Modeling of stationary mode for pipeline operation during hot oil pumping

D.D. Fugarov[1*], *O.A.* Purchina[1], and *A.D.* Purchina[1]

[1] Don State Technical University, 344002 Rostov-on-Don, Russia

Abstract. The objective of this work is to build and analyze a mathematical model of steady-state operation of a pipeline that conducts hot oil pumping in a particular section. Mathematical modeling of pipeline operation during hot pumping makes it possible to determine the pumping flow rate, temperature and pressure distribution along the length of the pipeline. The object and subject of the study is a horizontal pipeline route between two pumping stations with preheating of oil at the beginning of the section. Within the framework of this work, the dependence of oil viscosity on temperature was determined, and the following graphs were obtained: graph of oil temperature distribution in the considered pipeline section, graph of pressure distribution in the pipeline in the considered section, graph of hydraulic resistance coefficient distribution according to the Leibenzon model, graph of pressure distribution in the pipeline in the considered section according to the combined model of Shukhov and Leibenzon. The graphs obtained allowed us to assert that taking into account the real rheological properties of oil leads to a change in the law of pressure distribution along the pipeline route and to a significant change in the length of the linear section of the pipeline, where a significant pressure drop occurs.

1 Introduction

Pipeline transportation of oil is the most profitable and frequently used mode of transportation of large volumes of oil. The length of trunk oil pipelines in Russia is more than 70 thousand kilometers, through which about 90% of all produced oil is transported. Pumping high-viscosity oil due to its high viscosity leads to high friction losses and is economically inexpedient without the use of special methods [1]. In addition to this is the fact that half of the reserves of high-viscosity oil is located in the north, where the temperature in winter reaches -30 ° C, and in the northern regions up to -60 ° C. This affects the properties of oil and oil properties of the oil. This affects oil properties and imposes restrictions on the pumping process [2].

The main method of pipeline transportation of high-viscosity oils is heated transportation. Such pipelines pumping heated oil are called "hot" pipelines. Heating is carried out at thermal stations in steam and fire heaters. Heat stations are usually located together with pumping stations for ease of maintenance. The technology of pumping preheated oil is very energy-intensive, especially in the northern regions of Russia, where the transported liquid cools

* Corresponding author: ddf_1@mail.ru

down quickly due to low external temperature, and it is necessary to place more heating stations along the length of the pipeline [3].

The heated oil moving through the pipeline gradually releases its heat through the pipe walls and insulation to the environment, which causes the oil temperature to gradually decrease [4].

The heat flow from the oil to the ground passes through the above-mentioned thermal layer, through the metal of the pipe wall Δ_{m}, through the insulation layer Δ_{iz}, and then through a certain warmed up soil column l_r. The thickness of the heated ground with the pipeline laid in it is determined by the Forchheimer formula:

$$l_r = \frac{D_{iz}}{2}\ln[\frac{2h}{D_{iz}} + \sqrt{\left(\frac{2h}{D_{iz}}\right)^2 - 1}] \tag{1}$$

where D_{iz} defines pipeline diameter with insulation;
h is pipe axis embedment depth.

Heat flux through the laminar near-wall layer per unit time from oil to the pipe wall of length dx with inner diameter d is determined by Newton's formula:

$$q = \alpha_1 \pi d (T_m - T) dx \tag{2}$$

here α_1 is a heat transfer coefficient;
T_{m} – wall metal temperature.

Heat transfer coefficient characterizing thermal resistance of the laminar near-wall layer, which is determined by the ratio of the thermal conductivity coefficient of the fluid λ_j, and thickness of this layer is determined by the formula:

$$\alpha_1 = \frac{\lambda_j}{\Delta_l}. \tag{3}$$

The thickness of the thermal near-wall layer of liquid in the pipeline is determined by the experimental dependence according to the formula:

$$\Delta_l = \frac{d}{Nu} \tag{4}$$

where $\mathrm{Nu} = 0.021\mathrm{Re}^{0.8} \cdot \mathrm{Pr}^{0.43}$.

In (4) the dimensionless Nusselt, Prandtl and Reynolds numbers are determined by the following formulas:

$$Nu = \frac{\alpha_1 d}{\lambda_j} \tag{5}$$

$$Pr = \frac{v \rho C_v}{\lambda_j} \tag{6}$$

$$Re = \frac{ud}{\nu} \tag{7}$$

where C_v is specific heat capacity of liquid at constant volume;
u – average flow velocity;
ρ – liquid density;
n – fluid kinematic viscosity.

