Application of vibration insulating mats in tram track construction

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Abstract. The article deals with a feasibility study methodology for applying vibration insulating mats in tram track construction. The method for calculating railway track vibrations from the rolling stock with axial loads of 220-250 kN/axle at speeds of 50-90 km/h is presented. The dependences of the vibration amplitude obtained are approximated for tram loads, and the performed measurement errors are calculated. Based on the calculations, the dependence of vibration amplitudes on the distance to the tram track for frequencies of 31.5 and 63 Hz is obtained. In conclusion, a recommendation on the use of vibration isolating mats for ensuring acceptable values of the vibration velocity level is given.

1 Introduction

Vibration insulating mats are used for vibration mitigation in railway transportation [1;2;3]. They reduce transport-borne vibration effects generating resonance in walls and ceilings creating noise pollution that noise shields cannot protect against ("secondary noise"). They can reduce vibration level by 10-18 dBA and noise level by 5-10 dBA [4;5;6]. The sub-ballast mat parameters are determined by calculations and mathematical models taking into account the site specific conditions, the distance to the facilities, the type of rolling stock that will be running, and the predicted levels of vibration impact [7;8;9].

A feasibility study is to be conducted to prove their effectiveness on tram tracks. For this purpose, theoretical research has been carried out to find out how vibration mats can mitigate vibrations of different frequencies[10;11;12;13;14].

2 Method for calculating the induced vibrations

Calculations of transport-induced low-frequency vibrations in the tram track structure have been carried out based on the methodology for calculating the railway formation dynamics and stability [2].

As earlier studies show, the general character of vertical vibration amplitude reduction has a power dependence [15;16]:

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$$A_z = A_0 \delta^z \ \mu\text{m}$$

For practical calculations, it is convenient to use the formula in the following form

$$A_z = A_0 e^{z \ln \delta} \ \mu\text{m} \tag{1}$$

where

A_0 is the highest possible ground vibration amplitude at the sleeper foot level for the corresponding section, μm;

e is the base of the natural logarithm;

z is the vertical distance from the sleeper foot, m;

δ is the coefficient of vibration attenuation characterizing the vibration amplitude reduction per meter thickness of the considered layer.

During the experimental research for the railway transport, I.V. Prokudin and G.M. Stoyanovich have obtained empirical dependencies of the vibration amplitudes occurring in the subgrade formation propagating from the sleeper foot vertically and horizontally [17].

$$A_{zy} = A_0 \cdot e^{(z \cdot lg \cdot \delta_1 - f(y) \cdot \delta_2' - (y - 1.35) \cdot \delta_2'' + \delta_3 \cdot h)} \tag{2}$$

where

y is the distance from the track axis, m

$f(y)$ =7.65 when y > 9.0 m;

$f(y)$ =1.35 when 0.5B < y ≤ 9,0 m;

$f(y)$ = 6.65 when y < 0.5B m;

$h_i(y) =$ 0 when y < 0.5B m;

(y - 0.5B) .tgα when y > 0.5 m

A_{zy} is the induced amplitude at the point with z and y coordinates, μm;

A_0 is the induced amplitude of vibrations within the sleeper length, μm;

δ_1 is the coefficient of vibration attenuation in the vertical plane.

For an earth cut (as a rule, a tram track in an urban environment is built in an earth cut) δ_1, the following values are taken:

for sand, 0,18 - 0,21;

for sand loam, 0,21 - 0,36;

for clay loam, 0,27 - 0,33;

for clay, 0.24 - 0.2.7.

δ_2', δ_2'' are the coefficients of vibration attenuation in the horizontal plane, 1/m. Within the limits of (y ≤ 0,5·B), $\delta_2'' = 0$.

δ_3 is the attenuation coefficient on the slope.

For a tram track built in the cut, the value of the attenuation coefficient on the slope is assumed to be equal to zero.

For calculating the amplitudes of the train-induced vibrations, the field studies of the rolling stock vibrodynamic impact on the ground performed by G.M. Stoyanovich were used [18]. Based on the results of those studies, the vibration amplitude values depending on the axial load (220-250 kN/axis) and speed (50-90km/h) have been obtained.

The results of the above calculations are summarized in Table 1 and Figure 1.

