On the collision of railcars as an interaction of nonlinear shock wave-like perturbations

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Abstract. The article examines the collision of railcars based on the solution of hydrodynamic equations using shock waves during the transition from weak nonlinearity to perturbations of arbitrary amplitude. Several new mathematical issues have been set for the development of high-speed transport, which can be solved in the framework of hydrodynamics to describe the process of hydrodynamics, creating effective rolling stock dampers. which requires the improvement and development of the corresponding mathematical apparatus. In this work, we use a hydrodynamic approach to find the density distributions of matter during railcar collisions at high speeds, which is important in light of the problems of high-speed transport. In our approach, we found an analytical solution to the obtained hydrodynamic equations for the one-dimensional case. The equations under study were obtained taking into account nonequilibrium processes. To find a solution to the hydrodynamic equations, the shock wave approximation is used, similar to the soliton solutions we considered earlier. Taking into account possible deviations from the results of a one-dimensional problem is considered. Such a reduction of solutions of hydrodynamic equations to shock waves has not been considered previously and may be of interest for a wide variety of applied problems. The resulting consideration of railcar collisions is important for solving problems of transport safety and technospheric safety.

1 Introduction

The challenges associated with implementing the high-speed rail transport (see, for example, [1–4]) pose a large number of different physical and mathematical problems,which are solved using the apparatus of the equations of hydrodynamics.

The use of a hydrodynamic approach to the collision of railcars as a collision of rods was proposed by the great Nikolay E. Zhukovsky, the father of Russian aviation, and was continued, for example, in [5]. In [6–8], for simplicity, we carried out this consideration in flat one-dimensional geometry, and the problem of collision of layers-slabs was reduced to a description of the interaction of Korteweg–de Vries solitons. Here we have obtained a

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description for the propagation of shock waves during the transition from soliton perturbations to perturbations of arbitrary amplitude.

A large body of literature on shock waves has been published, concerned with the analytical solution [9] and numerical problems [10–11].

An approximate solution of one-dimensional hydrodynamic equations using shock waves can be used in calculations of rail dampers (see, for example, [5, 12–15]) and construction equipment [16–20].

These are calculations of hydraulic transmissions, which have been developed for a long time, but with the increase in the speed mode of rolling stock and with the consideration of friction received a new continuation [21]. To create effective wheel dampers, vibration dampers, and shock absorbers, hydrodynamics is also used [2-4], which requires improvement by taking into account the nonlinearity and non-equilibrium of the damping process at high speeds. Linear equations are used in [22], and simplified hydrodynamic equations are used in [23]. This can be developed further and refined in detail for a nonlinear compressible medium within our hydrodynamic approach.

The development of the hydrodynamic approach can be applied to describe nonlinear dynamics in the calculations of bridges on high-speed electric transport lines [16], and can also be used in the analysis of the stability of transport structures in extreme conditions [24].

Hydrodynamics is used in many areas of physics and technology [24-26] for shock waves in plasma [24,25], in collisions of heavy ions [25,26], and ending with the hydrodynamics of quantum dots . In our works [6-8], nonequilibrium hydrodynamics was proposed. And this can be extended to a wide area of technical applications and used in the design of wheel shock absorbers, pipes, transport structures, bridges and other objects of transport and construction engineering (see, for example, [1- 16.23]) in the light of the problems of high-speed transport, since we proposed a rigorous mathematical approach.

The current stage of development of railway transport in Russia and the World is characterized by an increase in the speed of passenger trains while the state of the railway infrastructure remains unchanged, which leads to increased risks to the life and health of passengers in the event of emergency situati0ons. The most dangerous accidents are longitudinal collisions of passenger trains with obstacles on the track, which reflect 99.2% of registered cases of emergency collisions on Russian railways. The adoption of the Strategy for the Development of Railway Transport until 2030, which provides for the production and commissioning of high-speed and high-speed rolling stock, makes the problem of ensuring the safety of railway passenger transportation increasingly urgent . In this regard, the task of increasing the safety of passenger railcars during longitudinal collisions is a priority direction for the development of new generation railway rolling stock . The most effective way to improve the safety of railway transportation is the development and implementation of mechanical safety systems for passenger railcars, based on the use of special destructible elements that absorb the energy of a train colliding with an obstacle Thus, the task of developing a methodology for determining the parameters of security systems passenger railcars and their rationale are relevant [27] .