Heat transfer coefficient α_4 which characterizes the thermal resistance of the ground, is equal to the ratio of the heat conductivity coefficient of the ground λ_r to the thickness of the heated soil by the formula:

$$\alpha_4 = \frac{\lambda_r}{l_r} \tag{8}$$

Heat losses from the underground pipe compartment can be determined using (3), but it is better to use the same Newton's formula and heat transfer coefficient K [5]. Heat flux to the media q_{oc} through the lateral surface of the pipeline in some arbitrary cross-section can be determined by the formula:

$$q_{amb} = \pi dK(T_r - T)dx \tag{9}$$

here d is the pipe inner diameter;

K – heat transfer coefficient from oil to ground;

T_r – ambient ground temperature;

T – oil temperature in the section under consideration.

The inverse value of the heat transfer coefficient K is equal to the sum of the inverse values of the heat transfer coefficients of the thermal wall layer α_1, pipe metal α_2, insulation layer α_3, and the layer of the warmed part of the ground α_4 according to the formula:

$$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} + \frac{1}{\alpha_4} \tag{10}$$

The value of the heat transfer coefficient of each layer is determined by a relationship similar to (3) and (8). The heat transfer coefficient K has the same dimension as the heat transfer coefficient α – $W/m^2 \cdot$ deg.

2 Temperature distribution along the transported oil stream

Under the steady-state hydraulic and thermal regime of oil pumping, the differential equation of heat flux balance in the pipe compartment of length dx is as follows:

$$GC_v dT = -\pi DK(T - T_{oc})dx \tag{11}$$

here G – fluid mass flow rate;

T – fluid temperature in the section with coordinate x;

T_{oc} – ambient temperature, e.g. ground temperature.

The left part of the last equation defines the convective component due to heat inflow into the considered pipe section due to fluid flow [6]. The right part determines heat losses through the lateral surface of the pipeline and insulation to the environment. This equation does not take into account the heat released in the flow due to the work of internal friction forces and due to the latent heat of paraffin crystallization [7].

The result of integration of equation (11) at a previously known temperature T_{init} at the beginning of the pipeline looks like this:

$$T(x) = T_{oc} + (T_{init} - T_{oc})\, exp\left(-\frac{\pi Kd}{\rho Q C_v}x\right) \tag{12}$$

here T_{init} – initial liquid temperature at the beginning of the pipeline section;

T_{amb} – ambient temperature;

ρ – liquid density;

d – pipe inner diameter;

K – heat transfer coefficient from the liquid flowing in the pipeline to the environment;

Q − fluid volume flow;

The fluid volume flow rate is determined by the formula:

$$Q = \frac{\pi d^2 u}{4} \tag{13}$$

Hence, we obtain the value of the liquid temperature T_k at the end of the pipeline [8]. If the required temperature T_k is set at the end of the calculation section, then the required temperature of oil heating at the beginning of the pipeline T_{init} can be determined from the formula:

$$T_{init} = T_{amb} + (T_k - T_{amb})\exp\left(-\frac{\pi K d}{\rho Q C_v}x\right) \tag{14}$$

Heat transfer through the pipe walls or reduces the (at $T > \mathrm{T_{amb}}$) or raises (at $T < \mathrm{T_{amb}}$**)** temperature of the transported medium [9]. Whereas dissipation of mechanical energy always leads to an increase in the temperature of the transported fluid.

Law of temperature change along the flow, taking into account dissipation of mechanical energy of the fluid flow moving under the action of hydraulic gradient $i = const$ has the form:

$$\frac{T(x) - T_{amb} - T_*}{T_{init} - T_{amb} - T_*} = exp\left(-\frac{\pi K d}{\rho Q C_v}x\right) \tag{15}$$

where $\mathrm{T_*}$ is constant value, which has the dimension of temperature:

$$T_* = \frac{\rho g Q i_0}{\pi D K}$$

here i_0 – hydraulic gradient of pumping at the investigated pipeline section.

The temperature of the pumped liquid, if good thermal insulation is selected, will remain constant and equal to the initial temperature $\mathrm{T_H}$ throughout the entire pipeline section [10]. To ensure such an effect, it is necessary that the heat transfer coefficient K satisfies the following condition:

$$K = \frac{gi\rho\nu\omega}{\pi D(T_{init} - T_{amb})} \tag{16}$$

Such technique (16) is used for oil transportation through the oil pipeline in Alaska (USA) [11]. Oil is pumped through this pipeline without heating due to good thermal insulation of the pipe.

