Influence of stationary porous media on fluid flow

Umurdin Dalabaev[1*], *Malika* Ikramova[2], and *Nusratilla* Latipov[3]

[1]University of World Economy and Diplomacy, 100007 Tashkent, Uzbekistan
[2]Scientific Research Institute of Irrigation and Water Problems, Tashkent, Uzbekistan
[3]National University of Uzbekistan, Tashkent, Uzbekistan

Abstract. The work examines the flow of an incompressible fluid containing a porous medium. A porous body is formed through various mechanisms. To describe the flow, a single equation is used that describes the flow of fluid in the porous and free zone. The flow is simulated based on Rakhmatulin's two-speed model, in a laminar mode with zero speed of a discrete phase. The results of numerical simulation of the hydrodynamic features of a two-dimensional viscous flow are presented. The Kozeny-Karman relation is used as the force of interaction with the porous layer. Computational experimental methods are used to study the effects of non-uniformity of the fluid velocity field arising from a porous body. For the numerical implementation of the resulting equation, which is a generalization of the Navier-Stokes equation, a SIMPLE-like algorithm with corresponding generalizations was used. A single algorithm is used for the entire area, without identifying the free and porous zones.

1 Introduction

In many environmental and technical processes, porous media play a major role in fluid flow. Especially in channel flows, interactions of the fluid flow with various porous inclusions are observed: flow in the presence of sediment, with vegetation, with an uneven bottom, etc. The study of flow patterns in the combined region is one of the areas of research in the field of mechanics of multiphase media [1-9].

There are two approaches to modeling the flow of liquid or gas in a combined domain. Modeling using the first approach: in a free zone, flow is described using systems of Navier-Stokes/Stokes equations, and filtration through a saturated porous layer can be modeled using Darcy's law, which introduces an average fluid velocity in a volume of a porous medium sufficiently large relative to its size por. To represent the filtration of a free flow through a porous medium, it is necessary to introduce appropriate interboundary coupling conditions between the Navier–Stokes and Darcy (or Forchheimer) equations through their common interface. The Beavers–Joseph or Safman conditions are used as interboundary conditions [1–5,12,13].

* Corresponding author: udalabaev@mail.ru

Second approach: the flow is modeled for the entire region (the region of the free zone and the zone with a porous layer) by a single equation. In this approach, the difficulty of defining appropriate interface interaction conditions and solving different types of equations in two subdomains of the domain of interest can be avoided [10,11,14-18].

This work represents a further development of the application of the two-speed model [14-18] for new problems.

2 Mathematical model and numerical method

Let us consider an interpenetrating model that describes the flow of two-phase media [8-9], where the velocity of the discrete phase is neglected. Then the flow of the liquid phase is described by a system of equations (two-dimensional case):

$$fu\frac{\partial u}{\partial x}+fv\frac{\partial u}{\partial y}=-\frac{f}{\mathrm{Re}}\frac{\partial p}{\partial x}+\frac{4}{3\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial u}{\partial x}\right)+$$
$$+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial u}{\partial y}\right)-\frac{2}{3\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial v}{\partial x}\right)-Cu-\frac{\sin\alpha}{Fr}, \quad (1)$$

$$fu\frac{\partial v}{\partial x}+fv\frac{\partial v}{\partial y}=-\frac{f}{\mathrm{Re}}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial x}\right)+$$
$$+\frac{4}{3\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial u}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial y}\right)-\frac{2}{3\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial v}{\partial x}\right)-Cv.-\frac{\cos\alpha}{Fr}, \quad (2)$$

$$\frac{\partial(fu)}{\partial x}+\frac{\partial(fv)}{\partial y}=0 \quad (3)$$

Here, $u,v-$ longitudinal and transverse flow velocities, $p-$ pressure, f volume concentration, Re - Reynolds number, C - interaction coefficient. In equations (1-3) the parameters are dimensionless ($Re=UH\rho/\mu$), U is the average volume velocity, L is the characteristic scale, $\rho-$ is the density of the liquid, μ is the viscosity). The Kozeny-Karman relation was used for the interaction coefficient: $C=\frac{D^2}{\mathrm{Re}}\frac{(1-f)^2}{f^2}$, $D=\sqrt{\beta}H/d$, where d is the characteristic size of the porous medium. Fr is the Froude number, α is the angle of inclination to the horizon, β is the coefficient in the Kozeny-Karman formula.