Consideration of the collision of high-speed railcars is important for problems of transport safety and technospheric safety. The purpose of the work is to develop and theoretically substantiate technical solutions for ensuring mechanical safety of passenger railcars in case of longitudinal collisions

Next, Section 2 examines the model used to describe the collision of rail cars within the framework of the hydrodynamic approach, then in Section 3 a solution to the proposed equations is found using shock waves and the results of the collision of cars are analyzed in order to determine the degree of impact of the collision of railcars on their condition. In Section 4, the main conclusions of the work are presented.

2 The Model

The solution of the kinetic equation for the distribution function $f(\vec{r},\vec{p},t)$ is found as

$$f(\vec{r},\vec{p},t) = f_1 q + f_0 (1-q)\,, \tag{1}$$

where $f_0(\vec{r},\vec{p},t)$ is locally equilibrium distribution function, $f_1(\vec{r},\vec{p},t)$ is nonequilibrium distribution function. In the nonequilibrium case, $q = 1$ the equations of long-range hydrodynamics [6] are obtained, which in the one-dimensional case have the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \upsilon)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial(m\rho\upsilon)}{\partial t} + \frac{\partial(m\rho\upsilon^2 + P)}{\partial x} = 0\,. \tag{3}$$

The equations of hydrodynamics for the relaxation factor $q=1$(non-equilibrium case) in the one-dimensional case are reduced to a system of equations (2)-(3) for finding the nucleonic density $\rho(x,t)$, velocity $\upsilon(x,t)$, thermal energy density $I(x,t)$.

From equations of hydrodynamics it follows that the thermal term $I = I_1 \left(\frac{\rho}{\rho_0}\right)^3$, where I_1 is an independent coefficient ρ. The joint solution of equations (2) and (3) is sought in the form $\upsilon = \upsilon(\rho)$ and we obtain two Korteweg-de Vries equations.

That is, the equations of hydrodynamics can be reduced to two Korteweg-de Vries equations This allows us to proceed to the representation of the collision of complex systems (nuclei), as a collision of solitons, if the simple wave of the Korteweg-de Vries equation is reintegrated over x_1. That is, to find

$$Z = \int_0^L \zeta \frac{dx_1}{L}\,, \tag{4}$$

where L-is the thickness of the layer, Z is a simple Korteweg-de Vries wave emitted by this layer $\zeta(x - x_1, t)$ -a one-soliton solution of the Korteweg-de Vries equation. This applies to each nuclear layer-the source of simple waves. Taking into account the multiple reflections of the Korteweg-de Vries waves from the boundaries of the system, one can consider the entire dynamics of the collision of the nuclear slab layers.

Let us consider the propagation of perturbations of arbitrary amplitude using the equations of hydrodynamics [6–8]:

For the energy density, we can use a simple expression

$$e = K(\rho - \rho_0)^2\ , \tag{5}$$

where ρ_0 is the equilibrium density and $K = 9mc_{s0}^2$ is the compression modulus. The pressure is then

$$P = -\frac{\partial(e/\rho)}{\partial(1/\rho)} = K(\rho^2 - \rho_0^2) - \alpha\left(\frac{\partial\rho}{\partial x}\right)^2 . \tag{6}$$

Here we have added a dispersion term with the coefficient α, where $\frac{\alpha}{2mc_{so}^2}\rho_0 =$ $(\mathrm{m})^2$, and the speed of sound is $c_{s0} \approx 3\ 10^3$ m/s. A collision between two railcars generates shock waves propagating at velocity D, which can be found from the hydrodynamic equations (2)–(3), assuming that $\frac{\partial}{\partial t} = -D\frac{\partial}{\partial x}$.

3 Problem Solution

Basically, the solution of this system of nonlinear partial differential equations is found numerically on a computer. Here we develop an approach to an approximate analytical solution of these equations, both in the case of weak nonlinearity, by reducing them to the Korteweg-de Vries equations, and in the case of large-amplitude perturbations, using soliton-like solutions. A generalization of this approach to the two-dimensional case in the case of density $\rho(x,t)$, depending only on coordinate x and time t has been carried out Such consideration has not been considered before and can be extended to arbitrary complex systems.