3 Hydraulic calculation of a "hot" oil pipeline

To calculate modes of oil pumping with preheating (hot pumping) use Shukhov's equation (12) and the equation of head balance for a section of oil pipeline, which takes into account the variability of coefficient λ along the length of the pipeline:

$$(p_p - p_k) + (z_{init} - z_k) + H(Q) = \int \lambda(Re, \xi)dx\frac{u^2}{2g}d \tag{17}$$

where p_p – head pressure upstream of the pumping station;
p_k – end of section head;
z_{init}, z_k – elevations at the beginning of the section and at the end;
$H(Q)$ – pump station characteristic.

The viscosity of oil depends on its temperature $\nu(T)$ [12]. In this case, the oil viscosity is not a constant value, but varies along the flow as the temperature (cooling) of the pumped oil changes [13]. For this reason, in this problem the Reynolds number and the coefficient λ are functions of the coordinate x.

The latter circumstance is taken into account when integrating the function λ in the right part of expression (17) along the length of the considered section under the following boundary conditions: the value of x from 0 to L.

$$(p_p - p_k) + (z_{init} - z_k) + H(Q) = \lambda_{ef}\frac{Lu^2}{d2g} \tag{18}$$

In this case λ_{ef} is the effective coefficient of hydraulic resistance determined by the formula:

$$\lambda_{ef} = \lambda_{amb} \frac{1}{m} [Ei(-b) - Ei(-b * exp^{-m})] \tag{19}$$

here

$$b = \frac{k}{4} (T_{init} - T_{amb}) \tag{20}$$

$$m = \frac{4KL}{\rho C_v u D} \tag{21}$$

$$\lambda_{amb} = \frac{0.3164}{\sqrt[4]{ud/v_{amb}}} \tag{22}$$

here v_{amb} – oil viscosity at ambient temperature T_{amb}.

Symbol $Ei(z)$ denotes the Euler function:

$$Ei(z) = \int_{-\infty}^{z} \frac{e^{\eta}}{\eta} d\eta \tag{23}$$

Special tables have been compiled for this function; some values of this function are given in Tab. 1.

Table 1. Some values of the Euler function.

Z	-1.0	-0.8	-.06	-0.4	-0.2	-0.1	-0.05
Ei	-0.22	-0.31	-0.45	-0.70	-1.22	-1.82	-2.47

4 Modeling of oil pipeline operation under steady-state hot pumping mode

Oil is pumped along the horizontal section of the pipeline. Pumping is carried out with heating by two centrifugal pumps of NM 3600-230 type connected in series [14]. It is required to determine the temperature distribution and pumping flow rate at the following data:

- pipeline outer diameter $D = 720\ mm$;
- pipe wall thickness $\delta = 10\ mm$;
- pump hydraulic characteristic $H = 273 - 0.125 \cdot 10^{-4} Q^2\ m$,
- fluid volume flow Q, m^3/h;
- section length $L = 120\ km$;
- oil density $\rho = 870\ km/m^3$;
- specific heat of oil $C_v = 2000\ J/(kg \cdot {}^\circ C)$;
- oil viscosity $v_1 = 0.000005\ m^2/s$ at the temperature $T_1 = 50^\circ C$ and $v_2 = 0.00004\ m^2/s$ at the temperature $T_2 = 20^\circ C$;
- initial oil temperature $T_0 = 50^\circ C$;
- ambient temperature $T_{init} = 10^\circ C$;
- section average heat transfer coefficient $K = 3.5\ W/(m^2 \cdot {}^\circ C)$.

To build the mathematical model, we will assume the fluid to be homogeneous, incompressible, and the flow regime to be stationary, which obeys the head balance equation (17), taking into account the hydraulic characteristics of the two working pumps:

$$H(Q) = \lambda_{ef} \frac{Lu^2}{d2g} \tag{24}$$

where u is pumping speed.

Since rheological properties of oil depend on temperature, kinematic viscosity values are more often used for calculation models [15]. In case of lack of experimental data, the

Reynolds-Filonov formula is most often used. Let us determine the dependence of viscosity on temperature taking into account the initial data. In this case we obtain:

$$40 = 5 \cdot exp^{-k(20-50)} \tag{25}$$

hence $k \cong 0.0693\ 1/^{o}\ C$.

Fig.1 shows the dependence of oil viscosity on temperature taking into account the obtained coefficient *k*.

