Numerical method for determining pressure distribution in tube cross-flow heat exchangers

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Abstract. This paper presents a numerical method for determining the pressure distribution along the fluid flow path in a cross-current high-temperature heat exchanger. The pressure drop across the superheater was determined using the momentum conservation equation, which was solved by the finite difference method. Local pressure losses at tube elbows were also taken into account. A new formula for calculating friction factor on rough tubes' inner surface is proposed. A straightforward model for the friction factor in tubes with a rough inner surface for Reynolds numbers in the interval 3000 - 10^8 has been proposed. The pressure distribution in the last stage of the live steam superheater in a 900 MWe supercritical boiler was calculated.

1 Introduction

Proper design of boiler steam superheaters is challenging. This is evidenced by the number of emergency boiler shutdowns caused by superheater failures [1]. About 40% of emergency boiler stops are caused by overheating of superheater tube material, which consequently causes superheater tube failures. Accurate determination of the steam pressure changes in the superheaters is essential for selecting the pressure after the feedwater pump in the power unit. The pressure distribution along the steam flow path is needed in thermal calculations of the individual superheater stages, especially in supercritical steam boilers. If the steam pressure is greater than the critical pressure, then the specific heat of the steam at constant pressure depends not only on the steam temperature but also on the pressure. In order to correctly determine the course of the steam temperature in the individual superheater stages, it is necessary to know the pressure distribution along the flue gas flow path [2-3].

2 Calculation method of pressure variations in the live steam superheater

The momentum conservation equation for one-dimensional steady-state fluid flow in a channel has the following form [4]

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$$\frac{\partial p}{\partial s} = -\rho\left[w\frac{\partial w}{\partial s} + g\sin\varphi + \frac{\xi}{d_{in}}\frac{w|w|}{2}\right], \tag{1}$$

where: w – velocity of the fluid, m/s; p – static steam pressure, Pa; s – curvilinear coordinate directed according to the direction of steam flow, m; ρ – fluid density, kg/m^3; g – acceleration due to gravity, m/s^2; φ – angle of the tube to the horizontal, rad; ξ – Darcy friction factor, -; d_{in} – tube's inner diameter, m.

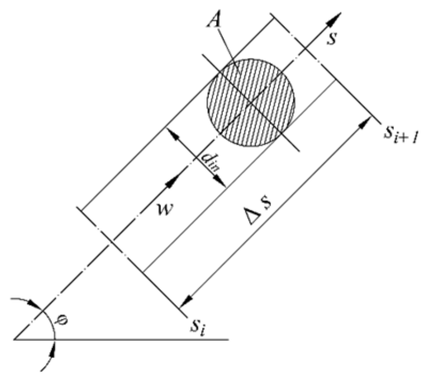

Fig. 1. Finite difference grid for a steam superheater tube sloped to the horizontal at an angle φ.

On the right side of Eq. (1), in square brackets, three terms represent pressure changes due to variations in fluid velocity, gravity and friction. The difference equation of the momentum conservation for the finite volume shown in Figure 1 has the following form:

$$\frac{p_{i+1} - p_i}{\Delta s_i} = -\rho_i\left[w_i\frac{w_{i+1} - w_i}{\Delta s_i} + g\sin\varphi_i + \frac{\xi_i}{d_{in}}\frac{w_i|w_i|}{2}\right], \tag{2}$$

where the symbol $\Delta s_i = s_{i+1} - s_i$ denotes the i-th control volume's length.

The lower subscript i denotes the inlet to the finite volume, and (i+1) refers to the outlet. Solving Eq. (2) for p_{i+1} gives:

$$p_{i+1} = p_i - \rho_i\,\Delta s_i\left[w_i\frac{w_{i+1} - w_i}{\Delta s_i} + g\sin\varphi_i + \frac{\xi_i}{d_{in}}\frac{w_i|w_i|}{2}\right] \tag{3}$$

Considering that at steady-state $\dot{m} = \text{const}$ throughout the entire length of the superheater, velocity w_i is determined using the following equation

$$w_i = \dot{m}/(\rho_i\,A_i) \tag{4}$$

The pressure value behind the individual superheater passes should be reduced by the local pressure losses occurring in two 90° elbows between the individual superheater passes. Local flow resistance is assumed to occur at the interface of two adjacent finite volumes.

