E3S Web of Conferences **549**, 05006 (2024) https://doi.org/10.1051/e3sconf/202454905006
TransSiberia 2024

Analysis of the thermally stressed state of the heat-generating element in the form of an ellipsoid

Alexandr Kanareykin[1*]

[1]Sergo Ordzhonikidze Russian State University for Geological Prospecting, 117997, Miklouho-Maclay St. 23., Moscow, Russia

Abstract. The paper considers the thermally stressed state of fuel rods when their geometric shape changes. The temperature fields for the sphere and the ellipsoid are determined. It is shown that, while maintaining volumetric heat release, maximum stresses occur in fuel rods of circular cross-section. Similar results were obtained when changing the geometric shape of a spherical fuel element: all other things being equal, maximum thermal stresses occur in spherical fuel elements.

1 Introduction

Among the internal stresses, temperature stresses occupy a special place. Their appearance is caused by an inhomogeneous temperature deformation. The latter arises during various technological operations of manufacturing products, as well as during the operation of structural elements of nuclear technology [1].

Structural elements of new technology (nuclear, space, rocket) are operated under conditions of significant temperature gradients [2-10]. This leads to the appearance of an inhomogeneous temperature deformation in the volume of a solid. Such heterogeneity of temperature deformation is accompanied by the occurrence of thermal stresses, which are a special case of internal stresses. The physical nature of thermal stresses is quite transparent. Under conditions of inhomogeneous temperature, the more heated regions of the solid tend to expand, while the neighboring less heated regions do not allow this. Therefore, the first of them are in a state of compression, since the cold regions prevented them from expanding in accordance with the set temperature. The less heated areas of the solid are in a different position: hot areas tend to stretch them beyond the natural temperature expansion. In this regard, the colder areas of the material are in a state of tension [11-20].

The mechanism of the occurrence and development of thermal stresses in the structural elements of nuclear technology can be illustrated by the following example. In a long solid cylinder with volumetric heat release (a model of the fuel element of nuclear reactors), the temperature decreases from the center to the surface according to the parabolic law. The central regions of the solid cylinder are located at a higher temperature compared to the near-surface areas. Therefore, the outer regions of the heat-generating cylinder are subject

* Corresponding author: kanareykins@mail.ru

to tensile stresses, and the central ones to compressive stresses. Such a distribution of thermal stresses is essential when analyzing the performance of the system. If there are cracks or recesses of arbitrary geometric shape on the surface of a solid cylinder, then these macroscopic defects become thermal stress concentrators. In addition, corrosion processes occur very intensively in the vicinity of thermal stress concentrators when a heat-generating cylinder comes into contact with a chemically active coolant. If volumetric heat absorption occurs in a solid cylinder (for example, due to chemical reactions or phase transitions with heat absorption), then the sign of thermal stresses changes. Now the areas of the solid cylinder adjacent to the surface are in a state of compression. In this case, the course of diffusion processes, corrosion cracking, as well as crack advancement slows down. All other things being equal, such a system (compression stresses in the near-surface region of a solid cylinder) retains its operability for a longer time.

Fuel elements (fuel rods) are the main structural elements of nuclear power plants of various functional purposes. Volumetric heat release in fuel element materials (UO_2, UC, UN) is caused by the conversion of kinetic energy from fission fragments of heavy elements (for example, uranium) into thermal energy. The shape and geometric dimensions of fuel rods depend on the purpose of the nuclear reactor. The most common form of fuel rod is considered cylindrical. A long heat-generating cylinder (most often of circular cross-section) is placed in a metal shell that interacts with the coolant. Some design schemes use spherical fuel rods (nuclear fuel in a metal shell). The strength reliability of fuel rods is determined by the magnitude and nature of the distribution of temperature stresses due to volumetric heat release [21-26].

