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Fast algorithm for obtaining mathematical relations for optimizing the operating mode of an electric power system

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Abstract. The article proposes a method for composing mathematical expressions by which it is possible to reduce the time for determining the parameters of flexible alternating current transmission system devices in the operating mode of the electric power system. The problem of quasi-stationarity of the steady-state regime is considered, taking into account the rapidity of its change. Criteria for the optimality of the regime are also put forward. The problem of optimizing the power system mode is solved by recording various sets of target functions. The article provides in detail an algorithm for obtaining objective functions based on the theory of sensitivity of electrical circuits. A method is also proposed for determining the success of choosing functional relationships and the optimal location of devices.

1 Introduction

Optimization of the established modes and control of the operation of the electric power system (EPS) using modern information technologies and methods [1, 2] is an urgent direction in the development of modern energy. The important tasks are to control node voltages and to reduce power losses in the power grid [3, 4]. To solve them, methods of regulating the mode of operation of the EPS are used. One of the methods is the load control method, which is widely used in the elimination of modes with reduced frequency and power shortage in power plants. The latter, in turn, is accompanied by a decrease in voltage at the terminals of the generators. This mode is eliminated by means of automation devices [5]. But despite the fact that the method is very effective in solving the problems described above, it is associated with significant economic losses associated with limiting consumer power supply.

Another, more resource-saving method is the use of generator excitation control. It helps to maintain the voltage on the tires of generator switchgear in case of its decrease due to various reasons (most often the occurrence of asymmetry in the EPS [6, 7]), but it is not effective in solving problems of optimizing the normal steady-state mode of the EPS, since it helps to regulate only the voltage of one node and some of its surroundings. By analogy with the previous method, it can be used to eliminate abnormal and emergency modes.

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To optimize the mode of operation of the power grid, flexible alternating current transmission system (FACTS) devices are used, which can change the capacity of power transmission lines (PTL), by changing their own reactance, and the voltage of nodes in the power grid [8], in order to reduce power losses in the network. FACTS, with their correct location and appropriate regulation, can be used to optimize the operating modes of the EPS, therefore, the article focuses on this technical solution. Accordingly, the task of determining the best position of FACTS and the task of determining their parameters that ensure the optimal mode are both meaningful and relevant.

For further reference, the term "optimal mode" requires a strict definition. In [9], the following fairly general definition of the optimal mode is proposed, as a mode in which power supply to consumers is provided at minimal cost, which is acceptable and satisfies the conditions of reliability and quality of electricity.

In this work, in accordance with the criteria put forward in the definition, [10] are established:

- deviation of voltages in the nodes of the EPS relative to the nominal value, as an indicator of the quality of electricity;
- power losses in PTL, as an indicator of the cost of electricity transmission;
- PTL capacity, as an indicator of the reliability of power supply.

Mathematical modelling methods are widely used to optimize the operation of the EPS according to these criteria [8]. However, their solution is associated with the repeated calculation of the mode of the EPS model at different values of the FACTS device parameters. The calculation of the mode is an iterative solution of a system of equations, including nonlinear ones [11]. In this connection, the duration of calculating device parameters increases, and the time spent is an order of magnitude higher than the time of maintaining a certain steady-state mode in the EPS. As a result, it is impossible to adjust the steady-state mode directly in real time using FACTS devices. Also, the solution of such a model requires large computing power, which affects economic efficiency. The improvement of computer technologies in the foreseeable future is not able to reduce the time difference between the determination of satisfactory parameters and the change in the steady-state mode of the EPS.

A large number of works in this field are based on the use of various optimization algorithms designed to speed up the determination of parameters [12-14]. The article proposes to focus on the description of the operation of the EPS using the functional relationship between the characteristics and parameters of the regime. The characteristics are the measured values characterizing the mode of operation of the EPS: currents, voltages, etc. The values characterizing the elements of the EPS are taken as parameters: active and reactive resistances, regulated using FACTS devices. This approach will simplify the recording of the target function. A genetic algorithm is proposed as an optimization method, which, copes with the task, despite its simplicity [15]. To further increase the speed of parameter determination, it is possible to use more advanced optimization algorithms. The purpose of the work is to compile an objective function based on the functional relations of the EPS that meets the criteria:

- an accurate description of the task of regulating the EPS regime;
- ease of calculation, which does not require large computing power;
- relevance in time and convenience for perception of the output data.

In the first part of the article the theoretical base of the article is considered. The task statement and initial assumptions of the model are resulted. Mathematical calculations showing the process of obtaining target functions based on the theory of sensitivity [16] have been made. In the second part of the article the results of work in the form of graphs and tables are given. Also, the analysis of the results is made. The conclusion summarizes the advantages of the algorithm and the disadvantages of the model that was used. The article also provides assumptions about methods of elimination of model defects, which can serve as a basis for more in-depth and thorough analysis of the task.

2 Theoretical foundations of the work

2.1 Setting the task

The article aims at creating mathematical expressions linking the FACTS parameters of devices and EPS characteristics. The characteristics are selected for the calculation:

- voltage deviation relative to the nominal value in EPS nodes;
- active power loss in PTL;
- deviations of the active power passing through the PTL from the permissible value of the throughput.

