

Movement of particles in shear flows

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Abstract. Some studies on the dynamics of particles in a viscous medium are presented. The influence of the effect of added mass when a particle moves in the Poiseuille and Couette flow field is shown. The behavior of the particle is studied based on the description of the Lagrangian approach. It is assumed that the presence of a particle does not have a noticeable effect on the flow. An analytical expression is obtained that determines the trajectory of the particle. The behavior of the particle taking into account the Saffman transverse force in the Poiseuille flow field was also studied using a numerical method.

1 Introduction

Problems of particle motion occur in many natural phenomena and in production. Examples include pneumatic transport of granular materials, oil and gas industry, water erosion, etc. Detection of patterns in particle behavior is one of the main tasks of the mechanics of multiphase problems.

To mathematically describe the motion of two-phase media, two approaches of Euler and Lagrange are used. Euler's mixture approaches consider interpenetrating continua. For each phase, hydrodynamic equations are derived with complications associated with the interaction between the phases [1-2].

The Lagrange approach considers the trajectories of many test particles in a carrier flow [3-5]. In this approach, the proportion of the dispersed phase should be no more than 10%. When the concentration of the dispersed phase is low, the mutual influence of the particles is negligible. In this case, the movement of each particle in the liquid can be considered as occurring independently of the presence of other particles.

Following the Lagrange approach, we believe that the concentration of the dispersed phase is low and the particles are small in size. We also neglect the influence of particle motion on the carrier flow. Then the carrier flow is described by the Navier-Stokes equation. The behavior of the particle is described using the Lagrange approach. Calculation of the trajectory of an individual particle in a fluid flow is carried out as a result of various forces acting on the particle (gravitational, drag, lifting, attached, etc.). Using different initial positions of particles and following their trajectories, it is possible to simulate the flow of a solid particle in a liquid [1-5].

Analysis of the patterns of motion of individual particles in a known velocity field of a carrier medium is of independent interest, since in many cases the approximation of a single particle is sufficient to describe the evolution of the state of the mixture [6-23].

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2 Modeling the behavior of particles in a flow

Let us consider a two-phase flow (liquid and particles), in which the fraction of particles and its size are relatively small. Under this assumption, the influence of the particle on the flow is neglected. In this case, by studying the movements of a single particle in a fluid flow, one can identify the main patterns of an ensemble of particles [5].

Modeling the flow while ignoring the influence of particles on the parameters of the carrier flow corresponds to the Lagrange approach. The Maxey–Riley equation is widely used to describe the particle flow [3]. The equation of particle motion described in the Lagrangian frame of reference is expressed by the following equations [3,5,8]:

$$m_p \frac{d\vec{U}_p}{dt} = \vec{F}, \quad (1)$$

In equation (1) m_p is the particle mass; \vec{U}_p is particle speed; \vec{F} is total force acting on the particle:

$$\vec{F} = \vec{F}_D + \vec{F}_g + \vec{F}_A + \vec{F}_S + \vec{F}_m + \vec{F}_B + \vec{F}_M.$$

Here \vec{F} is the total force, \vec{F}_D is the resistance force due to the effect of viscosity and the shape of the particle; \vec{F}_g is gravity; \vec{F}_A is the force of Archimedes; \vec{F}_S is lift force normal to the relative speed; \vec{F}_m is the force required to accelerate a certain amount of mass of the surrounding fluid due to the movement of the particle (force of added masses); \vec{F}_B is Basse force; \vec{F}_M Magnus force due to the rotation of particles.

The resistance force is calculated by the formula:

$$\vec{F}_D = 0,5C_D S \rho_f (\vec{V}_f - \vec{U}_p) |\vec{V}_f - \vec{U}_p|. \quad (2)$$

In equation (2) C_D is resistance coefficient; S is coefficient of the area of the midsection of the particle. The resistance force is the force caused by the mismatch between the speed of the fluid and the particle.

Gravity forces are defined as follows:

$$\vec{F}_g = m_p \vec{g}, \quad (3)$$

where m_p is the particle mass and \vec{g} is the free fall acceleration vector. In a gradient (uneven) flow, the Saffman force takes place

$$\mathbf{F}_S = \frac{C_S}{4} \pi d_p^2 \sqrt{\frac{\mu \rho_p}{|\mathbf{rot}(\mathbf{V}_p)|}} (\mathbf{V}_f - \mathbf{V}_p) \times \mathbf{rot}(\mathbf{V}_p), \quad (4)$$

where C_S is Saffman's constant; d_p is particle diameter.

