E3S Web of Conferences **548**, 08009 (2024) https://doi.org/10.1051/e3sconf/202454808009
AGRITECH-X 2024

Modeling the concentration profile of the solid phase during hydrotransport of ore in pipes

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Abstract. The authors consider the calculation of movement modes for unstructured suspensions in horizontal pipes. The movement mode in these cases is conventionally defined by the proportion of the concentration at the top of the pipe and in its middle. According to multiple authors, the flow carrying capacity is determined by its average velocity. This provides an opportunity to define the solid phase concentration profile along the vertical separately from solving the movement equations. This article presents an equation to determine the vertical distribution of the concentration obtained as a balance between turbulent pulses and the gravitation settling of solid particles. The analysis of solutions showed that there is a strong dependency between solid phase distribution and its average concentration in the flow. The comparison of movement mode calculation results obtained using the developed mathematical model and popular empirical dependencies showed good correlations. The obtained concentration profile can be used for the simplification of the general mathematical model of hydrotransport by predicting the motion mode and setting the concentration profile. This shall reduce the number of equations solved in the hydraulic fluid movement model and simplify the subsequent modeling of unstructured suspension movement in pipes.

1 Introduction

Currently hydraulic transport of ore concentrates, coal tailings and other bulk materials is becoming more widespread abroad and in our country. For Russia, the issues of economical transportation of minerals are especially relevant, since the main areas of their occurrence and consumption can be separated by significant distances. Pipeline transport is the most promising and environmentally friendly method of transporting ore and other minerals. It contains reserves of power and economic efficiency. With this type of transport, it becomes possible to fully automate the entire process of moving bulk cargo.

Controlling processes and designing optimal equipment for them requires exact knowledge of its operating modes that can be determined through physical or mathematical experiments [1]. Consider the hydrotransport of unstructured suspensions in this respect.

Concerning unstructured suspensions, there are four different hydraulic fluid motion modes (with increasing flow velocities): stationary layer mode, saltation mode,

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heterogeneous mode, and homogeneous mode [2, 3]. In practice, the important ones are the latter two. Prior to the calculations in practical problems, it is necessary to determine the motion mode, either homogeneous (where the solid phase is equally distributed across the flow cross-section) or heterogeneous (featuring a significant vertical distribution asymmetry of the dispersed phase).

The distribution of particles affects the velocity profile. In the heterogeneous mode, the presence of greater amounts of the dispersed phase at the bottom of the channel comparing to its top makes the flow movement in the area difficult. The thickening of the dispersed phase at the bottom of the pipe results in an asymmetric distribution of velocities along the vertical.

The transition between the heterogeneous and homogeneous flow modes is generally determined by the value of the simplex. It is obtained from the Izmail proportion [3] (all of the concentrations used are volumetric $\left[m^3\ solid / m^3\ mixture\right]$):

$$\lg \frac{C_{up}}{C_a} = -1.8 \frac{V_m}{0,4U^*} ,$$

where C_{up} are the solid phase concentrations in a point that is *0.08D* away from the top of the pipe; C_a are the dispersed phase concentration in the middle of the pipe; $U^* = \sqrt{\tau_0 / \rho_{mix}^0}$ is the dynamic hydraulic fluid velocity; τ_0 is the limiting viscous shear (in our case, on the pipe wall); $\rho_{mix}^0 = \rho_l \left(1 - C_0\right) + \rho_s C_0$ is the density of the hydraulic fluid for the average solid phase concentration; ρ_s is the solid phase density; ρ_l is the liquid phase density; C_0 is the average solid phase concentration; V_m is the particle movement velocity in liquid taking into account the tightness (the calculation is discussed below).

With $C_{up} / C_a > 0.8$ the mode is homogeneous, and otherwise, it is heterogeneous [3].