After that, use these ratios as an objective function to optimize the EPS operation regime. The result of the optimization will be the parameters of the FACTS devices with a minimum of the characteristics described above. To solve the optimization problem, it is proposed to use the optimal placement of devices presented in [8]. The placement of devices is shown in Figure 1.

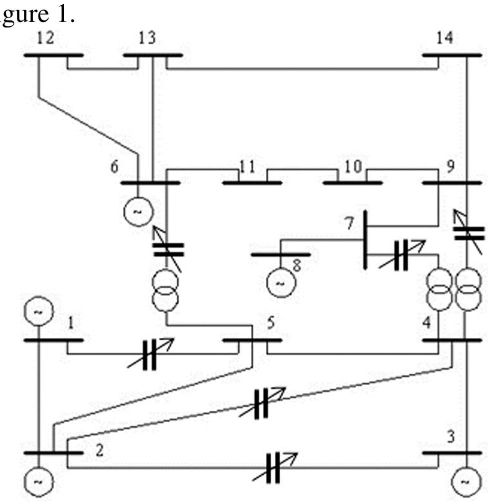

Fig. 1. Optimal placement of FACTS devices on the IEEE test circuit consisting of 14 nodes.

It is proposed to construct a set of Pareto, to graphically represent the result obtained. An important task is to determine the sensitivity of the function to the FACTS parameters of the devices. Such an analysis will determine the suitability of functions for solving such problems.

It is important to note that the article adopts the following assumption. The regime is accepted stationary, that is, the value of characteristics of EPS does not change in time, but is accepted by some constant. Generally speaking, in reality this is not possible, as in any established regime there are fluctuations of characteristics in the vicinity of the values accepted for it. These values are valid, they are chosen to characterize the established regimes in this work.

2.2 Mathematical relations

The theory of sensitivity of electrical circuits is used to record mathematical relations [16]. It is widely known that the provisions of this theory apply to EPS, taking into account the assumptions of the theory of electrical networks. In this study, the provisions of the bilinear theorem (BT) are applied. BT allows us to obtain a relationship between current or voltage on any element of an electric circuit depending on the resistance of any element of this circuit. At the same time, as it was proved in [17], it helps to do this quickly and with high accuracy, even taking into account the experimental error. The expression number (1) is a BT entry for the dependence of the voltage of some EPS node $\dot{U}_n$ on the reactance of PTL x_k.

$$\dot{U}_n = \frac{\dot{a}_{n,k} + \dot{b}_{n,k} x_k}{1 + \dot{c}_{n,k} x_k}., \tag{1}$$

where n is the node number in EPS, k is the PTL number, $\dot{a}_{n,k}$, $\dot{b}_{n,k}$, $\dot{c}_{n,k}$ are the complex constants.

Obviously, the function is approximative and to determine it, it is necessary to calculate the value of the constants $\dot{a}_{n,k}$, $\dot{b}_{n,k}$, $\dot{c}_{n,k}$. To do this, it is necessary to solve the system of equations presented in expression (2). To record the system, "experimental" voltage measurements are used at a given reactance value. Thus, the number of voltages corresponding to the PTL reactance required for solving the system is obtained. Thus, it is possible to determine the voltage at any node from any PTL reactance [3, 4].

$$\dot{U}_{n,m} = \frac{\dot{a}_{n,k} + \dot{b}_{n,k} x_{k,m}}{1 + \dot{c}_{n,k} x_{k,m}}, \ m = \overline{1,3}. \tag{2}$$

where m is the equation number corresponding to the experiment number.

As a result of solving the problem of optimal placement of FACTS devices, the problem of insufficient coverage arises. Namely, not all nodes of the power system are covered by the influence of devices. Therefore, the two-parameter BT form is used to increase sensitivity. The algorithm using two-parameter BT is described in detail in [8]. Here, for clarity, we will give only the general form of writing the corresponding functional ratio (3).

$$\dot{U}_n = \frac{a^{(0)}_{n,k_1',k_2'} + a^{(1)}_{n,k_1',k_2'} x_{k_1'} + a^{(2)}_{n,k_1',k_2'} x_{k_2'} + a^{(3)}_{n,k_1',k_2'} x_{k_1'} x_{k_2'}}{1 + c^{(1)}_{n,k_1',k_2'} x_{k_1'} + c^{(2)}_{n,k_1',k_2'} x_{k_2'} + c^{(3)}_{n,k_1',k_2'} x_{k_1'} x_{k_2'}}. \tag{3}$$

The expression is similar to (2), but here the strokes that appeared at the k index are introduced to distinguish the parameters of different devices. $a^{(t)}_{n,k_1',k_2'}, t = \overline{0,3}$ and $c^{(t)}_{n,k_1',k_2'}, t = \overline{1,3}$ are the corresponding complex constants. Obviously, it is possible to write a system of equations similar to (2) and determine the unknown constants of the two-parameter BT. The principle of composing equations is similar to that described above and is discussed in detail in [8]. In the future, indexes $\dot{U}^{(1)}_n$ and $\dot{U}^{(2)}_n$ will be used to distinguish between classical and two-parameter BT, respectively.