Integrating these equations over the density jump we obtain

$$D = -\frac{\rho_0 \upsilon_0}{\rho - \rho_0} , \tag{7}$$

where υ_0 is the initial velocity of colliding layer-cars. Assuming velocity D to be equal to the speed of sound $c_s = \sqrt{\frac{\partial P}{m\partial\rho}}$, and taking into account the expression for pressure (4), we obtain the equation for density ρ :

$$K(\rho^2 - \rho_1^2) - \alpha\left(\frac{\partial\rho}{\partial x}\right)^2 = mc_s^2(\rho_1)(\rho - \rho_1) , \tag{8}$$

where ρ_1 is the maximum compression density on a shock wave,

$$c_s^2(\rho_1) = 2K\rho_1 / m = \frac{(\rho_0\upsilon_0)^2}{(\rho_1 - \rho_0)^2} . \tag{9}$$

From here

$$\frac{\partial\rho}{\partial x} = \pm\sqrt{\frac{K}{\alpha}}(\rho - \rho_1) , \tag{10}$$

and at $x. > 0$

$$\rho - \rho_1 = (\rho_1 - \rho_0)\left(\exp\left(-\sqrt{\frac{K}{\alpha}}x\right) - 1\right). \tag{11}$$

Isolating the main terms, since we are not currently interested in the details of the wave front structure, we can approximate solution (9) with the soliton solution

$$\rho = \rho_0 + 4\frac{(\rho_1 - \rho_0)}{(\exp(-\lambda x / 2) + \exp(\lambda x / 2))^2}, \tag{12}$$

where $\lambda = \sqrt{\frac{K}{\alpha}}$. Expression (10) describes the main features of solution (8)–(9). Also, as we did earlier with the Korteweg–de Vries solitons, we can integrate expression (12) over the length of the layer and consider the propagation of the shock wave front and its reflection from the boundaries. As a result of integration we obtain

$$\rho = \frac{1}{L}\int_{l_1}^{l_2} \rho' dx = \rho_0 + 4\frac{(\rho_1 - \rho_0)}{\lambda L}\left[\frac{1}{1+\exp(\lambda(x - l_2 - Dt))} - \frac{1}{1+\exp(\lambda(x - l_1 - Dt))}\right], \tag{13}$$

where ρ' is formula (12), l_1 and l_2 are the boundaries of the railcar, and $L = l_2 - l_1$ is its size. Since at the maximum of the shock wave the velocity is $\upsilon = 0$, for the maximum, a wave equation is obtained from the equations of hydrodynamics that admits the d'Alembert solution.This is what we did. In this case, velocity can be found from the continuity equation, using expression (11) for density, taking into account possible reflections of shock waves from the boundaries of the system and the movement of the boundaries.

Figure 1 shows instantaneous photographs of the relative density (ρ / ρ_0) for the interaction of two rod cars in a system of equal speeds, when the train begins to move off at an initial velocity of $\upsilon_0 = 100$ m/s at times $t = 1;2;3;4;5;6;7;8;9$ ms.. Figure 2 shows instantaneous photographs of the relative density (ρ / ρ_0) for the interaction of two rod cars in a system of equal speeds, when the train begins to move off at an initial velocity of $\upsilon_0 = 300$ m/s at times $t = 1;2;3;4;5;6$ ms.

Figure 3 shows instantaneous photographs of the relative density (ρ / ρ_0) for the interaction of two rod cars in a system of equal speeds, when the train begins to move off at an initial velocity of $\upsilon_0 = 500$ m/s at times $t = 1;2;3;4;5$ ms. After the initial compression and formation of a hot spot, followed by expansion, at the expansion stage a rarefaction is observed in the center.

As for the size, the region of rarefaction turns out to be of the order of the railcar length (10 m) and, therefore, in accordance with the estimates of shock absorber parameters [5], does not result in destruction.

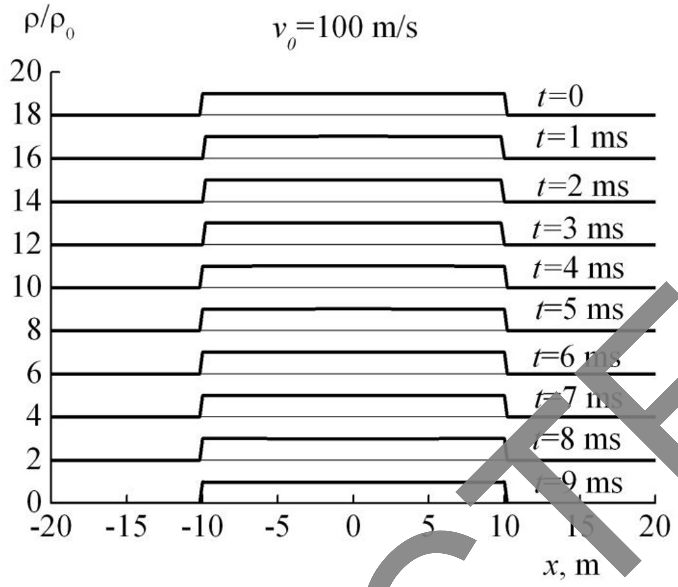

Fig. 1. Instantaneous photographs of the collision of rod cars at an initial velocity of $\upsilon_0 = 100$ m/s at times $t = 1;2;3;4;5;6;7;8;9$ ms