Today, the upward solid phase transfer mechanism is explained using the understanding of turbulence as a flow with the vortex movement of substances. The size of the largest vortices, which are the main force transferring solid particles, is determined by the fluid movement velocity and the cross-size of the flow [4, 5]. Thus, we can claim that the flow's ability to lift solid particles and even out the concentration profile is mainly determined by its average speed.

Therefore, we believe that mathematically, the hydraulic fluid movement mode can be determined without solving the full system of movement equations for the entire flow. It would suffice to review the movement of the solid phase and close them with some factors.

2 Problem statement

Analyze the mass conservation equation for a stationary isothermal flow of unstructured hydraulic fluid over a balanced flow section with a length L and a diameter D with the pressure drop p at the pipe ends.

To describe the movement of a multiphase hydraulic fluid, we used Euler's continual approach. The geometry of the problem considered, the coordinate system used, and some linear sizes are shown in Figure 1a.

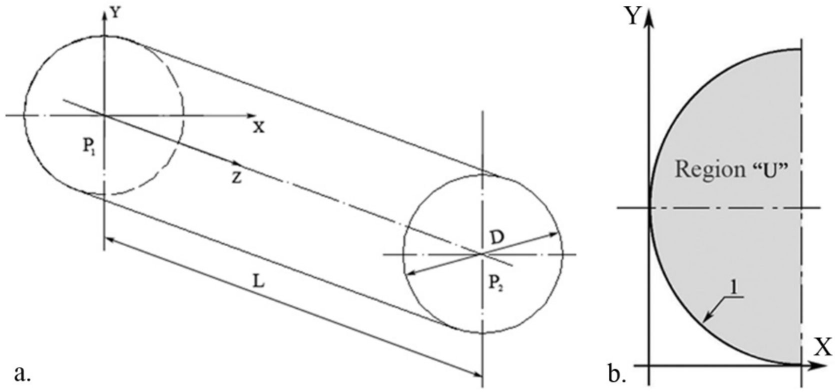

Fig. 1. The geometry of the problem in question with coordinate axes location (a) and modified computational domain (b).

Assume that interphase transitions are absent, and solid phase particles have the same diameter d_s that does not change during transportation. Ignore molecular transfer because it has little impact on the solid particles of the sizes in question. All derivatives of the z-axis, except for the pressure, are equal to zero due to the established flow character. Of mass forces, we consider gravity that is applied along the y-axis.

The pressure gradient in the stabilized section is written down as

$$\frac{\partial P(z)}{\partial z} = -\frac{\Delta P}{L} = -a = const \, .$$

Assume that changing the solid phase concentration along the x-axis is negligible [2, 6]. Particles move along the y-axis at a hindered falling velocity typical of a particle in a medium V_m. With these assumptions, the mass conservation equation can be written down as the dynamic balance between turbulent pulses and the gravitational settling of particles. This equation can be written as follows [6]:

$$\frac{dC_s}{dy} = -\frac{V_m^0 \left(1 - C_s\right)^{m-1}}{\varepsilon_s} C_s \tag{1}$$

where: $C_s\left(y\right)$ is the solid phase concentration; V_m^0 is the free-falling velocity of a particle in a clean fluid; ε_s is the turbulent solid phase transfer factor; the degree *m* accounts for the impact of particle tightness on their settling rate.

$$\frac{V_m}{V_m^0} = \left(1 - C\right)^m$$

It is determined depending on various factors (particle sizes, settling mode, etc.). We obtained it using the Wallis formula [6] obtained for spherical particles

$$m = \frac{4,7\left(1 + 0,15\,\mathrm{Re}_\infty^{0,687}\right)}{1 + 0,253\,\mathrm{Re}_\infty^{0,687}}$$

where $\mathrm{Re}_\infty = V_m^0 d_s / \mu_l$

The V_m^0 value can be determined using the Kravtsov formula [8]:

$$V_m^0 = -\frac{A\mu_l}{2\rho_l\left(K + C_d d_s\right)} + \sqrt{\left[\frac{A\mu_l}{2\rho_l\left(K + C_d d_s\right)}\right]^2 + \frac{4d_s\left(\rho_s - \rho_l\right)g}{3\rho_l\left(K + C_d d_s\right)}} \tag{2}$$

where: g is the free-falling acceleration; A, B_0, C_d are the experimental factors.