To solve the problem, we introduce the concept of voltage deviation. Its mathematical expression is presented in (4). Theoretically, voltage deviation is defined as the relative deviation of the effective voltage value from the nominal one [16]. Since in this formulation there are no restrictions on the method of determining the voltage at the node, there are no indexes in the record of expression (4) indicating the type of BT used. But due to the fact that there are several regulating devices and they simultaneously exert their influence on each of the nodes, the serial number of the device, denoted by k , is added to the voltage indexing. In this expression, the k index also implies pairs of devices used for two-parameter BT, adding the pair as one device to the set of devices used for one-parameter BT.

$$D_n = \frac{\sum_k \left(\left| \dot{U}_{n,k} \right| - U_0 \right)}{U_0}. \tag{4}$$

The next characteristic that requires mathematical modeling is the active power. BT is also used to record these expressions. According to the theory of sensitivity and the theory of electrical circuits, the active power is not related to the parameters bilinearly [16]. However, the total power is related, since it is the product of the voltage $\dot{U}$ by the complex conjugate current $\hat{I}$ [4]. According to sensitivity theory, the expression for current has the same type of notation as for voltage [16]. The output of the expression is given under the number (5). The resulting expression for the power leaving or entering node n , depending on the device number k , is given under the number (6).

$$\dot{S} = \dot{U}\hat{I} = \frac{\dot{a}_U + \dot{b}_U x}{1 + \dot{c}_U x} \cdot conj\left(\frac{a_I + b_I x}{1 + c_I x} \right) = \frac{\dot{a}_U \hat{a}_I + (\dot{a}_U \hat{b}_I + \dot{b}_U \hat{a}_I)x + \dot{b}_U \hat{b}_I x^2}{1 + (\dot{c}_U + \hat{c}_I)x + \dot{c}_U \hat{c}_I x^2}. \tag{5}$$

From the point of view of the theory of electric power networks, the power of the beginning $\dot{S}^{(b)}$ and the power of the end $\dot{S}^{(e)}$ are distinguished [16]. Therefore, the power expression for one PTL has two corresponding forms at once, given under the numbers (6) and (7). The resulting expression for the power leaving or entering node n , depending on the device number k , is given under the number (6).

$$\dot{S}_{l,k}^{(b)} = \frac{\dot{a}_{l,k}^{(b)} + \dot{b}_{l,k}^{(b)} x_k + \dot{c}_{l,k}^{(b)} x_k^2}{1 + \dot{d}_{l,k}^{(b)} x_k + \dot{e}_{l,k}^{(b)} x_k^2}. \tag{6}$$

$$\dot{S}_{l,k}^{(e)} = \frac{\dot{a}_{l,k}^{(e)} + \dot{b}_{l,k}^{(e)} x_k + \dot{c}_{l,k}^{(e)} x_k^2}{1 + \dot{d}_{l,k}^{(e)} x_k + \dot{e}_{l,k}^{(e)} x_k^2}., \tag{7}$$

where the l index denotes the PTL number in which the power is determined; $\dot{a}_{l,k}^{(e)}$, $\dot{b}_{l,k}^{(e)}$, $\dot{c}_{l,k}^{(e)}$, $\dot{d}_{l,k}^{(e)}$, $\dot{e}_{l,k}^{(e)}$ are the corresponding complex constants; indices b and e are pointers of the beginning of PTL and the end, respectively.

Since it is impossible to calculate the mutual influence for power losses for this functional using the overlay method, a unique algorithm based on sensitivity theory was organized. It can be seen from the coverage algorithm that devices only affect certain nodes. Based on this, we will take into account the power loss in the line from the device that affects the start and end node. Thus, it is possible to match each index l with a pair of index n from the set of all nodes. Therefore, the index k is replaced in the formula (8), since the device number is determined by the node number. For nodes controlled by two devices, a power expression record is used depending on the two devices (8). The same expression can be written for the power of the PTL end.

$$\dot{S}_{l,n} = \frac{\dot{a}_{l,n}^{(0)} + \dot{a}_{l,n}^{(1)} x_{k_1'} + \dot{a}_{l,n}^{(2)} x_{k_2'} + \dot{a}_{l,n}^{(3)} x_{k_1'} x_{k_2'} + \dot{a}_{l,n}^{(4)} x_{k_1'}^2 + \dot{a}_{l,n}^{(5)} x_{k_2'}^2 + \dot{a}_{l,n}^{(6)} x_{k_1'}^2 x_{k_2'} + \dot{a}_{l,n}^{(7)} x_{k_1'} x_{k_2'}^2 + \dot{a}_{l,n}^{(8)} x_{k_1'}^2 x_{k_2'}^2}{1 + \dot{c}_{l,n}^{(1)} x_{k_1'} + \dot{c}_{l,n}^{(2)} x_{k_2'} + \dot{c}_{l,n}^{(3)} x_{k_1'} x_{k_2'} + \dot{c}_{l,n}^{(4)} x_{k_1'}^2 + \dot{c}_{l,n}^{(5)} x_{k_2'}^2 + \dot{c}_{l,n}^{(6)} x_{k_1'}^2 x_{k_2'} + \dot{c}_{l,n}^{(7)} x_{k_1'} x_{k_2'}^2 + \dot{c}_{l,n}^{(8)} x_{k_1'}^2 x_{k_2'}^2}. \quad (8)$$