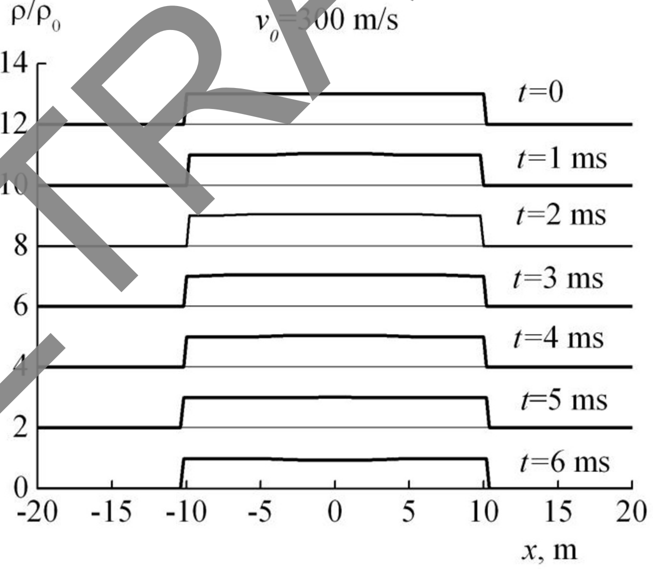

Fig. 2. Instantaneous photographs of the collision of rod cars at an initial velocity of $\upsilon_0 = 300$ m/s at times $t = 1;2;3;4;5;6$ ms

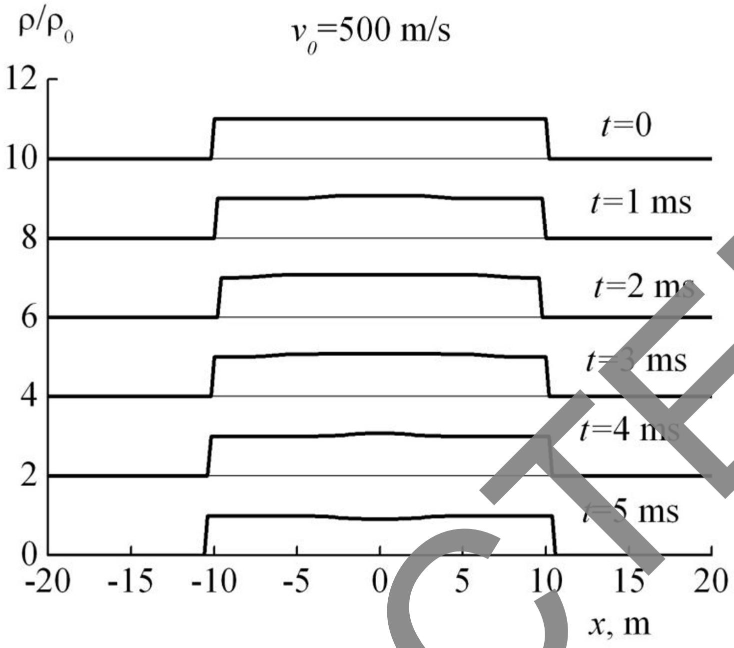

Fig. 3. Instantaneous photographs of the collision of rod cars at an initial velocity of $\upsilon_0 = 500$ m/s at times $t = 1;2;3;4;5$ ms

We also can be found a simplified solution to the issue in the two-dimensional case. The equations are obtained from the hydrodynamic equations by integrating the hydrodynamic equations over the transverse coordinate, assuming that the density ρ *(x,t)* does not depend on the coordinate *y*. The solution is given by formula (11) by replacing $\rho_1 \to \rho_1 S(x,y,t)$ and then dividing the perturbation by *S*, where $S(x,y,t) = (y_0(x) + \upsilon_y t)^2 / y_0^2$, and υ_y coincides with the speed of sound.