Table 1. Vibration amplitude *A* (μm)

V(km/h) *P* (kN/axis)	50	60	70	80	90
220	165	195	220	245	270
230	200	225	250	275	300
250	280	310	345	385	420

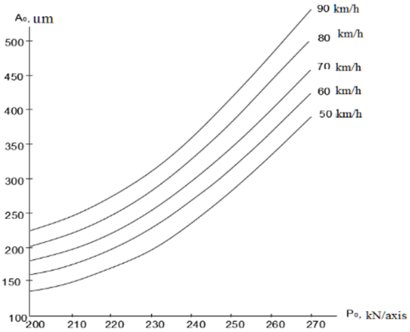

Fig. 1. Dependence of vibration amplitude (μm) on axial loads at different speeds.

3 Approximation of the dependence for tram loads and speeds

For a tram track, it is necessary to establish the dependencies of vibration amplitudes on axle loads between 70 kN/axle and 200 kN/axle and for speeds of 20-50 km/h. Thus, the formulas for the relations between vibration amplitude, axial load, and speed as shown in Table 1 has to be derived. Initially, the formulas for the one-variable functions in Table 1 were derived from the lines $A_f = f(V, \tilde{a}, \tilde{b})$ and columns $A_g = g(P, \tilde{c}, \tilde{d})$, where $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$ are one-dimensional function parameters. Figures 2, 3, and 4 for three different types of $f(V, \tilde{a}, \tilde{b})$ function show the parameter calculations obtained in the EXCEL programme by the least squares method.

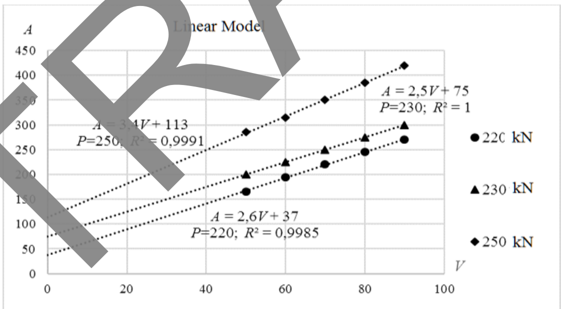

Fig. 2. Calculation of the linear function parameters $A = \tilde{a}V + \tilde{b}$

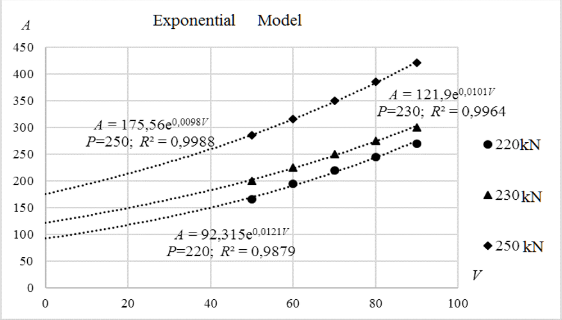

Fig. 3. Calculation of the exponential function parameters $A = \tilde{a}V + \tilde{b}$

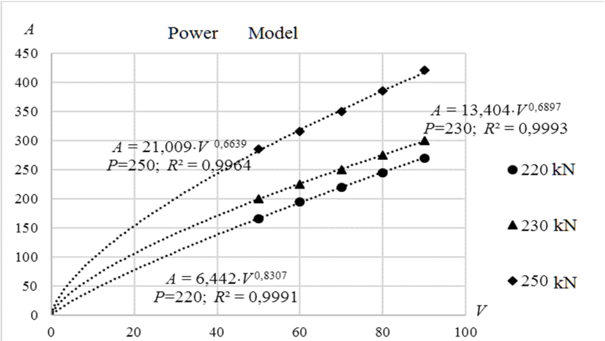

Fig. 4. Calculation of the power function parameters $A = \tilde{a} \cdot V^{\tilde{b}}$

The R^2 approximation reliability in all three cases is practically equal to one. This means that, from the mathematical point of view, any model is suitable for describing this experiment. However, the amplitude values of the linear and exponential models are not equal to zero at $V = 0$, which does not correspond to the physical essence of the experiment. The power function $A = \tilde{a} \cdot V^{\tilde{b}}$ is to be taken as the most suitable for the approximation.

A similar model $A = \tilde{c} \cdot P^{\tilde{d}}$ (Figures 5 and 6) was chosen for the $g(P, \tilde{c}, \tilde{d})$ function.