The Saffman force [9] is due to the uneven profile of the surrounding fluid, which in turn leads to an uneven pressure distribution around the particle.

The force associated with the pressure gradient is defined as follows:

$$\mathbf{F}_A = m_f \left(\frac{D\mathbf{V}_f}{Dt} - \mathbf{g} \right), \quad (5)$$

Where m_f is the mass of liquid displaced by a particle; D/Dt is the Lagrangian time derivative for the fluid. If $\mathbf{V}_f = 0$, then we have the well-known Archimedes law for a fluid at rest.

The so-called added mass force [3] has the form:

$$\mathbf{F}_m = \frac{m_f}{2} \left(\frac{D\mathbf{V}_f}{Dt} - \frac{d\mathbf{V}_p}{dt} \right). \quad (6)$$

The Basse force is negligible for particles with a density significantly higher than the density of the liquid [1].

The Magnus force, which arises as a result of the rotation of a particle with angular velocity $\boldsymbol{\omega}$, due to a pressure drop, is directed perpendicular to the flow,

$$\mathbf{F}_M = C_\omega m_p \left[\mathbf{V}_f, \boldsymbol{\omega} \right], \quad (7)$$

where C_ω is a dimensionless coefficient (the symbol [] means a vector product).

3 Results and discussions

We apply the equation of particle motion (1) to specific problems. First, consider the motion of a particle in an upward flow. We assume that the upward flow is laminar and corresponds to Poiseuille's law. We neglect the influence of the particle on the flow.

3.1 Motion of a particle in an upward flow

Let us introduce the notation: let $x(t)$, $y(t)$ represent the coordinates of the particle trajectory at time t , $u(t)$, $v(t)$ be the components of particle velocity at time t , the fluid velocity obeying Poiseuille's law. Then the system of equations of particle motion in dimensionless variables takes the form;

$$\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{du}{dt} = D(u_l - u) - DW, \quad \frac{dv}{dt} = -Dv. \quad (8)$$

Choice of coordinate system: the x axis is directed against gravity and is located in the middle of the channel, the y axis is particular to it. The relationship between the coefficients in equation (1) and the flow parameters is as follows:

$$D = \frac{A}{\rho + 0.5}, \quad A = \frac{9}{2\varepsilon^2 R}, \quad W = \frac{2r^2(\rho_p - \rho_l)g}{9\mu U}$$

In these relationships, r is particle size, R is Reynolds number ($\varepsilon = r/L$, $R = UL/\nu$). U is characteristic speed, L is characteristic size, ν is kinematic viscosity of the carrier fluid, $\rho = \rho_p/\rho_l$ - ρ_p is particle density, ρ_l density of the carrier medium, ($D = 1/Stk$) Stk is Stokes number.

The initial position and initial velocity of the particle are known $y(0) = y_0$, $x(0) = 0$, $u(0) = u_0$, $v(0) = v_0$. Then the particular solution (1) has the form:

$$\begin{aligned} x(t) &= \frac{1}{D} \left(u_0 - 1.5 + 6y_0^2 + 12y_0 \frac{v_0}{D} + W \right) (1 - \exp(-Dt)) - \left[1.5 - 6 \left(y_0 + \frac{v_0}{D} \right)^2 - W \right] t + \\ &+ 12 \left(y_0 + \frac{v_0}{D} \right) \left[\frac{1}{D^2} - \frac{t \exp(-Dt)}{D} - \frac{\exp(-Dt)}{D^2} \right] - \frac{6v_0^2}{2D^3} (1 - \exp(-2Dt)), \\ y(t) &= -\frac{v_0}{D} \exp(-Dt) - y_0 + \frac{v_0}{D}, \quad v(t) = v_0 \exp(-Dt), \\ u(t) &= 1.5 - 6 \left(y_0 + \frac{v_0}{D} \right)^2 - W - \left[12 \left(y_0 + \frac{v_0}{D} \right) v_0 + u_0 - 1.5 + 6y_0 \frac{v_0}{D} \right] \exp(-Dt) + \frac{6v_0^2}{D^2} \exp(-2Dt). \end{aligned}$$

The resulting analytical solution allows us to obtain some details of the influence of the added mass effect.