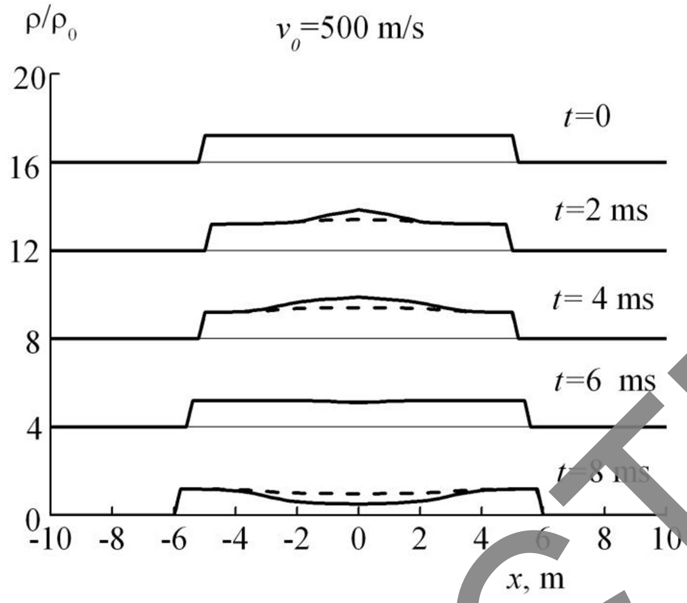

Fig. 4. Instantaneous collision profiles of identical slabs (solid lines) at velocity $\upsilon_0 = 500$ m/s various points $t = 0; 2; 4; 6; 8$ ms for the two-dimensional case, the dashed lines are density profiles for one-dimensional layers.

Thus, Figure 4 shows the density profiles for collisions of identical nuclei with a longitudinal dimension of $L = 5$ m with velocity $\upsilon_0 = 500$ m/s at time moments $t = 0; 2; 4; 6; 8$ ms. In this case, the results are indicated by solid lines The dashed lines correspond to the one-dimensional case. It can be seen that in the two-dimensional case, the oscillations of compression and rarefaction are stronger.

4 Conclusion

Thus, in the present work, the nonequilibrium hydrodynamic approach has been further developed to describe complex systems on the example of slab collision. The non-equilibrium approach to the hydrodynamic equations allows describing the experimental data better than the equation of state corresponding to traditional hydrodynamics, assuming the establishment of local thermodynamic equilibrium. In this description, the isolation of the hot spot was essential. In this paper, we show that the introduction of dispersion terms does not violate this representation. During the expansion stage, a rarefied region is formed in the center of the system. This consideration was carried out in one-dimensional case and may be carried in two-dimensional case too.

The reduction of the hydrodynamic equations to the solution of two Korteweg-de Vries equations in the form of solitons makes it possible to find an analytical solution to the issue. The approach itself based on hydrodynamic equations for perturbations of arbitrary amplitude is of independent fundamental interest for physics in general. It can be extended to a wide range of technical applications and used in the design of wheel dampers, pipes, transport structures, bridges and other transport and construction equipment (see, for example, [12–20]) in the light of high-speed transport problems.

To further develop the approach, it is necessary to take into account the dissipative terms in the hydrodynamic equations and the possible deviation from the one-dimensional problem. As a result of our assessment of the modeling results, a refined hydrodynamic computer model of railcar collisions can be selected for further research. In the third part of the work, we proposed an approach for selecting the parameters of energy absorption devices [27]. Thus, on the basis of the "Universal Mechanism" program complex, as a result of calculations and numerical experiments, for example, the development of a energy absorption devices design can be carried out