To determine the loss of active power in the line, it is necessary to take the real part of the difference between the complex capacities of the beginning and the end (9). For the convenience of further designations, the indices of the beginning and end of the line have been moved to the node index.

$$\Delta P_l = real\left(\dot{S}_{l,n^{(b)}} - \dot{S}_{l,n^{(e)}}\right). \quad (9)$$

In the future, it is proposed to use indexing similar to voltage deviation to distinguish between one-parameter and two-parameter functions. Accordingly, $\dot{S}_{l,n^{(b)}}^{(1)}$ and $\dot{S}_{l,n^{(b)}}^{(2)}$. For the power of the end of the line, the entry is similar. To compare with the line capacity, it is necessary to write an expression for the active power passing through the line (10). The starting power will be used. Which, accordingly, also has two forms of recording, depending on the form of recording power $P_l^{(1)}$ and $P_l^{(2)}$:

$$\dot{S}_{l,k}^{(b)} = \frac{\dot{a}_{l,k}^{(b)} + \dot{b}_{l,k}^{(b)} x_k + \dot{c}_{l,k}^{(b)} x_k^2}{1 + \dot{d}_{l,k}^{(b)} x_k + \dot{e}_{l,k}^{(b)} x_k^2}. \quad (10)$$

2.3 Getting target functions

To write the objective function, the following task is proposed: it is necessary to increase the voltage in one of the EPS nodes, while in all other nodes it should not deviate more than the value allowed by the standard. In addition, to optimize the mode, it is necessary to reduce the power loss in PTL. Based on this, a system of three objective functions is obtained (11). When composing objective functions, it is necessary to introduce some elements of set theory:

– the set N includes all node numbers, N' includes the numbers of only those nodes that are controlled by two devices at once (the coverage was performed by a two-parameter BT). It is implied that $N' \subseteq N$;

– the set K includes the device FACTS numbers, the set K' includes the device numbers involved in the two-parameter BT. It is implied that $K' \subseteq K$;

– the set L includes the numbers of all lines.

Due to the fact that the sets N and K are mutually unambiguously mapped into each other, we will not specify the set K to simplify writing.

$$
\begin{cases}
f_1 = \begin{bmatrix} \dfrac{1}{D_n^{(1)}}, n = \tilde{n} \in N \setminus N', \\ \dfrac{1}{D_n^{(2)} + D_n^{(1)}}, n = \tilde{n} \in N'; \end{bmatrix} \\
f_2 = \begin{bmatrix} \sqrt{\dfrac{\sum_n \left(D_n^{(1)}\right)^2}{\max(n) - 1}}, \forall n \neq \tilde{n};\ n \in N \setminus N', \\ \sqrt{\dfrac{\sum_n \left(D_n^{(2)} + D_n^{(1)}\right)^2}{\max(n) - 1}}, \forall n \neq \tilde{n};\ n \in N'; \end{bmatrix} \\
f_3 = \sqrt{\dfrac{\sum_l \Delta P_l^2}{\max(l)}}, \forall l \in L;\ \Delta P_l = real \begin{bmatrix} \dot{S}_{l,n^{(b)}}^{(1)} - \dot{S}_{l,n^{(e)}}^{(1)};\ n^{(b)}, n^{(e)} \in N \setminus N' \\ \dot{S}_{l,n^{(b)}}^{(1)} - \dot{S}_{l,n^{(e)}}^{(2)};\ n^{(b)} \in N \setminus N', n^{(e)} \in N' \\ \dot{S}_{l,n^{(b)}}^{(2)} - \dot{S}_{l,n^{(e)}}^{(1)};\ n^{(b)} \in N', n^{(e)} \in N \setminus N' \\ \dot{S}_{l,n^{(b)}}^{(2)} - \dot{S}_{l,n^{(e)}}^{(2)};\ n^{(b)} \in N', n^{(e)} \in N' \end{bmatrix}.
\end{cases}
\tag{11}
$$

Further analysis is performed in accordance with this objective function. In addition to the optimization task, the task of analyzing the sensitivity of these functions to parameters is also set. Let's give the sensitivity functions to a certain parameter x_k for a one-parameter function and $x_{k_1'}$ for a two-parameter function. Expression for the first function number (12).

$$
f_1' = \begin{bmatrix}
-\dfrac{1}{U_0\left(D_n^{(1)}\right)^2}\left(\dfrac{\dot{b}_{n,k}}{1 + \dot{c}_{n,k}x_{k,m}} - \dfrac{\dot{c}_{n,k}\left(\dot{a}_{n,k} + \dot{b}_{n,k}x_{k,m}\right)}{1 + \dot{c}_{n,k}x_{k,m}}\right), n = \tilde{n} \in N \setminus N', \\
-\dfrac{1}{U_0\left(D_n^{(2)} + D_n^{(1)}\right)^2}\left(\dfrac{\dot{a}_{n,k_1',k_2'}^{(1)} + \dot{a}_{n,k_1',k_2'}^{(3)}x_{k_2'}}{1 + \dot{c}_{n,k_1',k_2'}^{(1)}x_{k_1'} + \dot{c}_{n,k_1',k_2'}^{(2)}x_{k_2'} + \dot{c}_{n,k_1',k_2'}^{(3)}x_{k_1'}x_{k_2'}} - \right. \\
\left. -\dfrac{\left(\dot{a}_{n,k_1',k_2'}^{(0)} + \dot{a}_{n,k_1',k_2'}^{(1)}x_{k_1'} + \dot{a}_{n,k_1',k_2'}^{(2)}x_{k_2'} + \dot{a}_{n,k_1',k_2'}^{(3)}x_{k_1'}x_{k_2'}\right)\left(\dot{c}_{n,k_1',k_2'}^{(1)} + \dot{c}_{n,k_1',k_2'}^{(3)}x_{k_2'}\right)}{1 + \dot{c}_{n,k_1',k_2'}^{(1)}x_{k_1'} + \dot{c}_{n,k_1',k_2'}^{(2)}x_{k_2'} + \dot{c}_{n,k_1',k_2'}^{(3)}x_{k_1'}x_{k_2'}}\right), n = \tilde{n} \in N';
\end{bmatrix}
\tag{12}
$$