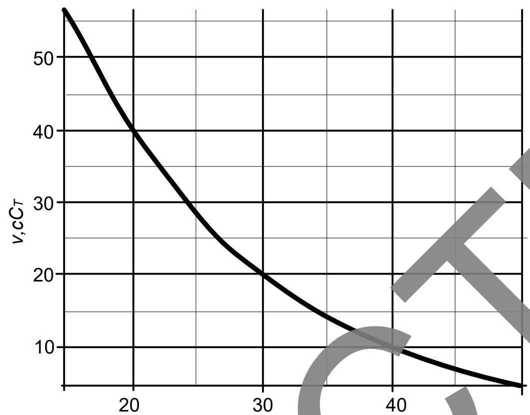

Fig. 1. Graph of viscosity dependence on temperature

Equation (24) taking into account the initial data takes the form:

$$2 \cdot [273 - 0.125 \cdot 10^{-4} Q^2] = \lambda_{ef} \cdot \frac{120000}{0.7} \cdot \frac{u^2}{2 \cdot 9.81} \tag{26}$$

Since the fluid volume flow rate is determined according to formula (13), then, taking into account (25) and (26), we obtain:

$$546 = u^2 \cdot \left(8737.4 \cdot \lambda_{ef} + 47.94\right) \tag{27}$$

We solve the obtained equation by iteration method (successive approximations) [16].

As a first approximation let us assume $\lambda_{ef}^{(1)} = 0.02$. Then from equation (27) we find the fluid flow velocity: $u^{(1)} = 1.566\ m/s$. Next, we check the validity of the assumption made.

Viscosity at ambient temperature

$T = 10^{\circ}\ C$:

$$\nu_{10} = 5 \cdot exp[-0.0693 \cdot (10 - 50)] = 0.00007995\ m^2/s.$$

Accordingly, the coefficient of hydraulic resistance at a given temperature will be according to the formula (22):

$$\lambda_{10} = \frac{0.3164}{\sqrt[4]{1.556 \cdot \frac{0.7}{79.95 \cdot 10^{-6}}}} \approx 0.0292$$

From formulas (20) and (21) we calculate the value of the corresponding coefficients:

$$b = \frac{1}{4} \cdot 0.0693 \cdot (50 - 10) = 0.693$$

$$m = \frac{4 \cdot 3.5 \cdot 120000}{1.566 \cdot 0.7 \cdot 870 \cdot 2000} \cong 0.881$$

The effective coefficient of hydraulic resistance determined by formula (19), taking into account the found coefficients and values of the Euler function (23), will be as follows:

$$\lambda_{ef} = 0.0292 \cdot \frac{1}{0.881} \cdot [-0.379 - (-0.939)] \cong 0.0186$$

Since between the accepted $\lambda_{ef} = 0.02$ and the obtained $\lambda_{ef} = 0.0186$ there is a difference between the values of effective hydraulic resistance, let us make a second approximation [17].

As a second approximation let us assume $\lambda_{ef}^{(2)} = 0.0186$. Then from equation (3.4) we find fluid velocity: $u^{(2)} = 1.611\ m/s$. Then we check the validity of the made assumption.

Viscosity at ambient temperature $T = 10°\ C$:

$$v_{10} = 0.00007995\ m^2/s.$$

Accordingly, the coefficient of hydraulic resistance at a given temperature will be according to the formula (22):

$$\lambda_{10} = \frac{0.3164}{\sqrt[4]{1.611 \cdot \frac{0.7}{79.95 \cdot 10^{-6}}}} \cong 0.0290$$

From formulas (20) and (21) we calculate the value of the corresponding coefficients:

$$b = \frac{1}{4} \cdot 0.0693 \cdot (50 - 10) = 0.693$$

$$m = \frac{4 \cdot 3.5 \cdot 120000}{1.611 \cdot 0.7 \cdot 870 \cdot 2000} \cong 0.856$$

The effective coefficient of hydraulic resistance, determined by formula (19), taking into account the found coefficients and values of the Euler function, will be as follows:

$$\lambda_{ef} = 0.0290 \cdot \frac{1}{0.856} \cdot [-0.379 - (-0.921)] \cong 0.0184$$

Since for the accepted $\lambda_{ef} = 0.0186$ and the obtained $\lambda_{ef} = 0.0186$ values of effective hydraulic resistance obtained a good match, the process of successive approximations ends. Thus, oil flow velocity and fluid volume flow rate are respectively equal to $u \cong 1.611\ m/s$ and $Q = 2231\ m^3/h$.