Equations (1)-(3) make it possible to study flows both inside and outside a porous medium, since at f = 1 we obtain the Navier-Stokes equations for an incompressible fluid. Moreover, these equations are suitable for the entire region under consideration. To numerically solve (1)-(3), we use the control volume method [19,20] with a non-uniform mesh. The uneven mesh was built so that their concentrations formed around the porous medium. The SIPMLE algorithm [19] is generalized for equations (1)-(3).

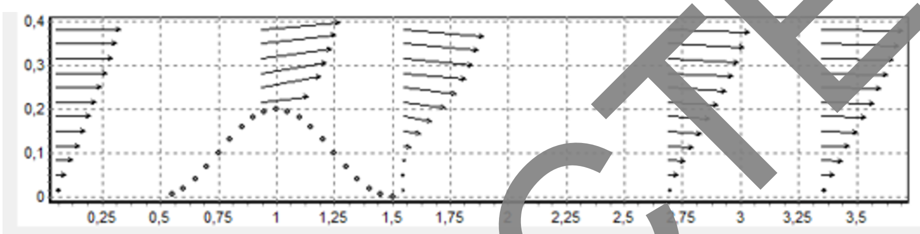
3 Results and discussion

3.1 Problem of flow around a porous barrier

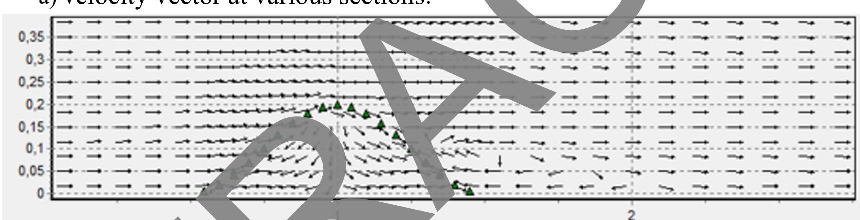
Let part of the open channel be filled with a porous layer. Equation (1)-(3) is considered in the area: $0\le x\le L,\ 0\le y\le 1.$ The x axis is directed along the lower wall of the channel, and the y axis is perpendicular to it. Upon entering the channel, a parabolic velocity profile is given. Part of the channel $0,5\le x\le 1.5,\ 0\le y\le g(x)$ is filled with a porous layer: $g(x) = \frac{h_0}{2}\left[1 + \cos\frac{2\pi}{L_0}\left(x - d - \frac{L_0}{2}\right)\right]$, here d is the distance from the entrance to the channel, h_0 is the fullness of the channel, L_0 is the width of the porous layer.

At the boundaries of the computational domain, a no-slip condition is specified on the solid wall; hydrostatic pressure and Poiseuille flow are specified at the channel inlet; and at the output - a soft boundary condition. On the free surface we use the boundary condition $\frac{\partial u}{\partial y} = 0,\ v = 0.$

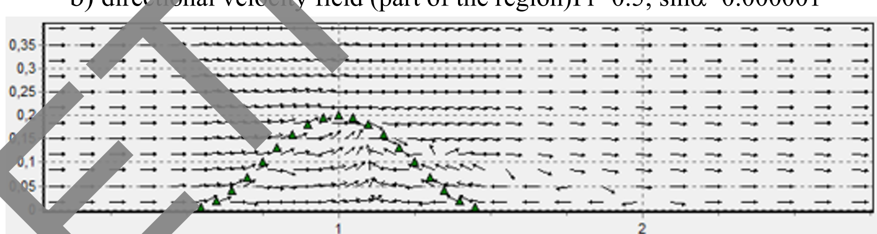
The main assumptions in the study: 1) the flow is laminar, 2) the porous body does not influence the level of the free surface.