In expression (13), it is given for the second function, while the calculations are shortened, since they include actions from expression (12).

$$
f_2' = \begin{bmatrix}
\dfrac{1}{\sqrt{\max(n) - 1}} \dfrac{\sum_n \left(D_n^{(1)}\right)'}{\sqrt{\sum_n \left(D_n^{(1)}\right)^2}}, \forall n \neq \tilde{n};\ n \in N \setminus N', \\
\dfrac{1}{\sqrt{\max(n) - 1}} \dfrac{\sum_n \left(D_n^{(2)} + D_n^{(1)}\right)'}{\sqrt{\sum_n \left(D_n^{(2)} + D_n^{(1)}\right)^2}}, \forall n \neq \tilde{n};\ n \in N';
\end{bmatrix}
\tag{13}
$$

For the third function, the general form of the derivative is given in expression (14). We present the derivative activity only for the case of single-parameter BT (15). Since in general the expression for two-parameter BT is very cumbersome. They do not use the division into beginning and end, since the general appearance of the function, as shown earlier, does not depend on the place of consideration. For the same reason, the indices of node, device and PTL numbers are omitted. But the indices r and i have been added to denote the real and imaginary parts, respectively.

$$f_3' = \frac{1}{\sqrt{\max(l)}} \frac{\sum_l \Delta P_l'}{\sqrt{\sum_l \Delta P_l^2}}, \forall l \in L;\ \Delta P_l' = \left[\begin{array}{l} \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(1)}_{l,n^{(b)}}\right)}{dx_k} - \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(1)}_{l,n^{(e)}}\right)}{dx_k};\ n^{(b)}, n^{(e)} \in N \setminus N' \\ \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(1)}_{l,n^{(b)}}\right)}{dx_k} - \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(2)}_{l,n^{(e)}}\right)}{dx_{k_1'}};\ n^{(b)} \in N \setminus N', n^{(e)} \in N' \\ \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(2)}_{l,n^{(b)}}\right)}{dx_{k_1'}} - \dfrac{\mathrm{Re}\left(\dot{S}^{(1)}_{l,n^{(e)}}\right)}{dx_k};\ n^{(b)} \in N', n^{(e)} \in N \setminus N' \\ \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(2)}_{l,n^{(b)}}\right)}{dx_{k_1'}} - \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(2)}_{l,n^{(e)}}\right)}{dx_{k_1'}};\ n^{(b)} \in N', n^{(e)} \in N' \end{array} \right. . \tag{14}$$

$$\begin{aligned} \frac{d\,\mathrm{Re}\left(\dot{S}^{(1)}\right)}{dx} &= \frac{(c_i e_i + c_r e_r)x^4 + (b_i e_i + c_i d_i + b_r e_r + c_r d_r)x^3}{\left(e_i^2 + e_r^2\right)x^4 + (2d_i e_i + 2d_r e_r)x^3 + \left(d_i^2 + d_r^2 + 2e_r\right)x^2 + 2rd_r x + 1} + \\ &+ \frac{(a_i e_i + b_i d_i + a_r e_r + b_r d_r + c_r)x^2 + (a_i d_i + a_r d_r + b_r)x + a_r}{\left(e_i^2 + e_r^2\right)x^4 + (2d_i e_i + 2d_r e_r)x^3 + \left(d_i^2 + d_r^2 + 2e_r\right)x^2 + 2rd_r x + 1}. \end{aligned} \tag{15}$$

Let us present another set of objective functions (17). For them, the task will be to minimize deviation, minimize power losses and reduce deviations from the nominal capacity of the line. This set will be similar to the previous one, however, for the last function it is necessary to exclude some set T from the set L. Set T includes EPS transformers. P_{nom} is the rated throughput of the PTL at a given voltage class. Since the functions f_1 and f_2 are the same as for the previous set (11), the functions f_2 and f_3, respectively, we will not give derivatives for them. In turn, for the function f_3, the derivative of the external function will be similar to the derivative f_3 of the previous set (14). However, the internal function has only the power of the PTL beginning in its composition, as shown in (16).