In this paper, the pressure distribution in a cross-current superheater with two passes, shown in Fig. 2, was determined. The number of finite volumes along the length of one superheater pass is n. The pressure $p_{2,1}$ at the inlet to the n-th finite volume in the second pass is equal to the pressure $p_{1,n+1}$ at the outlet of the first pass minus the local pressure drops at the two 90° elbows and the pressure drop at the straight section of the tube connecting the two elbows.

The average temperatures of the flue gas at the inlet to the control volume in the first pass are T_g', and at the outlet $T_{g,i}''$. The outlet temperatures $T_{g,i}''$ represent the inlet temperatures of the control volumes in the second pass. The symbols $T_{g,i}'''$ denote the average flue gas temperature at the exit of the i-th control volume in the second pass.

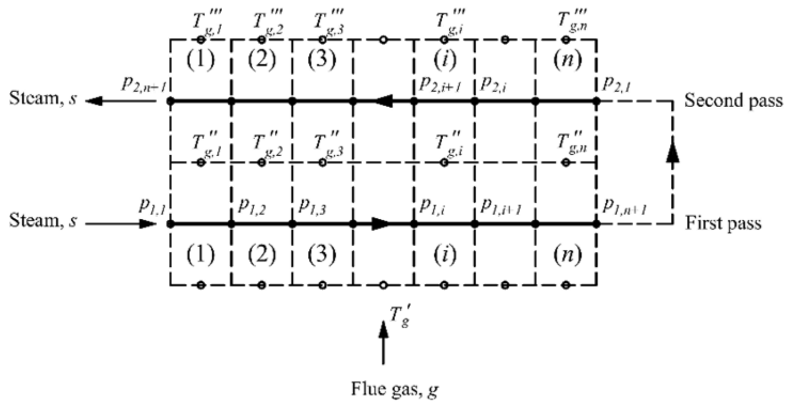

Fig. 2. Finite volume grid for the two-pass superheater of the live steam.

In superheater calculations, the steam pressure $p_{1,1}$ and temperature $T_{s,1}$ are assumed to be known. The symbols $p_{1,i}$ and $p_{1,i+1}$ denote the inlet and outlet pressures at the i-th finite volume in the first pass. The corresponding quantities in the second pass are denoted by $p_{2,i}$ and $p_{2,i+1}$, respectively.

Elbow pressure drops between the first and second pass are calculated as follows:

$$p_{1,n+1} - p_{2,1} = \sum_{j=1}^{2} \zeta_j \frac{\rho_j w_j^2}{2} \tag{5}$$

The symbol ζ_j stands for the pressure loss coefficient at a 90° elbow, and the lower subscript j indicates the inlet of the first and second bend, respectively.

After considering that the pressure $p_{1,n+1}$ is known from the first tube row (first pass) calculation, the pressure $p_{2,1}$ at the inlet to the second tube row (second pass) is calculated.

Values for local ζ_j flow resistance coefficients can be found in many books on heat exchanger design [5-7]. The friction factor ξ_i for tubes with a rough inner surface is given by Moody's diagram [5-7]. The implicit Colebrook-White formula [4-6] is commonly used to calculate the friction factor in superheater tubes [4]:

$$\frac{1}{\sqrt{\xi}} = -2\log\left(\frac{2.51}{\mathrm{Re}\sqrt{\xi}} + \frac{\varepsilon}{3.7}\right), \tag{6}$$

where: $\mathrm{Re} = w\, d_{in}/\nu$ – Reynolds number, -; ν - kinematic viscosity, m²/s; $\varepsilon = R_a/d_{in}$ - relative roughness of the tube surface, -; R_a - the average absolute surface roughness, m; ξ - friction factor, -.

In order to determine the ξ value using Eq. (6), a nonlinear algebraic equation with one unknown must be solved iteratively, which increases computer calculation time. For this reason, an explicit form of the relationship for calculating the friction factor ξ is proposed:

$$\xi = \left\{ \left[\log\left(0.392645\,\mathrm{Re}^{1.2776}\right) \right]^{-6.915062} + \frac{1}{69.6364} \left[\log\left(\frac{3.7}{\varepsilon}\right) \right]^{-6.121769} \right\}^{0.326879} \tag{7}$$

Equation (7) is valid in the following ranges of Re and ε:

$$3 \cdot 10^3 \le \mathrm{Re} \le 10^8 \qquad 10^{-5} \le \varepsilon \le 5 \cdot 10^{-2} \tag{8}$$

The coefficient of determination for the relationship (7) determined by the least squares method is $r^2 = 0.9996$.