Oil temperature distribution along the pipeline design section using Shukhov's formula (12):

$$T(x) = 10 + (50 - 10) \cdot \exp(-0.856) \cong 27°\ C$$

Fig. 2 shows the graph of oil temperature distribution according to Shukhov's formula.

We calculate the pressure distribution in the pipeline at the considered section [18]. Taking into account the data of the set problem and taking into account the formulas (18) and (20-22) we obtain:

$$P = 2 \cdot (273 - 0.125 \cdot 10^{-4} \cdot Q^2) \cdot \rho \cdot g - \frac{x\lambda u^2 \rho}{2d}$$

$$P = 3.597036736 \cdot 10^6 - 29.99820316x$$

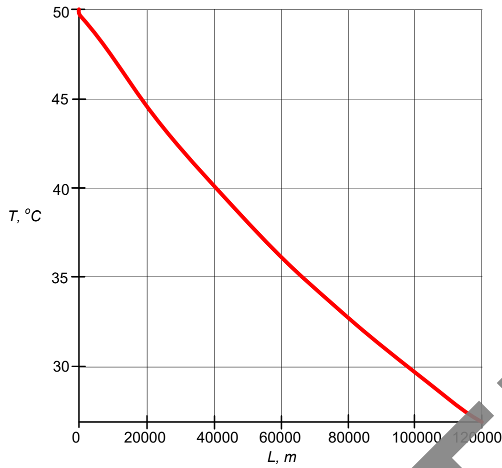

Fig. 2. Graph of oil temperature distribution at the considered pipeline section

The graph of pressure distribution in the pipeline at the considered section is shown in Fig. 3.

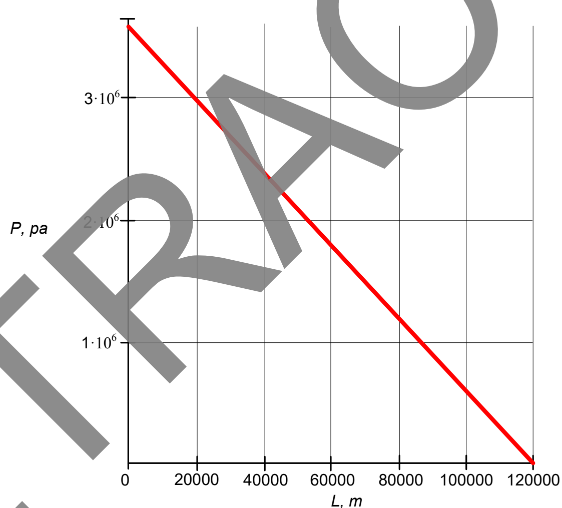

Fig. 3. Graph of pressure distribution in the pipeline at the considered section

Further refinement of non-isothermal oil flow in the pipeline includes taking into account the real physical properties of oil, which include nonlinear dependence of oil viscosity on temperature and its compressibility [19]. On this basis, further refinement of the mathematical model of non-isothermal oil flow consists in the calculation of oil flow taking into account the properties of its viscosity and compressibility [20].

The density of oil and oil products as a function of temperature is determined by the formula:

$$(T) = \rho_{20}\big(1 + \xi(T_{standard} - T)\big) \tag{28}$$

where $T_{standard} = 293.16\ K$ – is standard oil temperature;
ρ_{20} – oil density at standard conditions;
ξ – coefficient of thermal expansion of oil.

Table 2 with values of the coefficient of thermal expansion of domestic oils ξ is as follows.

Table 2. Values of the coefficient of thermal expansion of different oils

Density $\rho_{20}, (kg/m^3)$	Coefficient $\xi, 1/K$
700–719	0.001225
720–739	0.001183
740–759	0.001118
760–779	0.001054
780–799	0.000995
800–819	0.000937
820–839	0.000882
840–859	0.000831
860–879	0.000782

The density of oil and oil products is also determined by a generalized formula, which takes into account the simultaneous dependence of the density of oil (oil products) on pressure and temperature:

$$\rho(P,T) = \rho_{20}\left(1 + \xi(T_{standard} - T) + \frac{P-P_{atm}}{K}\right) \quad (29)$$

where $P_{atm} = 0.101325\ MPa$ – is standard oil pressure;

K – volume elastic compression coefficient.

Average values of modulus K for gasoline are $\sim 1000\ MPa$, for oil, kerosene and diesel fuel $\sim 1500\ MPa$.

To calculate the coefficient of hydraulic resistance, we will use the Leibenzon model relation, the formula of which is written as follows:

$$\lambda = A/Re^m \quad (30)$$

where A is a constant model parameter equal to 0.3764;

m – coefficient, taking values from 0 to 1 depending on the nature of flow in the pipeline.