$$f_3' = \frac{1}{\sqrt{\max(l)}} \frac{\sum_l P_l'}{\sqrt{\sum_l P_l^2}}, \forall l \in L;\ P_l' = \left[\begin{array}{l} \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(1)}_{l,n^{(b)}}\right)}{dx_k};\ n^{(b)} \in N \setminus N' \\ \dfrac{d\,\mathrm{Re}\left(\dot{S}^{(2)}_{l,n^{(b)}}\right)}{dx_{k_1'}};\ n^{(b)} \in N' \end{array} \right. . \tag{16}$$

$$
\begin{cases}
f_1 = \left[\begin{array}{l} \sqrt{\dfrac{\sum_n \left(D_n^{(1)}\right)^2}{\max(n)}}, \forall n \in N \setminus N', \\ \sqrt{\dfrac{\sum_n \left(D_n^{(2)} + D_n^{(1)}\right)^2}{\max(n)}}, \forall n \in N'; \end{array}\right. \\
f_2 = \sqrt{\dfrac{\sum_l \Delta P_l^2}{\max(l)}}, \forall l \in L; \ \Delta P_l = real \left[\begin{array}{l} \dot{S}^{(1)}_{l,n^{(b)}} - \dot{S}^{(1)}_{l,n^{(e)}};\ n^{(b)},\ n^{(e)} \in N \setminus N' \\ \dot{S}^{(1)}_{l,n^{(b)}} - \dot{S}^{(2)}_{l,n^{(e)}};\ n^{(b)} \in N \setminus N',\ n^{(e)} \in N' \\ \dot{S}^{(2)}_{l,n^{(b)}} - \dot{S}^{(1)}_{l,n^{(e)}};\ n^{(b)} \in N',\ n^{(e)} \in N \setminus N' \\ \dot{S}^{(2)}_{l,n^{(b)}} - \dot{S}^{(2)}_{l,n^{(e)}};\ n^{(b)} \in N',\ n^{(e)} \in N' \end{array}\right. \\
f_3 = \sqrt{\dfrac{\sum_l p_l}{\max(l)}}, \forall l \in L \setminus T; \\
p_l = \left[\begin{array}{l} \left[\begin{array}{l} P_l^{(1)} - 0.8 P_{nom}, n \in N \setminus N' \\ P_l^{(2)} - 0.8 P_{nom}, n \in N' \end{array}\right., P_l \geq 0.8 P_{nom} \\ \left[\begin{array}{l} 0.4 P_{nom} - P_l^{(1)} - 0.8, n \in N \setminus N' \\ 0.4 P_{nom} - P_l^{(2)}, n \in N' \end{array}\right., P_l \leq 0.4 P_{nom} \end{array}\right.
\end{cases}
\quad (17)
$$

3 The results of the study

Figure 2 and Figure 3 show the Pareto sets for the first (11) and second (17) sets of functions, respectively. In these graphs, it can be observed that the fronts are planes with pronounced bends at the edges. In the projections on the axes presented to the right of the surfaces, it can be observed that for the first set of functions there is a quadratic relationship between f_1 and f_2. And for the second set of functions, a similar dependence can be observed between f_1 and f_3.

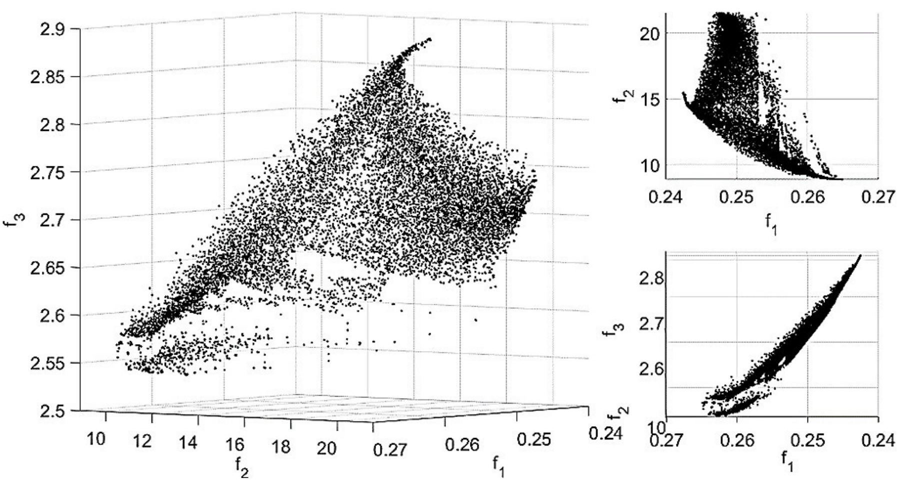

Fig. 2. The Pareto front for the first set of functions.

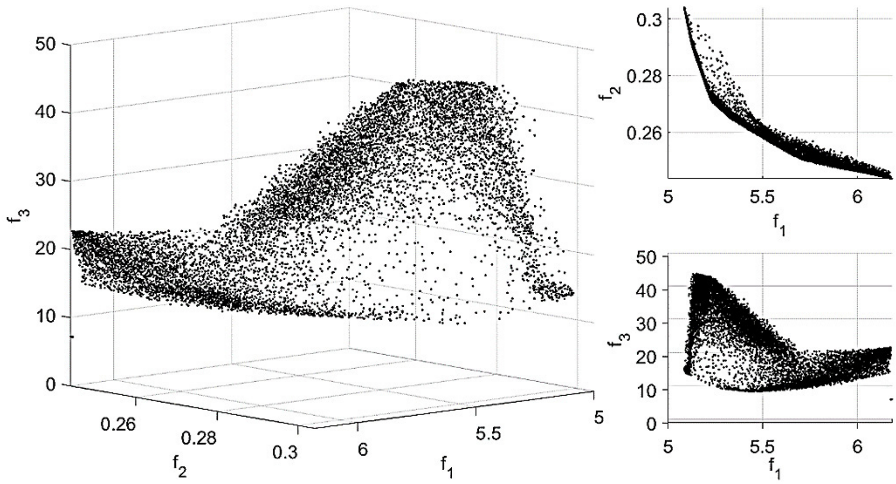

Fig. 3. The Pareto front for the second set of functions.