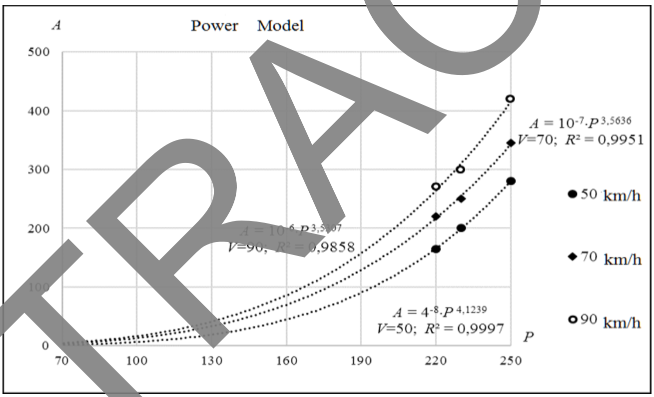

Fig. 5. Calculation of the power function parameters $A = \tilde{c} \cdot P^{\tilde{d}}$

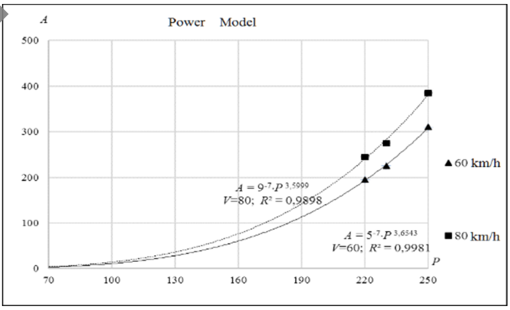

Fig. 6. Calculation of the power function parameters $A = \tilde{c} \cdot P^{\tilde{d}}$

A general formula approximating all values of the two-dimensional Table 1 can be presented as

$$A(P,V)=\tilde{k}\cdot f(P,a,b)\cdot g(V,c,d)=\tilde{k}\cdot a\cdot P^{b}\cdot c\cdot V^{d}=k\cdot P^{b}\cdot V^{d} \tag{3}$$

where
k, a, d are the function parameters of the *P* and *V* variables. In this case, after one of the variables *P* or *V* is fixed, the $A(P,V)$ function becomes similar to those of the variables $f(P, \tilde{a}, \tilde{b})$ or $g(V, \tilde{c}, \tilde{d})$. To calculate the function parameter values, we should find a function logarithm

$$\ln A=\ln k+\ln V^{a}+\ln P^{c} \tag{4}$$

The values in Table 1 are replaced with their logarithms in Table 2.

Table 2. Logarithms of the ln*A* vibration amplitude

ln*V* / ln*P*	3.91	4.09	4.25	4.38	4.50
5.39	5.11	5.27	5.39	5.50	5.60
5.44	5.30	5.42	5.52	5.62	5.70
5.52	5.63	5.74	5.84	5.95	6.04

Indicating

$$y=\ln A,\ a_0=\ln k, a_1=a, x_1=\ln V,\ a_2=c, x_2=\ln P$$

Let us transform (4) into a linear function of two variables x_1 и x_2:

$$y=a_0+a_1x_1+a_2x_2 \tag{5}$$

Next, the least squares method should be used to determine the parameter values a_0, a_1, a_2 at which the function (5) best approximates the values in Table 2. For the linear function (5) in the EXCEL "Data Analysis" section, it is possible to calculate these parameters:

$$y=-16.3962+3.459198\cdot x_1+0.739534\cdot x_2 \tag{6}$$

Referring back to formula (1), we obtain

$$A=7.57\cdot P^{3.46}\cdot V^{0.74}\cdot 10^{-8} \tag{7}$$

To evaluate the reliability of the approximation, Table 3 shows the experimental data (*A* Table) taken from Table 1, the predicted amplitude values (*A* prediction) calculated by Formula (7) and the absolute and relative prediction errors.

The relative error does not exceed four percent, which proves good approximation results. Although extrapolation is less reliable than approximation, this formula can be assumed to be valid for tram speeds and axle loads. Table 4 and Figure 7 show the estimated vibration amplitudes for the tram speeds and loads calculated using formula (7).

Table 3. Errors of prediction for *A* amplitude

A Table	*A* Prediction	Absolute error	Relative error
165	167.82	-2.82	1.7%
200	202.00	-2.00	1.0%
280	269.53	10.47	3.7%
195	192.05	2.95	1.5%
225	231.16	-6.16	2.7%
310	308.44	1.56	0.5%
220	215.24	4.76	2.2%
250	259.07	-9.07	3.6%
345	345.68	-0.68	0.2%
245	237.58	7.42	3.0%
275	285.96	-10.96	4.0%
385	381.56	3.44	0.9%
270	259.20	10.80	4.0%
300	311.98	-11.98	4.0%
420	416.29	3.71	0.9%