The Reynolds number in terms of oil volume flow rate is expressed as follows:

$$R_e = \frac{4Q}{\pi d v} \quad (31)$$

For this problem the following value of the coefficient m is obtained:

m=0.155

Taking into account the calculations and formulas (30), (31) we obtain the distribution of the hydraulic resistance coefficient according to the Leibenzon model [21]:

$$\lambda = \frac{0.01909413375}{\left(\frac{1}{e^{-0.0693T+3.4650}}\right)^{0.155}}$$

Fig. 4 shows a graph of the distribution of the hydraulic resistance coefficient according to the Leibenzon model.

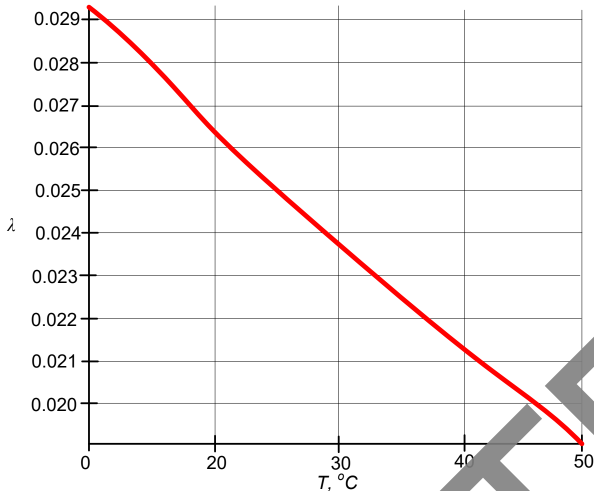

Fig. 4. Distributions of hydraulic resistance coefficient by Leibenzon model

Let's find the pressure distribution in the pipeline at the considered section according to the combined model of Shukhov and Leibenzon [22]. Taking into account the previously performed calculations and formulas: (12), (18), (20-22), (30), (31) we obtain:

$$P = 3.597036736 * 10^6 - \frac{30.79514534x}{\left(\frac{1}{e^{-0.0693T+3.4650}}\right)^{0.155}}$$

The graph of pressure distribution in the pipeline at the considered section according to the combined model of Shukhov and Leibenzon is presented in Fig. 5.

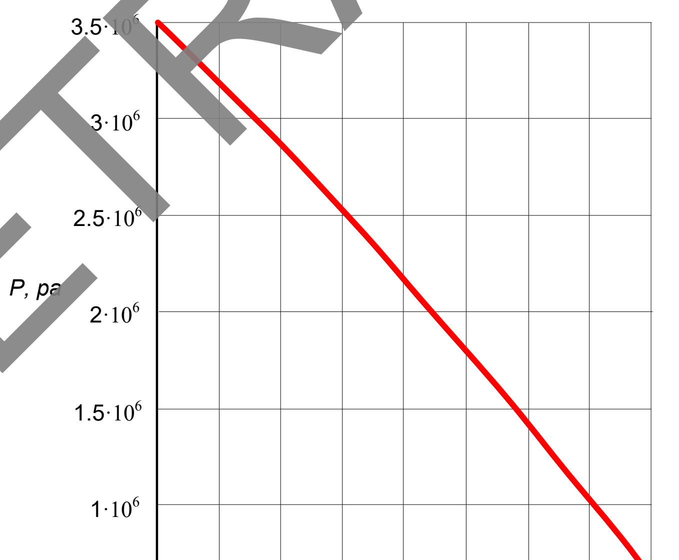

Fig. 5. Pressure distributions in the pipeline at the considered section according to the combined model

Thus, the pressure drop along the pipeline route differs from the linear distribution law [23].

5 Discussion

The presented graphs show that taking into account the real rheological properties of oil, in particular, taking into account the dependence of oil on temperature, leads to a change in the law of pressure distribution along the pipeline route and to a significant change in the length of the linear section of the pipeline (less than 120 km, which was found for normative calculations), where a significant pressure drop occurs [24].

The graph shows that taking into account the rheological properties of oil leads to the specification of the real length of the linear section of the pipeline (length of about 80 km) between the pumping stations, which are located on the horizontal section of the pipeline route [25].

Thus, when designing linear sections of oil pipeline it is necessary to take into account the rheological properties of the pumped product, because the pressure (and temperature) distribution along the pipeline route changes significantly and depends on the rheological properties of oil and oil products [26].

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