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References

1. Musk E., Hyperloop Alpha white paper (2013)
2. Evans B., Rose C., J Automob Eng : **228**, 1127–1141 (2014) DOI: 10.1177/0954407013511071
3. Alexander N.A., Kashani M.M., Exploring Bridge Dynamics for Ultra-high-speed, Hyperloop, Trains, Structures, **14,** 69–74 (2018); https://doi.org/10.1016/j.istruc.2018.02.006
4. Olivier B., Connolly D.P., Alves Costa P., Kouroussis G., Int J Rail Transp, **4**, 229–246 (2016); DOI: 10.1080/23248378.2016.1220844
5. Myamlin S.V., Pysmennyi Ye. O., Yalovyi O.I., Manashkin L.A.,*Mathematical modeling of shock absorbers*, Science and Transport Progress Bulletin of Dnipropetrovsk National University of Railway Transport 2004 ; DOI: 10.15802/stp2004/20689
6. D'yachenko A.T., Mitropolsky I.A., Bulletin of the Russian Academy of.Science.: Physics **86**, 8 , 962-965 (2022); DOI: 10.3103/S1062873822080068
7. D'yachenko A.T., Mitropolsky I.A., AIP Conf. Proc., **2624(1),** 050007 (2023); https://doi.org/10.1063/5.013224
8. D'yachenko A.T., Mitropolsky I.A., Bulletin of the Russian Academy of.Science.: Physics, 2023, **87,** 0. 8, 1169-1173; DOI: 10.3103/S1062873823703057.
9. Landau L.D. , Lifshitz E.M., Fluid Mechanics (Pergamon, Oxford,1987).
10. Sewell E.G., Ferguson K.J., Krivets V.V, Jacobs J.W., J. Fluid Mech. , 2021, **917,** A41;doi:10.1017/jfm.2021.258
11. Johnson R.F., Kercher A.D., Journal of Computational Physics , 2020, **423,** 109826. doi:10.1016/j.jcp.2020.109826.
12. Lu Z., Wang Z., Masri S.F., Lu X., Control Heal. Monit., 2018, **25**, 1–25; https://doi.org/10.1002/stc.2058
13. Gagnon L., Morandini M., Ghiringhelli G. L., J. Sound Vib., 2019, **459**, 114865; https://doi.org/10.1016/j.jsv.2019.114865
14. Choi Y., Ju S., Chae S.H., et al., Smart Materials and Structures , 2015, **24(6),** 65029-12; DOI: 10.1088/0964-1726/24/6/065029
15. Wang W.., Zhou Z., Yu D., Qin Q., Iwnicki S., Mechanical Systems and Signal Processing, 2017, **95,** 138-157; DOI: 10.1016/j.ymssp.2017.03.031

16. Benin A., Guzijan-Dilber M., Diachenko L., Semenov A., E3S Web of Conferences, , 2020, **157**, 6, 06018; DOI:10.1051/e3sconf/202015706018
17. Egorov V., Abu-Khasan M., Shikova V., IOP Conference Series: Materials Science and Engineering, 2020, **753**, 2, 022021; DOI: 10.1088/1757-899X/753/2/022021.
18. Burkov A. T., Valinsky O. S., Evstaf'ev A. M., Maznev A. S., Tretyakov A. V. Russian Electrical Engineering, 2019, **90**, 10, 692–695; https://doi.org/10.3103/S106837121910002X
19. Evstaf'ev A. M., Boronenko Yu. P., Izvarin M. Yu. , Andreev V. E., Russian Electrical Engineering, 2017, **88**, 10, 672–675; https://doi.org/10.3103/S1068371217100042
20. Kim K.K., Panychev A.Y., Blazhko L.S.,, Solid State Phenomena, 2020, v**299** SSP , pp. 381-385; DOI: 10.4028/www.scientific.net/SSP.299.381
21. Khromova G., Makhamadalieva M., E3S Web of Conferences, 2024, **474**, 01027 ; https://doi.org/10.1051/e3sconf/202447401027
22. Khromova G., Makhamadalieva M., Khromov S., E3S Web of Conferences. – EDP Sciences, 202, **264**, 5029; https://doi.org/10.1051/e3sconf/202126405029
23. Kazakov Yu.B., Morozov N.A. Nesterov S.A., Journal of Magnetism and Magnetic Materials. 2017, **431**, 269—272; DOI:10.1016/j.jmmm.2016.10.006
24. Fortov V.F, Lomonosov I.V., Ya.B. Zeldovich, Physics- Uspekhi 2014, **57**, 3, 219–233; DOI: 10.3367/UFNe.0184.201403b.0231
25. Fortov V.F, Sharkov B.Yu., Stöcker H., Physics- Uspekhi 2012, **55**, 6, 582–602; DOI: 10.3367/UFNe.0182.201206c.062
26. Stöcker H, Greiner W., Physics Reports. 1986, **137**, 5-6, 277-392; https://doi.org/10.1016/0370-1573(86)90133-X
27. Kobishanov V.V., Lozbinev V.P., Sakalo V.I., Antipin D.Y., Shorohov S.G., Vysocky A.M., *Passenger car safety prediction*, World Applied Sciences Journal 24 (Information Technologies in Modern Industry, Education & Society), 2013, p. 208-212; DOI: 10.5829/idosi.wasj.2013.24.itmies.80041