Factors A, B_0, C_d were determined in [7] for different materials. Therefore, the values of these constants shall be selected depending on the mixed material. We used the following constant values: $A = 32.8; B_0 = 10.8; C_\partial = 1.07$. They were provided in [7] for the specific shape of natural sediment particles. The main advantage of this formula is that one calculation procedure is used for the entire parameter range in question. Additionally, the formula, although indirectly, accounts for the particle shape factor.

The multiplier $(1 - C_s)^{m-1}$ in equation (1) accounts for the impact of particle tightness on their settling rate.

The problem is symmetrical relative to the vertical axis. Considering this and the lack of directional values of values that change along the z-axis in formula (1), the computational domain can be reduced to a semi-circle and simplified to a quasi-two-dimensional problem. The new computational domain is shown in Figure 1b.

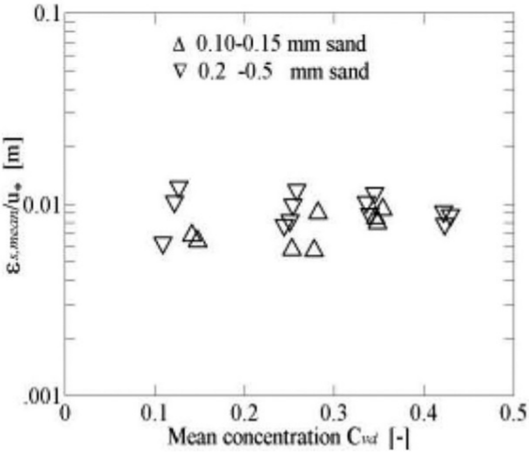

Fig. 2. Experimental turbulent transfer factor value [8].

We cannot remove the x-axis from the review due to its presence in the problem-closing integral. Equation (1) has all of the values identified except for the turbulent transfer factor. Review its calculation: this problem was addressed in multiple papers.

The numerical value of the turbulent transfer factor is quite frequently obtained from the vortex size/particle proportion.

It is normally assumed equal to the kinematic turbulent viscosity factor of the carrying fluid. The same value is used for the turbulent transfer factor of the solid phase [9]. This approach requires solving a general hydraulic fluid movement equation together with the turbulent model. In Matousek's experiments [10] (see Figure 2), the average value of the turbulent solid phase transfer factor determined in experiments for a wide range of pipe flows was equal

$$\varepsilon_s \approx 0,01U^* \tag{3}$$

We used this proportion in our model.

Consider the calculation procedure for the dynamic velocity U^*. Its formula includes the viscous shear on the well edge. We determined it by considering the balance of forces applied to the hydraulic fluid in the computational domain (Figure 1): the pressure forces are equal to the friction forces on the hard wall:

$$\Delta p \pi \frac{D^2}{4} = \tau_0 L \pi D, \ \tau_0 = \frac{\Delta p}{L}\frac{1}{4D} = \frac{a}{4D}, \ u_* = \sqrt{\frac{\tau_0}{\rho^0_{mix}}} = \sqrt{\frac{a}{4D\rho^0_{mix}}},$$

There are no common limiting conditions in the reviewed simplifications for our problem. However, there is an additional constraint applied to the concentration profile. It comes from the mass conservation equation in the integral form and the symmetry of the domain

$$\iint_U C_s dxdy = \frac{\pi D^2 C_0}{8} .$$

The concentration transfer equation (1) and its closing relations only include the parameters that can be clearly defined with the initial problem data. Therefore, it is possible to solve equation (1) separately from the general movement model, thus determining the hydraulic fluid movement mode and solid phase distribution profile before the main movement equations are solved.