The friction coefficient ξ as a function of Reynolds number Re and relative roughness ε is shown in Figure 3.

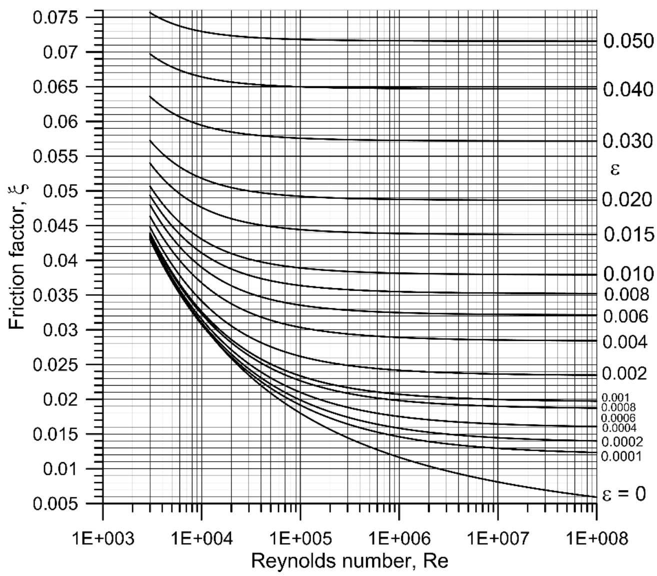

Fig. 3. Friction factor ξ in circular tubes for rough tubes versus Reynolds number Re and relative roughness ε.

The relationship (7) was determined using the method of least squares so that the sum of the squares of the differences of the friction factors determined from the Colebrook-White formula (6) and the friction factors calculated from formula (7) was the smallest. The sum of squares included several dozen data points for the given different values of Re and ε. For high Reynolds number values, when $\mathrm{Re} \ge \mathrm{Re}_{limit}$, the friction factor ξ depends only on the relative roughness ε, which is given by [7]:

$$\xi_g = \frac{1}{4\left[\log\left(\frac{\varepsilon}{3.7}\right)\right]^2} \qquad \mathrm{Re} \ge \mathrm{Re}_{limit}\,. \tag{9}$$

The limiting value of the Reynolds number Re_{limit} at which the value of the ξ coefficient reaches a value ξ_{limit} is given by :

$$\xi_{\text{limit}} = \left(1 + \frac{e}{100}\right)\xi_g \, . \tag{10}$$

The symbol e, % shows how much the actual value of the friction factor ξ is larger than the value of the ξ_g. The following relations were obtained for calculating the Reynolds number limits Re_{limit}:

$$\mathrm{Re}_{limit,0.5\%} = 10^{(3.508588 - 0.43375 \ln\varepsilon)} \tag{11}$$

$$\mathrm{Re}_{limit,1\%} = 10^{(3.207446 - 0.43321 \ln\varepsilon)} \tag{12}$$

The limiting Reynolds number values $\mathrm{Re}_{limit,0.5\%}$ and $\mathrm{Re}_{limit,1\%}$ are shown in Figure 4.

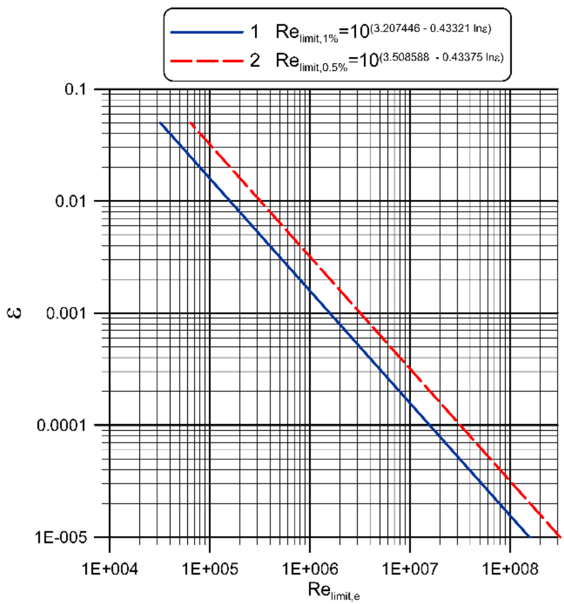

Fig. 4. Limiting Reynolds number $\mathrm{Re}_{limit,e}$ for a given value of relative roughness ε when $e = 0.5\%$ or $e = 1\%$.