Fuel elements (fuel rods) are integral elements of the structures of nuclear power plants of various fuels in a metal shell form independent micro-valves. In case of emergency structural failure, individual microtubules retain their integrity and retain the fission products of nuclear reactions. Volumetric heat release in fuel rods of different geometric shapes is accompanied by the appearance of thermal stresses. Their determination is considered one of the main tasks in substantiating the operability of structural elements of nuclear technology. During operation, it is possible to change the shape of the fuel rods. This leads to a violation of the thermally stressed state. Therefore, it seems interesting to investigate the thermally stressed state of a spherical fuel rod when its geometric shape changes. Solving thermoelasticity problems for fuel rods of arbitrary geometric shape encounters certain mathematical difficulties. They are caused by the impossibility of obtaining a solution to the problem in the class of known functions or their combinations. In such cases, numerical methods (mathematical experiment) are used. However, for some geometric shapes, it is possible to obtain an accurate analytical solution to thermoelasticity problems. This is possible if the geometry of the surface is described by curves of the second order: circle, ellipse, ellipsoid. The purpose of this article is to conduct a similar study for a sphere and an ellipsoid [27].

2 Main Part

We will focus on the analysis of the thermally stressed state of the ellipsoid. The latter should be considered as a violation of the geometric shape of a spherical fuel element. At the same time, the volume and power of internal heat generation are preserved. This implies a connection between the radius of the sphere and the semi-axes of the ellipsoid.

As is known, the determination of thermal stresses in a heat-generating element is reduced to solving a planar thermoelasticity problem. The stress function F is found from the solution of the equations

$$\Delta\Delta F = \frac{\alpha E q_v}{\lambda(1-\nu)} \tag{1}$$

where λ is the coefficient of thermal conductivity of concrete, α is the coefficient of linear expansion, E is the Young's modulus, v is the Poisson's ratio.

To find the temperature distribution, it is necessary to solve the Poisson equation with volumetric heat dissipation q_v

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -\frac{q_v}{\lambda} \tag{2}$$

Let's imagine the desired temperature in the form

$$T = A\left(\frac{x^2+y^2+z^2}{R^2} + B\right) \tag{3}$$

where R is the radius of the sphere.

To determine the constant B, we use the boundary condition: for $a^2+b^2+c^2=R^2$ T=0, from where B=-1. Then the desired temperature (3) will take the form

$$T = A\left(\frac{x^2+y^2+z^2}{R^2} - 1\right) \tag{4}$$

To define the constant A, substitute expression (4) in (2), then

$$\frac{6A}{R^2} = -\frac{q_v}{\lambda} \tag{5}$$

where from

$$T = \frac{q_v R^2}{6\lambda}\left(1 - \frac{x^2+y^2+z^2}{R^2}\right) \tag{6}$$

The temperature of the ellipsoid is found in the form

$$T = C\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} + D\right) \tag{7}$$

where a, b and c are the corresponding semi—axes of the ellipsoid along the coordinate axes x, y and z.

To determine the constant D, we use the boundary condition: for $x^2/a^2+y^2/b^2+z^2/c^2=1$ T=0, from where D=-1. Then the desired temperature (7) will take the form

$$T = C\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1\right) \tag{8}$$

To define the constant C, substitute expression (8) in (2), then

$$2C\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) = -\frac{q_v}{\lambda} \tag{9}$$

then

$$T_e = \frac{q_v}{2\lambda}\frac{a^2b^2c^2}{a^2b^2+a^2c^2+b^2c^2}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}\right) \tag{10}$$

The temperature difference between the center of the region and its surface is equal for both cases

$$T_c = \frac{q_v R^2}{6\lambda} \tag{11}$$

$$T_e = \frac{q_v}{2\lambda}\frac{a^2b^2c^2}{a^2b^2+a^2c^2+b^2c^2} \tag{12}$$

Let's find the ratio of temperature differences for both cases

$$\frac{T_e}{T_c} = \frac{3}{R^2}\frac{a^2b^2c^2}{a^2b^2 + a^2c^2 + b^2c^2} \quad (13)$$

Let's use the connection between the radius of the sphere and the semi-axes of the ellipsoid R^3=abc, then expression (13) will take the form

$$\frac{T_e}{T_c} = \frac{\left(a^2b^2c^2\right)^{4/3}}{a^2b^2 + a^2c^2 + b^2c^2} \quad (14)$$

At a=b=c=R, the temperature distribution corresponds to the heat-generating sphere.