In particular

$$\max |R_v| = \left| \frac{v_0}{2\rho} \exp \left((2\rho + 1) \ln \frac{\rho}{\rho + 0.5} \right) \right|.$$

$$\max_t |R_u| = \left| \frac{1}{2\rho} (u_0 - 1.5 + 6y_0^2 + W) \exp \left((2\rho + 1) \ln \frac{\rho}{\rho + 0.5} \right) \right|.$$

Here the notation is introduced: R_v and R_u are the differences between the transverse and longitudinal velocities, respectively, taking into account without it.

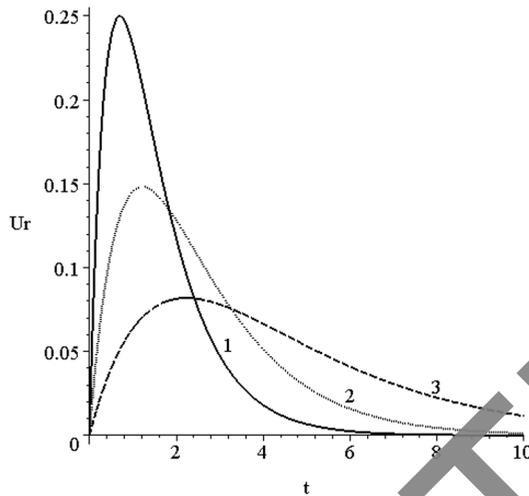


Fig.1. Influence of the added mass effect in the updraft

The maximum difference for longitudinal and transverse speed occurs at

$$t_* = -\frac{2\rho+1}{A} \ln\left(\frac{\rho}{\rho+0.5}\right)$$

From this relationship it follows that, at $\rho \ll 1$, the effect of the added mass is significant, and at $\rho \gg 1$, i.e. when the carrier flow is a gaseous medium, the effect of the added mass can be neglected.

In the case of $W = 0$, we can obtain an expression for the ratio of the force of the added

mass to the Stokes force:
$$\frac{|F_m|}{|F_h|} = \frac{1}{2\rho+1}.$$

(9)

From this we can conclude that the influence of the added mass effect is comparable to the hydrodynamic Stokes force at moderate.

We can obtain a condition for the initial transverse velocity at which the particle does not leave the layer:

excluding added mass

$$\left(-\frac{1}{2}-y_0\right)\frac{A}{\rho} \leq v_0 \leq \left(\frac{1}{2}-y_0\right)\frac{A}{\rho},$$

taking into account the added mass

$$\left(-\frac{1}{2}-y_0\right)\frac{A}{\rho+0.5} \leq v_0 \leq \left(\frac{1}{2}-y_0\right)\frac{A}{\rho+0.5}.$$

It follows from this that, when taking into account the effect of the added mass, the range of changes in the initial transverse velocity narrows, and the degree of compression is more noticeable, the smaller .

Figure 1 shows changes in the difference in longitudinal velocities related to $u_0 - 1.5 + 6y_0^2 + W$: $Ur = \left| R_u / (u_0 - 1.5 + 6y_0^2 + W) \right|$, curves numbered 1, 2, 3 correspond to the values $\rho = 0.5$, $\rho = 1$, and, $\rho = 2$, respectively, at $A = 1$. This shows that an increase in particle density reduces the influence of the effect of added mass.

3.2 Particle in Couette flow

One of the simplest cooling flows is the Couette flow. It is known that Saffman forces arise in gradient defects. Let us consider the motion of a single particle in Couette flows taking into account the Saffman forces. Such a system in dimensionless form has the form:

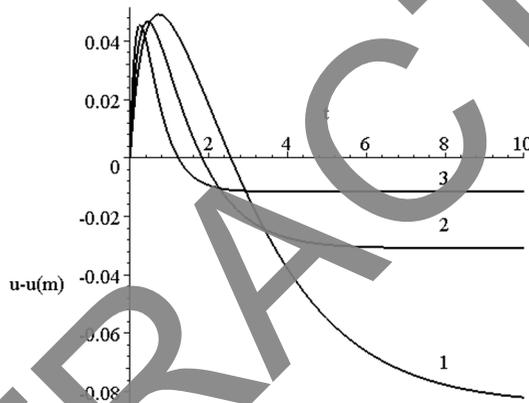


Fig.2. Influence of the added mass effect in the Couette flow

$$\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{du}{dt} = D(y - u), \quad \frac{dv}{dt} = -Dv + a(y - u). \tag{10}$$

Coefficient D is similar to that in equation (1), and $a = \frac{a_* \sqrt{A}}{\rho + 0.5} \left(a_* = \frac{1,615}{\pi} \sqrt{2} \right)$.

The solution to systems of equations (3) under initial conditions similar to (1) for the case has the form::

$$\begin{aligned} v &= C_1 \exp((-D + \sqrt{a})t) + C_2 \exp((-D - \sqrt{a})t), \\ y &= \frac{C_1}{-D + \sqrt{a}} \exp((-D + \sqrt{a})t) - \frac{C_2}{D + \sqrt{a}} \exp((-D - \sqrt{a})t) + C_3, \\ u &= y - \frac{1}{\sqrt{a}} \left[C_1 \exp((-D + \sqrt{a})t) - C_2 \exp((-D - \sqrt{a})t) \right] \end{aligned}$$

Based on the initial condition, the coefficients C_1, C_2 and C_3 are determined as follows:

$$C_1 = 0.5[v_0 - \sqrt{a}(u_0 - y_0)], \quad C_2 = 0.5[v_0 + \sqrt{a}(u_0 - y_0)],$$

$$C_3 = y_0 + \frac{C_1}{D - \sqrt{a}} + \frac{C_2}{D + \sqrt{a}}.$$

In the case $D^2 > a$ under the asymptotic condition $y(t)$, the solution at $t \rightarrow \infty$ becomes finite and calculating the difference of the asymptotic solution gives:

$$R_{y_\infty} = [v_0 \sqrt{A} - a_*(u_0 - y_0)] \frac{0.5A^{1.5}}{(A^{1.5} - a_*(\rho + 0.5))(A^{1.5} - a_*\rho)}$$

Here $R_{y_\infty} = y_{m\infty} - y_\infty$, $y_{m\infty}, y_\infty$ is an asymptotic solution taking into account the effect of the added mass and without it, respectively. Based on the expression, it follows that R_{y_∞} the asymptotic level is significantly different when taking into account the added mass; especially for small values of the difference $A^{1.5} - a_*(\rho + 0.5)$. It should be noted that the provided $a_*\rho < A^{1.5} < a_*(\rho + 0.5)$ value $y_{m\infty}$ will be finite and y_∞ will be infinite.

For this problem, the question of the condition for the particle to be in layers is defined as follows:

excluding added mass

$$\frac{a_*u_0}{\sqrt{A}} - y_0 \frac{A}{\rho} \leq v_0 \leq \frac{A}{\rho}(1 - y_0) + \frac{a_*(v_0 - 1)}{\sqrt{A}},$$

taking into account the added mass

$$\frac{a_*u_0}{\sqrt{A}} - y_0 \frac{A}{\rho + 0,5} \leq v_0 \leq \frac{A}{\rho + 0,5}(1 - y_0) + \frac{a_*(v_0 - 1)}{\sqrt{A}}.$$

Thus, the condition under which the particle does not leave the layer, if the added mass is taken into account, narrows.

Figure 2 shows the change in the longitudinal velocity difference. Designation: $u(m)$ – longitudinal velocity taking into account the added mass; u – without it.

Curves numbered 1, 2, 3 correspond to the values $A = 2$, $A = 3$, and, $A = 5$, respectively, for the values $v_0 = 0$, $u_0 = 0$, $y_0 = 0.3$ and $\rho = 1$.

It is not difficult to establish the expression (at $W = 0$)

$$\frac{|F_m|}{|F_h|} = \frac{1}{2\rho + 1} \frac{\sqrt{(y-u)^2 + (v-\lambda)^2}}{\sqrt{(y-u)^2 + v^2}},$$

Where $\lambda = a_* / \sqrt{A}$.