3 Solving the model

Mathematically, the problem of establishing the concentration profile requires solving a differential equation obtained by inserting expression (3) into (1)

$$\frac{dC_s}{dy} = -\frac{V_m^0 \left(1 - C_s\right)^{m-1}}{0,01U^*} C_s \tag{4}$$

with an additional condition to determine the singularity of the solution

$$\iint_U C_s dxdy = \int_{-r}^{r} \sqrt{r^2 - y^2} C_s dy = \frac{\pi D^2}{8} C_0 \tag{5}$$

Introduce a dimensionless variable $\overline{y} = y/D$. In this case, equations (4) and (5) look as follows

$$\frac{dC_s}{d\overline{y}} = -Ro\left(1 - C_s\right)^{m-1} C_s \tag{6}$$

$$\int_{-0,5}^{0,5} \sqrt{0,25 - \overline{y}^2} C_s d\overline{y} = \frac{\pi}{8} C_0 \tag{7}$$

where $Ro = V_m^0 D/\left(0,01U_*\right)$ is the similarity criterion completely determined by the input values. This is an alternative to the Rose number used in the sediment flow theory in open channels [4, 6]. It is a proportion between the particle flow due to gravitational settling to the flow caused by turbulence. Thus, the concentration profile only depends on the average concentration C_0 and the dimensional simplex Ro.

It is possible to obtain an analytical solution to equation (6) using the classic variable separation method but only for the integer values of *m*. This also leads to a transcendental equation that can only be solved using numerical methods. However, similar methods, especially if we consider the non-classic limiting condition (7), are more complex than finding the numerical solution to the initial equation (6).

Consider the solving procedure that satisfies condition (7). Depending on the limiting condition applied to the numerical solution of equation (6) $C_s(\overline{y} = -0,5) = C_s^0$, the integral

$$I\left(C_s^0\right) = \int_{-0,5}^{0,5} \sqrt{0.25 - \overline{y}^2} C_s d\overline{y}$$

can take various values (the value at $\overline{y} = -0.5$ was selected for the comfort of computational solving). In this case, we need to find a value of C_s^0 (and the respective concentration profile) that will satisfy the following condition:

$$f\left(C_s^0\right)=I\left(C_s^0\right)-\frac{\pi}{8}C_0=\int_{-1/2}^{1/2}\sqrt{\frac{1}{4}-\overline{y}^2}\,C_s d\overline{y}-\frac{\pi C_0}{8}=0 \tag{8}$$

i.e., we need to determine the root of the function $f\left(C_s^0\right)$.

We solved the problem in the SciLAB as follows.

1. Setting the value of $C_s^0=C_0$.
2. Calculating equation (6) considering the limiting condition $C_s(\overline{y}=-0.5)=C_s^0$.
3. Determining the value of the function (8) using the profile obtained in Step 2.
4. If the value of the function (9) is not equal to zero with a certain tolerance (we used the tolerance value of 0.0001), the value of C_s^0 is adjusted, and the steps above are repeated from the second one.

4 Modeling results and practical recommendations

Examples of solid phase concentration profiles at different values of C_0 and Ro are shown in Figures 3 and 4.

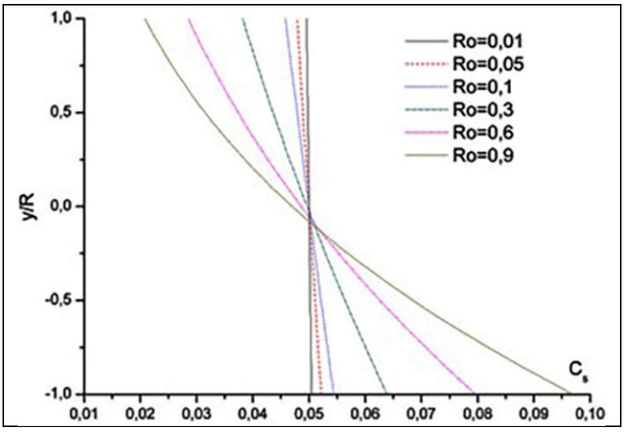

Fig. 3. The concentration profiles for C_0 =0.05 and various values of the Ro number.