a) velocity vector at various sections: $x = 0,05; 0,937; 1,551; 2.689 : 3.349.$



b) directional velocity field (part of the region)Fr=0.5; sinα=0.000001



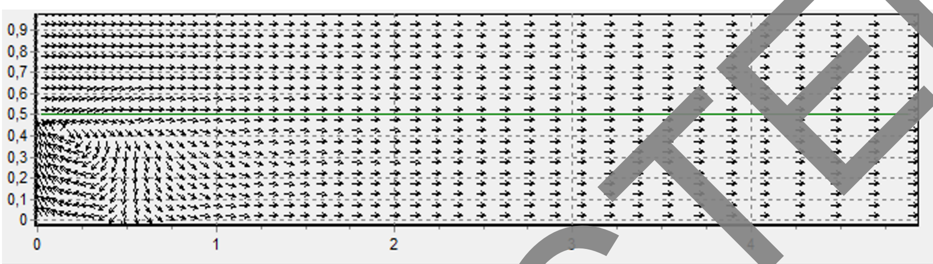
c) direction of the velocity field without taking into account gravity (part of the area)

Fig.1 Calculation parameters:: D=100; Re=500;f = 0,1; L=4; h_0=0.5; L_0=1; d=0.5;

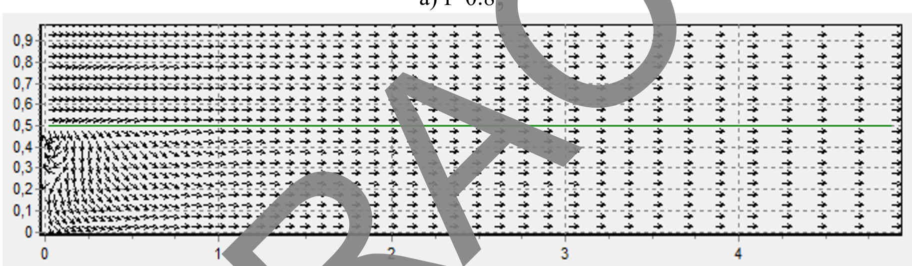
Figure 1 shows the velocity vector in various sections of the channel and the velocity field (the boundary of the porous layer is highlighted). From Fig. 1 it is clear that a small secondary flow is observed behind the layer. A comparison of Fig. 1 b) and c) shows the strong influence of gravity on the formation of the direction of the velocity field, especially in the porous region.

3.2 Flow in a horizontal porous layer

Let us consider a pressure flow in a flat channel, the lower half of which is filled with a porous medium. Equation (1)-(3) is considered in the area: $0 \le x \le L,\ 0 \le y \le 1.$ The direction of the axes is the same as in the previous problem. At the entrance to a part of the channel, $x = 0,\ 0.5 \le y \le 1$ a parabolic velocity profile with a unit flow rate is given, and at the boundary there are no-slip conditions. The no-slip conditions are met on the lower and upper walls. On the right boundary, we assume that the boundary conditions are satisfied similarly to the previous problem.



a) f=0.8



b)f=0.5

Fig.2 Velocity field at different porosity values: and with parameters: D=100; Re=100

Figure 3 shows the nature of the change in transverse velocity at the interface at different porosity values: f=0.9; 0.8; 0.5; 0.2. The bottom graph corresponds to a porosity of 0.9. On the one hand, an increase in porosity will lead to an increase in seepage through the interphase boundary, and on the other hand, to an increase in the seepage zone.

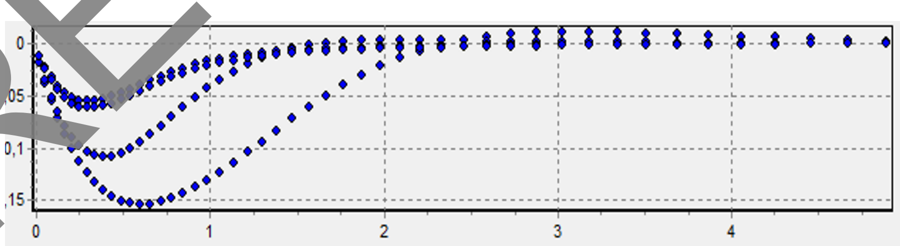


Fig. 3 Transverse velocity profiles at the interface at different porosity values with parameters: D=100; Re=100.