3 Conclusions

It can be shown that the ratio (14) takes on a maximum value at a=b=c, which corresponds to a spherical region. An arbitrary value of the ellipsoid axes (naturally, while maintaining the volume and power of heat generation) leads to a decrease in the value of this expression. This means that when the shape of the heat-generating sphere changes, the temperature difference between the centre and the surface decreases. This leads to a decrease in the level of thermal stresses that occur. Physically, this means that, all other things being equal, the level of thermal stresses in the heat-generating ellipsoid is reduced compared to those in the heat-generating sphere of equal volume.

References

1. Kanareikin A 2023 E3S Web of Conferences **460** 07024
2. Kanareikin A 2023 E3S Web of Conferences **402** 05024
3. Kanareikin A 2023 E3S Web of Conferences **402** 05025
4. Geelhood K J, Luscher W G, Beyer CE, Cuta J M 2011 FRAPTRAN: a computer code for the transient analysis of oxide fuel rods. Washington (DC): United States Nuclear Regulatory Commission
5. Thouvenin G, Ricaud J M, Michel B, Plancq D, Thevenin P 2006 ALCYONE: the PLEIADES fuel performance code dedicated to multidimensional PWR studies. Top Fuel 2006 International Meeting
6. Anders Logg, Kent-Andre Mardal, Garth N 2011 Wells Automated Solution of Differential Equations by the Finite Element Method. The FEniCS Book
7. Amosova E V, Shishkin A V 2017 Calculation of Distribution Dynamics of Inhomogeneous Temperature Field in Range of Fuel Elements by Using FreeFem++/. IOP Conference Series: Materials Science and Engi-neering **262** 7
8. Wilcox D C 2006 Turbulence modeling for CFD. DCW industries **522**
9. Shvetsov Yu E, Kouznetsov I A 2011 Analytical Studies on Thermal-Hydraulic Parameters of Fast Reactor Taking into Account Effect of Interwrapper Space. Journal of Materials Science and Engineering B **1 (7)** 938−946
10. Yamano H, Fujita S, Tobita Y, Sato I, Niwa H 2008 Development of a threedimensional CDA analysis code: SIMMER-IV and its first application to reactor case. Nuclear Engineering and Design **238** 66−73
11. Ramirez J C, Stan M, Cristea P 2006 Simulations of heat and oxygendiffusion in UO2 nuclear fuel rods. Journal of nuclear materials **359(3)** 174-184
12. Mihaila B 2009 Journal of Nuclear Materials **394(2)** 182-189
13. Hales J D 2015 Computational Materials Science **99** 290-297

14. Rashid Y, Dunham R, Montgomery R 2004 Fuel analysis and licensing code: FALCON MOD0. EPRI Report **1011308**
15. Philip B 2015 Journal of Computational Physics **286** 143-171
16. Geelhood K J, LuscherW G, Beyer CE, Cuta J M 2011 FRAPTRAN: a computer code for the transient analysis of oxide fuel rods. Washington (DC): United States Nuclear Regulatory Commission
17. Thouvenin G, Ricaud J M, Michel B, Plancq D, Thevenin P 2006 ALCYONE: the PLEIADES fuel performance code dedicated to multidimensional PWR studies. Top Fuel 2006 International Meeting
18. Semenovich, O.V. Modeling of Thermohydrodynamic Processes in Fuel Rod Assemblies of Water Cooled Reactors. MTA Energiatudományi Kutatóközpont, Budapest Part 2. P. 649– 665.
19. Kanareikin A 2023 E3S Web of Conferences **371** 05014
20. Kanareikin A 2023 E3S Web of Conferences **371** 03025
21. Kanareikin A 2023 E3S Web of Conferences **376** 01066
22. Kanareikin A 2023 E3S Web of Conferences **376** 01064
23. Kanareikin A 2023 E3S Web of Conferences **411** 01056
24. Kanareikin A I 2023. BIO Web of Conferences **71** 02018
25. Kanareikin A I 2022 AIP Conference Proceedings **2762** 020007
26. Kanareikin A I 2022 IOP Conference Series: Earth and Environmental Science **990** 012012
27. Kanareikin A I 2022 IOP Conference Series: Earth and Environmental Science **1045** 012070