The method proposed was applied to determine the pressure distribution in the two-pass supercritical steam superheater. If $\mathrm{Re} \geq \mathrm{Re}_{limit,0.5\%}$ then the ξ value determined by solving Eq. (6) is no greater than 1.005 ξ_g. If $\mathrm{Re} \geq \mathrm{Re}_{limit,1\%}$ is assumed, then the ξ value is no greater than 1.01 ξ_g.

3 Example of superheater calculation in a supercritical boiler

The procedure proposed in the paper was applied to determine the steam pressure changes along the steam path in the final live steam superheater. The electrical output of the supercritical unit is 900 MWe. The grid of finite volumes is shown in Figure 2. The superheater design and thermal calculations of the analyzed superheater are discussed in more detail in [4].

The mass flow rates of steam and flue gas were 643.5 kg/s and 1002 kg/s, respectively. Inlet steam and flue gas temperatures were 517.5 °C and 961.1 °C, respectively. In each of the 22 panels, steam flows in parallel through 38 tubes. The longitudinal and transverse pitch of the tube spacing is 75 mm and 880 mm, respectively. The outer diameter of tubes with a wall thickness of 8.8 mm is 42.4 mm. The relative roughness of the inner tube surface was $e = 0.064/(42.4 - 2 \cdot 8.8) = 0.00258$. The superheater is made of high-alloy steel. The steam pressure, flue gas and steam temperature distribution calculations were carried out for n=20. The physical properties of the steam were calculated according to the standard IAPWS-IF97 [8].

The results of calculations of the steam pressure across the superheater length are depicted in Figure 5.

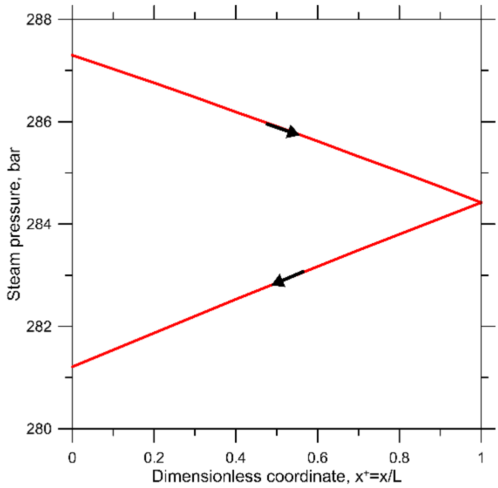

Fig. 5. Steam pressure in two-pass cross-flow live steam superheater at rated load.

It can be seen from the analysis of the results shown in Figure 5 that the pressure in the superheater varies almost linearly. The total pressure drop in the final superheater is 6.09 bar. The resulting pressure drop value agrees with the measured pressure drop.

4 Conclusion

The method presented in this paper for calculating pressure distribution in steam superheater tubes can be used to calculate pressure drop across superheater stages. A complete procedure for calculating the friction factor in tubes has been proposed. The calculation of the friction factor is carried out iteration-free, thus significantly reducing the computer calculation time of the superheater.

References

1. D. N. French, Metallurgical Failures in Fossil Fired Boilers, 2nd ed. Hoboken: John Wiley & Sons (1993)
2. G. F. Hewitt, Handbook of Heat Exchanger Design. New York: Begell House Inc (1992)
3. R. J. Martin, Thermal Systems Design. Fundamentals and Projects, 2nd ed. Hoboken: John Wiley & Sons (2022)
4. K. Węglarz, D. Taler, J. Taler, M. Marcinkowski, General numerical method for hydraulic and thermal modelling of the steam superheaters, Energy **291**, 130371 (2024)

5. R. K. Shah, D. Sekulić, Fundamentals of heat exchanger design. Hoboken: John Wiley & Sons (2003)
6. S. Kakac, H. Liu, A. Pramuanjaroenkij, Heat Exchangers. Selection, Rating, and Thermal Design, Boca-Raton: CRC Press (2020)
7. P. Stephan, editor, VDI Heat Atlas, 2nd ed. Berlin Heidelberg: Springer (2010)
8. Steam Properties for Industrial Use Based on IAPWS-IF97. Professional Version. New York: ASME (1999)