4 Conclusion

Using the proposed model, which is obtained on the basis of a two-speed interpenetrating model, it is possible to obtain a characteristic of the hydrodynamic fields of joint flow with porous inclusions. The extended Navier-Stokes model can be used to study hydrodynamic structures for flow in complex regions using a single equation without explicitly identifying the interregional boundary. Within the limits of applicability, the Koseny-Karman coefficient can be used as a drag force in porous structures.

References

1. Donald A. Nield, Adrian Bejan, Convection in Porous Media, Springer International Publishing AG (2017)
2. Matthias Ehrhardt, An Introduction to Fluid-Porous Interface Coupling, Progress in computational physics, **2:** 3–12, (2000).
3. Fernando A. Moralesa, Ralph E. Showalterb, Journal of Mathematical Analysis and Applications **452** 1332–1358, (2017)
4. Ochoa-Tapia, J.A., Whitaker, S., J. Porous Media **1,** 201–217, (1998)
5. Yu, P., T.S., Zeng, Y. Low, Y. T., Int. J. Numer. Meth. Fluids **53**, 1755-1775, https://doi.org/10.1002/fld.1383
6. Gol'dshtik M.A. Protsessy perenosa v zernistom sloye. Novosibirsk: Institut teplofiziki SO AN SSSR, 1984.
7. Kirillov V.A., Kuz'min V.A., P'yanov V.I., Khanayev V.M. *O profile skorosti v nepodvizhnom zernistom sloye* /DAN SSSR **245**, 1. 159-162 (1979)
8. Faizullaev, Dzharulla F, Laminar Motion of Multiphase Media in Conduits, , Springer US, (1969)
9. Nigmatulin, R. I., Fundamentals of the mechanics of heterogeneous media, Moscow, Izdatel'stvo Nauka, (1978)
10. F.Cimolin, M.Discacciati, Applied Numerical Mathematics **72**, 205-224 (2013) https://doi.org/10.1016/j.apnum.2013.07.001
11. F. J. Vald_es-Parada1 and D. Lasseux. Phys. Fluids **33**, 022106 (2021); doi: 10.1063/5.0036812
12. P.V. Bulat, K.N. Volkov Incompressible flows in channels with fluid/porous regions. Industrial and Systems Engineering, 2020 Vol. **34** No. 3, pp. 283 – 300, https://doi.org/10.1504/IJISE.2020.105739
13. Potapov I.I., Snigur K.S., Tsoy G.I. O modelirovanii obtekaniya pologikh peschanykh dyun turbulentnym potokom // Vychislitel'nyye tekhnologii. 2019. T. **24**, . 6. 99–107. DOI: 10.25743/ICT.2019.24.6.012
14. Dalaboyev U. Struktura potoka pri techenii cherez nepodvizhnyy zernistyy sloy Inzhenerno-fizicheskiy zhurnal, Minsk., **70,** №3, 1997, pp. 390-394
15. Dalabaev U. Turkish Journal of Physics, **21**, 5, 649-356 (1997)
16. *Numerical investigation of the character of the lift on a cylindrical particle in poiseuille flow of a plane channel.* Journal of Engineering Physics and Thermophysics. **84**, 6 (2011)
17. Dalabaev Umurdin. AIP Conference Proceedings. 2021. **2365**. 060015, https://doi.org/10.1063/5.0057568

18. Umurdin Dalabaev and Nusratilla Latipov. E3S Web of Conferences **431**, 04017 (2023) ITSE-2023
19. Patankar S. Numerical Heat Transfer and fluid Flow, ISBN 9780891165224 Published January 1, by CRC Press, (1980)
20. Blazek J. Computational Fluid Dynamics: Principles and Applications, Elsevier, pp 470, (2001)