrations.

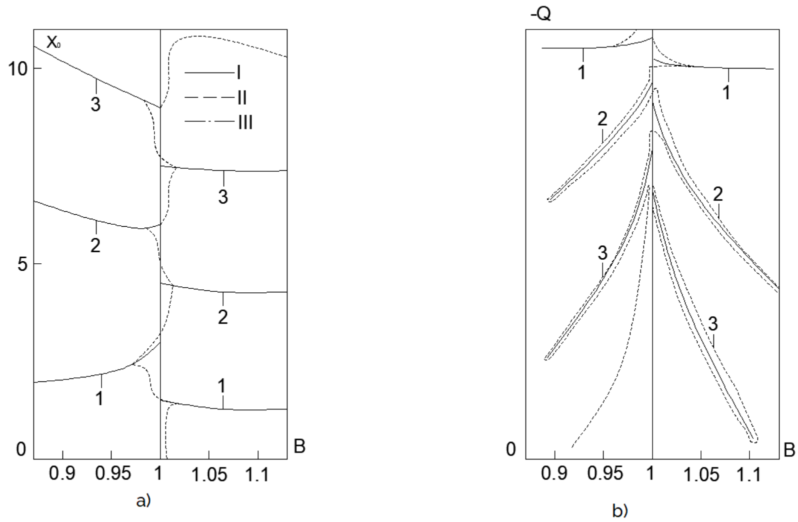

Fig. 2. Dimensionless natural frequencies and torsional vibration damping coefficient (n=2).

The solution of the transcendental equations of the day of torsional, spheroidal and radial oscillations by the analytical method seems difficult and they should be solved numerically. Here we present the results of a numerical solution for a low-contrast inclusion, a sliding contact, when the difference in the transverse velocities of the inhomogeneity and the host medium is no more than 25%, in the longitudinal - no more than 6% and in density no more than 0.5% (for torsional vibrations). The ratio of the transverse and longitudinal velocities for the enclosing medium (parameter γ) will be taken equal to 0.59. Dependence of the dimensionless frequency of natural radial oscillations (χ0=Reχ) on the ratio of transverse waves in the medium and inhomogeneity at η = 1, for various values of the ratio of the velocities of longitudinal waves is shown in Figure 2 on the left. In the same figure, on the right, the corresponding dependence of the attenuation coefficient corresponding to each frequency is shown. In this case, the damping coefficient is defined as the number of complete oscillations corresponding to a decrease in the amplitude in ep times, and is equal in our case Q=χ0/2χ1(χ1=lmχ). The numbers in the figures indicate the frequency number.

5 Conclusion

Thus, new problems of eigenoscillations of piecewise homogeneous spherical systems in an infinitely elastic or acoustic medium have been solved. It is shown that the problem under consideration has a discrete spectrum located in the lower complex plane ImC1<0 and symmetrical with respect to the imaginary C1 - axes. The spectrum of complex natural frequencies for spherical mechanical systems in an elastic medium is obtained depending on various geometric and mechanical parameters of the system. The found frequency spectrum makes it possible to ensure seismic resistance (vibration resistance) of underground structures such as spherical shells.

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