For the first set of functions the dependence is obvious, since the characteristic they control is the same. Not everything is so simple for the second one. Firstly, although the resulting set of points is approximated by a polynomial of the second degree, the accuracy of the approximation is not as accurate as in the case of the first set. And secondly, based on the logic described above, one would expect a similar relationship between f_2 and f_3, but there is none.

For further sensitivity analysis, a cluster analysis was performed in the field of function parameters. Due to the fact that it is impossible to graphically display the six-dimensional function, clusters were displayed on the surfaces of the Pareto front in Fig. 4 and Fig. 5, respectively. Typical representatives of clusters, which are their centers, were also identified. They are marked with pentagrams. It was experimentally determined that five partitions are sufficient to cluster the set of parameters of the first set of functions. And for the second set of functions, there are six partitions. Selecting typical representatives allows you not to consider the sensitivity at each point of the Pareto set. It is assumed that within each cluster, the function values at each point do not differ from the value at the typical representative point more than the error of measuring instruments [17]. Therefore, only the point of the representative can be considered. As seen in Figure 4 and Figure 5 clusters selected in the parameter area correspond in the values area to the "density" partition.

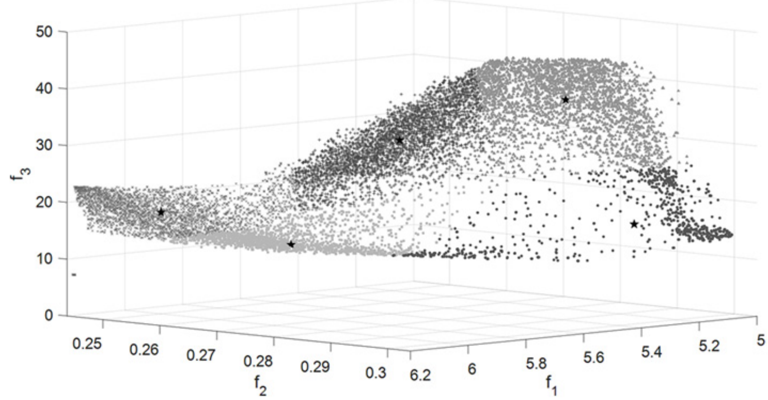

Fig. 4. Cluster analysis of parameters for the first set of functions.

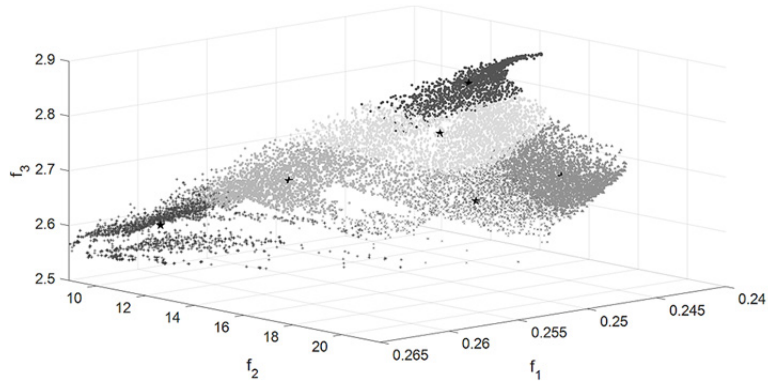

Fig. 5. Cluster analysis of parameters for the second set of functions.

Table 1 and Table 2 show the values of derivative functions at typical representative points. Rounding is done to two real decimal places. Moreover, if the sensitivity value is lower than 10^{-4}, then the function is not sensitive to changes in the parameter and is denoted const. The positivity or negativity of the derivative indicates, respectively, the increase or decrease of the function as the argument increases. The line number corresponding to the diagram in Fig. 1 is indicated in parentheses next to the parameter number, in accordance with [8]. In Table 1, it can be observed that the first function is most sensitive to the third and fourth parameters, the second to the first and fourth, and the third to the first, second and third. In turn, the functions show the least sensitivity to changes in the sixth parameter. The third function turned out to be the most sensitive, its sensitivity exceeds the others by several orders of magnitude.

Table 1. Sensitivity of the first set of functions.