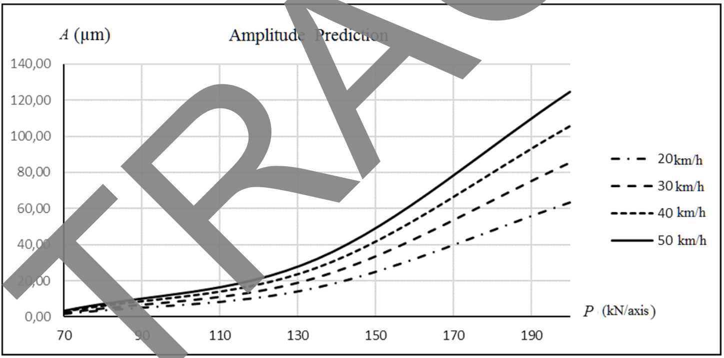

Fig. 7. Extrapolation of vibration amplitude values

Table 4. Extrapolation of vibration amplitude values

V(km/h)	*P* (kN/axis)	*A* (µm)
20	70	1.67
20	135	16.24
20	200	63.25
30	70	2.26
30	135	21.92
30	200	85.37
40	70	2.80
40	135	27.12
40	200	105.61
50	70	3.30
50	135	31.98
50	200	124.56

The above parameters were accepted for further studies. Currently available methods were adjusted for lower tram axle loads and used for calculating the amplitude dependence in different tram track structures with and without vibration isolating mats. The analysis of the obtained data shows that the use of vibration insulating mats (of different stiffness and thickness) does not affect the low-frequency vibration amplitude[6].

4 Calculation of the tram-induced vibration amplitude

To analyze high-frequency vibrations, calculations of tram-induced vibration levels were performed in accordance with SP (Code of Rules) 23-105-2004 "Vibration Assessment in the Design, Construction and Operation of Metro Facilities"[20].

- The theoretical assumptions are grounded on the following parameters:
- The scheme of using vibration-insulating mats is shown in Figure 8.
- Since the vibration insulating mats are laid on B7.5 concrete, the calculations are performed according to Clause 3.2 of SP (Code of Rules) 23-105-2004.
- Poisson's ratio is ν = 0.4.
- Average density of the ground surrounding the track structure is ρ = 1800 kg/m3.

Calculations were carried out in the octave frequencies of 16 ; 31.5 and 63 Hz.

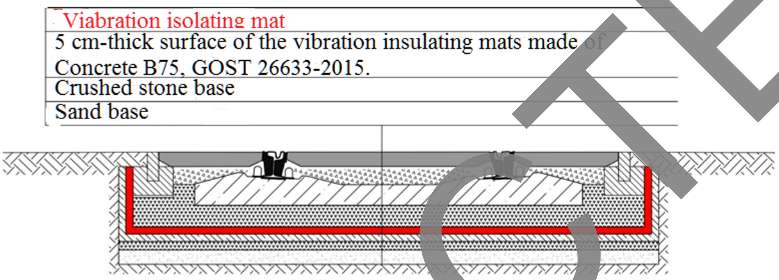

Fig. 8. Application of vibration insulating mats in a tram track structure with sleepers

Initial data:

- Longitudinal wave velocity in the ground is 600 m/s

- Speed of transverse waves in the ground is 200 m/s.

- Ground density is 1800 kg/m3

- Concrete lining thickness of a trough-type tram track formation made of B7,5 concrete is 0,1 m.

- The radius of the lining is accepted equal to half of the minimum width of a single-track section according to SP 98.13330.2016 and is 1.9 m.

Table 5. Standard vibration levels

Frequency, Hz	16	31.5	63
Highest level of the horizontal vibration velocity, (m/s)	0.00011	0.00096	0.00083
Highest level of the vertical vibration velocity, (m/s)	0.0001	0.00096	0.00083
Equivalent level of vertical vibration velocity, (m/s)	0.00006	0.00055	0.00048
Equivalent level of vertical vibration velocity (m/s)	0.00006	0.00055	0.00048

- $l_2 = 3.8\ m$ is the width of the formation bed.
- $E = 16\,000$ MPa is the elasticity modulus of B7.5 concrete
- $m = (1 + 3.8 + 1) \cdot 0.1 \cdot 2400 = 1392$ кг/m is the concrete casing weight per unit of the tram track trough.
- $\omega = 2\pi f$ is the cyclic frequency
- *f (Hz)* is the vibration frequency. It is accepted equal to 16; 31,5 and 63 Hz
- $К_у = \frac{6\sqrt{l_2 \cdot k \cdot \mu_г}}{2 + \mu_т/(\lambda_г + 2\mu_г)}$

where

$$k = 0.7\ m^{-1}$$

$$К_с = -\rho c l_2 = -1800 \cdot 600 \cdot 3.8 = -4\,104\,000$$

$J = 0{,}24\, m4$ is the moment of inertia of the concrete-case section of the tram track trough.