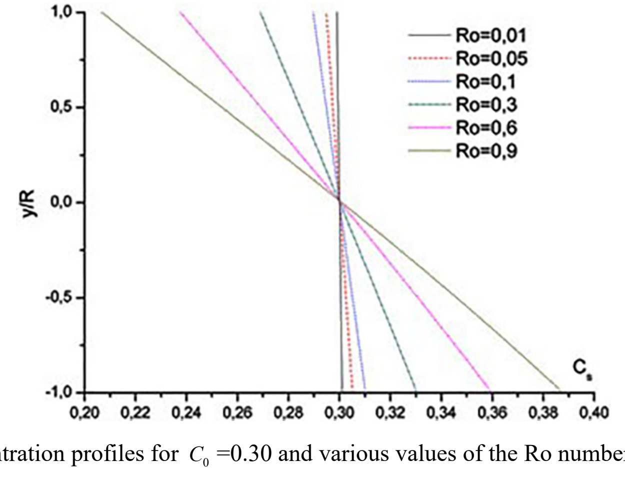

Fig. 4. The concentration profiles for C_0 =0.30 and various values of the Ro number.

As expected, the concentration profile depends not only on the hydrodynamic conditions determined by the number of, but also on the average solid phase concentration. This clear dependency is determined by the significant impact of concentration on the settling rate of particles in a fluid. As we can see from the solutions, the concentration profile tends to a more even state when the number of is reduced (i.e. when the impact on turbulence is increased) and when the average concentration is increased (which leads to the deceleration of the hindered movement of particles due to gravitational forces).

As mentioned above, the transition between the homogeneous and heterogeneous modes of hydraulic fluid motion modes is normally determined using the value of expression. For the obtained calculation profile, we can easily obtain the simplex value. We compared our calculation data with the Izmail correlation [3] and the experiments carried out in [6], [8], and [10]. However, the data from the experiments with the transition of the homogeneous regime into the heterogeneous are scarce. The majority of these data is related to the calculation of the pressure drop during hydraulic fluid movement.

The comparison of our data with the Izmail correlation and available experimental data showed that the modes corresponded in the majority of calculated cases. Under the maximum concentration of $C_0 > 0.35$, no heterogeneous flow mode was observed in the calculation range, which also correlates with the experimental data.

Note that our solutions produced slightly exaggerated values of simplex C_{up}/C_a compared to the available experiments. The discrepancies increase as the pipeline diameter is reduced, which may indicate an increased impact of the horizontal solid phase flow components on the concentration profile, which we ignored. Another factor that may cause the deviation of our solution from small-diameter experiments is our failure to account for the impact of pipe walls on the settling rate. This impact increases as the proportion of the pipe diameter and particle diameter is reduced.

The correlations of the calculated mode forecasts indicate that the initial concentration transfer equation and its solution methods are correct.

5 Conclusion and model development

We demonstrated that the mass conservation equation for the stationary movement of unstructured hydraulic fluids can be solved without finding the carrier phase velocity profile and calculating its turbulent parameters.

The concentration profile is determined by the Ro number (which characterizes hydrodynamics in our case) and the average concentration.

The calculation produces greater deviations with small diameters compared to the experimental data. We attribute this to the failure to account for the impact of pipe walls on the settling rate.

The obtained concentration profile can be used for the simplification of the mathematical model of flow movement by setting the concentration profile in advance.

The further development of the model can include, first of all, the replacement of the multiplier $\left(1-C_s\right)^{m-1}$ that accounts for the impact of the solid phase concentration on the settling rate on $\left(1-C_s/C_s^{\max}\right)^{m-1}$ as suggested in [7]. This shall facilitate a more accurate accounting for the impact of the concentration of the settling rate up to its zero value when the concentration approaches the maximum limit. We can assume that it can help describe the modes that are close to saltation.

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