The number of a typical representative \ Number of parameters		**1 (10)**	**2 (8)**	**3 (3)**	**4 (2)**	**5 (4)**	**6 (9)**
For the first function	1	-0.0024	-0.0007	-0.0192	-0.0138	-0.0054	-0.0003
	2	-0.002	-0.0006	-0.0137	-0.0085	-0.0043	-0.0002
	3	-0.0017	-0.0005	-0.0125	-0.0069	-0.0029	-0.0002
	4	-0.0022	-0.0006	-0.0156	-0.0114	-0.0048	-0.0002
	5	-0.0016	-0.0005	-0.0124	-0.007	-0.0031	-0.0001
For the second function	1	0.0005	0.0002	0.0002	0.0005	0.0002	0.0001
	2	0.0005	0.0002	0.0002	0.0003	0.0002	0.0001
	3	0.0004	0.0002	0.0002	0.0003	0.0002	0.0001
	4	0.0005	0.0002	0.0002	0.0004	0.0002	0.0001
	5	0.0004	0.0002	0.0002	0.0003	0.0002	const
For the third function	1	-0.57	0.15	0.27	-0.0075	0.0047	0.0052
	2	-0.35	0.09	0.16	-0.0015	0.0013	0.0022
	3	-0.27	0.07	0.13	-0.0012	0.0002	0.0014
	4	-0.9	0.24	0.42	-0.0097	0.0061	0.0049
	5	-0.63	0.17	0.30	-0.0047	0.0006	0.0018

As a result of analyzing the data in Table 2, it can be concluded that the first function, which is the second in the first set, does not change its relative sensitivity to the parameters. Although the sensitivity to all parameters has increased slightly, in relation to the first set of functions. The second function, which is the third in the first set, also does not change its behavior, there is a similar increase in sensitivity for it. The third function turned out to be the most sensitive to all parameters, compared to the previous ones. However, it shows the least relative sensitivity to the fourth parameter.

Table 2. Sensitivity of the second set of functions.

The number of a typical representative \ Number of parameters		1 (10)	2 (8)	3 (3)	4 (2)	5 (4)	6 (9)
For the first function	1	0.0004	0.0001	0.0002	0.0004	0.0002	const
	2	0.0004	0.0001	0.0002	0.0004	0.0002	const
	3	0.0004	0.0002	0.0002	0.0005	0.0002	0.0001
	4	0.0004	0.0002	0.0002	0.0005	0.0002	const
	5	0.0004	0.0001	0.0002	0.0005	0.0002	const
	6	0.0004	0.0001	0.0002	0.0004	0.0002	const
For the second function	1	-0.79	0.21	0.37	-0.011	0.006	0.0044
	2	-1.04	0.28	0.49	-0.011	0.0065	0.0044
	3	-0.72	0.19	0.34	-0.01	0.0058	0.0046
	4	-0.60	0.16	0.28	-0.01	0.0049	0.0033
	5	-0.51	0.14	0.24	-0.009	0.0041	0.0026
	6	-0.82	0.22	0.39	-0.011	0.006	0.0041
For the third function	1	-1.68	-1.34	0.19	-0.0028	0.0022	0.0014
	2	-1.8	-1.32	0.24	-0.0029	0.0014	0.0012
	3	-1.63	-1.39	0.15	-0.0029	0.0023	0.0015
	4	-1.62	-1.34	0.15	-0.0025	0.0023	0.0011
	5	-1.6	-1.29	0.16	-0.0023	0.0022	0.0008
	6	-1.71	-1.3	0.21	-0.0027	0.002	0.0012

If we compare the sensitivity of the sets of functions presented above, we can conclude that the second set of functions is more sensitive to parameter changes. This suggests that this set is more suitable for determining the parameters of the optimal mode. Moreover, it includes all the criteria of optimality put forward: quality, cost-effectiveness and reliability of power supply. In addition, the first set has a clearer relationship between the functions, which may affect the stability of solutions that can be observed from the surfaces of the Pareto front. However, the first set was put forward to control the mode, as an additional assistance from FACTS devices to the operation of emergency automation. Therefore, the information obtained from it is also important. It is assumed that both functional sets can be used to analyze the mode. Or combine them by adding the first function of the first set to the second. The sensitivity results suggest that the choice of the device installation location made in [8] is satisfactory. However, this does not mean that it is impossible to choose a more optimal position. Using the sets of functions described in the work, it is possible to more correctly determine the optimal position of the devices than in [8].

4 Conclusion

In the work, mathematical functions were obtained that allow approximation methods to simplify the task of choosing the parameters of FACTS devices. This reduces the running time of the algorithm and makes the calculation faster than changing the EPS mode. Moreover, the functions help to determine the optimal position of devices in EPS from sensitivity analysis. In order to determine recommendations for choosing device parameters, it is necessary to analyze the Pareto set in more detail, because it is possible to complete the task of narrowing it.

A serious assumption was accepted. The regime was adopted stationary. This did not take into account the changing characteristics of the established regime over time. And to approximate functions to transmit acting, not instantaneous values. If you consider the quasi-fastness of the mode, a number of difficulties will arise. If the variation amplitude of the characteristic determined by BT is sufficiently high, then the error of determination of optimal parameters in time can make an impressive value. Then there is the task of writing the target functions to account for fluctuations. However, this approach will greatly complicate the function relations and as a result, the algorithm will run longer. This can cause the algorithm to fall behind the fixed mode of EPS. Then, in order to accelerate the algorithm, it is possible to apply not instantaneous values, but displacement of the value of the characteristic relative to the acting in the area where it is necessary. There is a need to solve the problem of determining the value of the amplitude at which the offset is applied. This will be addressed in subsequent work.

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