When $A \gg 1$ the influence of the added mass effect is commensurate with the Stokes forces. Calculation of the ratio of the Safman force to the Stokes force gives:

$$\max \frac{|F_S|}{|F_h|} = \lambda.$$

Hence it can be argued that at large values of the Reynolds number the Safman force prevails over the Stokes force

3.3 Particle in Poiseuille flow taking into account the Saffman force

Let us consider the flow of a particle in an upward flow taking into account the Saffman force. Fluid flow obeys Poiseuille's law. Then the system of dimensionless equations has the form.

$$\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{du}{dt} = \varphi D(u_l - u) - DW, \quad \frac{dv}{dt} = -\varphi Dv + a\sqrt{|12y|}(u_l - u).$$

(11)

Equations (11), under initial conditions similar to previous problems, are solved numerically.

In equation (11) $\phi = 1 + 0.15 \text{Re}^{0.687}$, Re is the particle Reynolds number

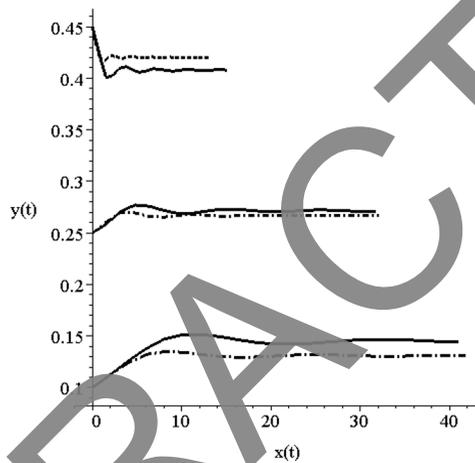


Fig.3. The influence of the effect of added mass on the trajectory of a particle ($u_0 = 1, v_0 = 0, \varepsilon = 0,01$).

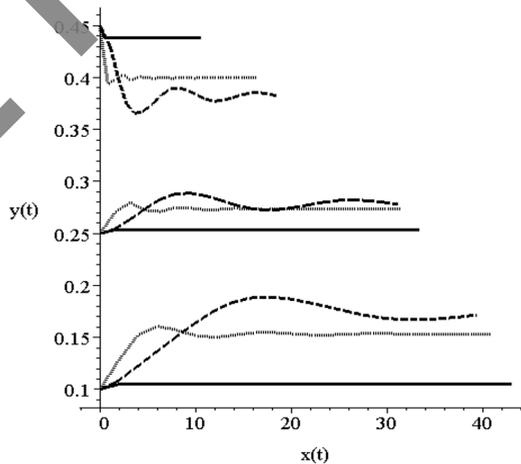


Fig.4. The influence of coefficient A on the particle trajectory ($u_0 = 1, v_0 = 0, \varepsilon = 0,01$). Solid lines $A=10$, dotted lines – $A=0.1$; dashed – $A=0.001$

Figure 3 shows the particle trajectories at various positions; the dotted lines correspond without taking into account ($A = 0.01$; $\rho = 0.5$, $W = 0$) the effect of added mass. Taking into account the added mass stimulates the Segre-Silberberg effect.

The influence of the coefficient A on the particle trajectory is presented in Fig. 4 ($\rho = 1$, $W = 0$, $\varepsilon = 0.01$). An increase in the Reynolds number will stimulate the Segre-Silberberg effect. The oscillatory nature of the behavior of the particle trajectory at the initial stage of the process is typical for small values of the parameter A , because A characterizes the inertia of the particle, which occurs at large values of the Reynolds number. It is easy to establish that the ratio of the forces due to the added mass to the hydrodynamic force is ($W = 0$)

$$\frac{|F_m|}{|F_h|} = \frac{1}{2\rho + 1} \frac{\sqrt{(u_l - u)^2 + (v - \lambda\sqrt{12}|y|(u_l - u)/\sqrt{\rho})^2}}{\sqrt{(u_l - u)^2 + v^2}}$$

This shows that, for $A \gg 1$, but finite, the force ratio is determined similarly to (9)

4 Conclusion

Thus, in shear flows, the effect of added mass significantly influences the behavior of the particle in this case, the influence of the effect of the added mass is more significant, the lower the particle density is compared to the density of the carrier flow.

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