The predicted values of the tram-induced ground surface vibrations were calculated using the formulas:

$$\nu = \sqrt{\nu_R^2 + \nu_{1.2l}} \tag{8}$$

$$\nu_R = \sqrt{\frac{R_0}{H_0}} \cdot \nu_{max} \exp(-\beta k_R x) \tag{9}$$

Where

ν_R is the vibration velocity calculated by Rayleigh wave

$\beta = 0{,}15$ is the ground attenuation coefficient accepted in accordance with Annex A of SP 23-105-2004

$k_R = \frac{C_1}{\omega} = \frac{600}{\omega}$ is Rayleigh wave number

$$\nu_{1.2l} = \sqrt{\frac{R_0}{\sqrt{x^2 + H_0^2}}} \cdot \sqrt{\nu_{1max}^2 + \nu_{2max}^2} exp(-\beta k_i \sqrt{x^2 + H_0^2}$$

where

$H_0 = 1$ m is the excavation depth

X = 5 m is the distance from the longitudinal axis of the track

L=20Lg(ν/ν0).

The results of the performed calculations are presented in Figure 9.

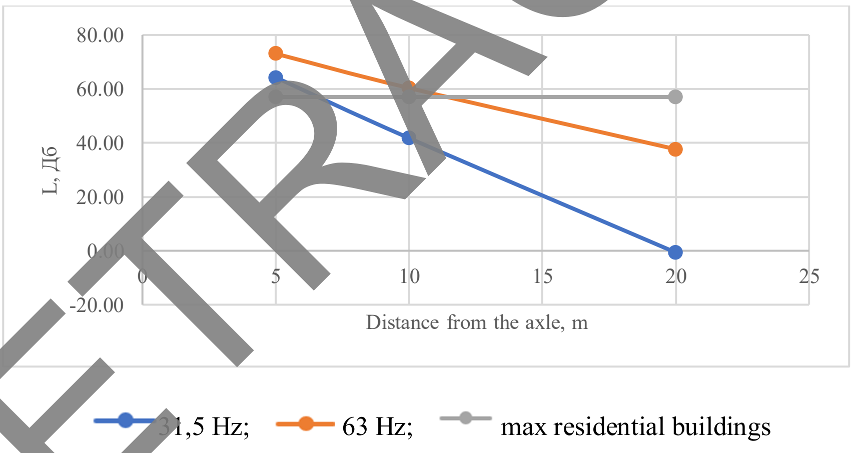

Fig. 9. Vibration levels at a distance from the tram track axle (31.5 and 63 Hz frequencies), (57 dB permissible level).

5 Conclusion

The results obtained show that for the considered tram track design without vibration isolating mats, the permissible levels of vibration velocity appear at a distance of more than 7 m from the track axle at the frequency of 31.5Hz and more than 12m at the frequency of 63Hz. In accordance with Clause 5.5 of SP 98.13330.2018, the minimum distance from the straight tram track axle to buildings and structures should be at least 2.8m. Therefore, if a building or facility with a permissible vibration level of 57 dB is located at a distance between 2.8 m and 7 m (from the tram track axle), the vibration velocity level should be reduced by 7.25 dB at 31.5 Hz and by 16.06 dB at 63 Hz.

To ensure the acceptable level, certain ways of reducing vibration velocity are recommended in Appendix "G" (Ж) of SP 23-105-2004. According to those recommendations, vibration level mitigation can be achieved by using 30mm-thick vibration isolating mats, which will reduce vibration by 0-3 dB at 16 Hz frequency, by 7-10 dB at 31.5 Hz frequency, and by 15-20 dB at 63 Hz frequency.

Thus, the preliminary calculations performed using the methodology of SP 23-105-2004 "Vibration Assessment in the Design, Construction and Operation of Metro Facilities" prove the anti-vibration mat efficiency in the tram track construction. However, the final decision on the application of anti-vibration mats should be taken after the experimental studies. When planning and carrying out experimental studies, the above-mentioned SP 23-105-2004 methodology should be adjusted for tram tracks, especially those with sleeperless (slab) structures largely applied in urban areas.

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