Numerical Simulation of Elastoplastic Problems in Strains and Displacements

Abduvali Khaldjigitov[1], *Umidjon* Djumayozov[2*], *Dilnoza* Sagdullaeva[3]

[1]National University of Uzbekistan, Tashkent, Uzbekistan
[2]Samarkand branch of Tashkent University of Information Technologies, Samarkand, Uzbekistan
[3]Institute of Mechanics and Seismic Stability of Structures, Tashkent, 100125, Uzbekistan

Abstract. Within the framework of Saint-Venant's compatibility conditions, plane problems of the theory of plasticity with respect to strains and displacements are formulated in this paper. Grid equations for displacements and strains for a rectangular plate are compiled using the finite-difference method. Difference equations for displacements and strains were solved, respectively, by the iterative method and the method of alternating directions. By comparing the numerical results for a rectangular plate, the validity of the formulated plastic problem with respect to deformations is shown.

1. Introduction

Mathematical and numerical modeling of the process of nonlinear deformation of structures and their elements is an important task in the theory of plasticity from the point of view of assessing the safety factor of solids. Mathematical and numerical modeling of plastic deformations based on modern information technologies is due to the widespread use of new structural materials in construction, mechanical engineering, aircraft manufacturing, medicine, the textile industry, and other areas. Numerical modeling and problems of developing effective algorithms for calculating plastic deformations of modern structures are relevant in such industrialized countries as the USA, Canada, Great Britain, EU countries, the Russian Federation, Japan, India, China, South Korea, Indonesia and Malaysia, etc.

The construction of mathematical models that describe the process of elastoplastic deformation of isotropic and anisotropic bodies is a fundamental problem in mathematical modeling and plasticity theory. These questions were studied in the classical works of famous scientists such as Saint-Venant, Genki, Drucker, Ilyushin, Nahdi, Kachanov and others.

Typically, plastic problems are formulated with respect to displacements within the framework of the deformation theory of plasticity [1] and the theory of plastic flows [2]. The formulation of plastic boundary value problems regarding stresses [3-5] and deformations [6-8] is a poorly studied area of mathematical modeling and mechanics of a deformable solid.

The main numerical methods for solving elastoplastic problems are the method of elastic solutions [9], which is usually considered in combination with well-known methods such as the finite element method, variational-difference method and finite-difference method, as well as the boundary element method.

This work is devoted to the formulation of boundary value problems of the theory of plasticity with respect to deformations. This study is a fairly new direction in the theory of plasticity and is based on the well-known Saint-Venant compatibility condition. The plastic problem of stretching a rectangular plate is solved numerically, formulated with respect to deformations and displacements. Discrete equations are compiled using the finite-difference method. Grid equations for deformations and displacements are solved by the method of sweeping along the corresponding directions and the iterative method, respectively. A comparison of numerical results shows the validity of the new formulation of plastic problems with respect to deformations.

[*]Corresponding author: djumayozov@bk.ru

2. Formulation of plastic problems regarding strains and displacements

Typically, plastic problems according to deformation theories are formulated in relation to displacements and consist of equilibrium equations [3]:

$$\sum_{j=1}^{3}\sigma_{ij,j}+X_i=0,\quad x_i\in V,\quad i=1,2,3. \tag{1}$$

and, the defining relation of the theory of plasticity, representing a nonlinear function between the stress and strain tensor [10] i.e.

$$\sigma_{ij}=F(\varepsilon_{ij}), \tag{2}$$

Cauchy relations [4]

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), \tag{3}$$

and boundary conditions

$$u_i\big|_{\Sigma_1}=u_i^o,\ \sum_{j=1}^{3}\sigma_{ij}n_j\big|_{\Sigma_2}=S_i^o,\quad i=1,2,3. \tag{4}$$

where $\sigma_{ij}-$ stress tensor, $\varepsilon_{ij}-$ strain tensor, u_i- displacement components, X_i- volumetric forces, $\lambda-$, $\mu-$ elastic Lame constants, $\delta_{ij}-$ Kronecker symbol, n_j- outer normal to surface Σ_2, S_i- surface load.

In the boundary value problem (1-4), instead of relation (2), we can consider the following nonlinear relationship between the stress and strain tensor, the so-called deformation theory of Ilyushin's plasticity [1]

$$\sigma_{ij}=\sigma\delta_{ij}+\frac{\sigma_u}{\varepsilon_u}e_{ij}, \tag{5}$$

where

$$\sigma=K\theta,\quad K=\lambda+\frac{2}{3}\mu, \tag{6}$$

$$\sigma_u=\sigma_u(\varepsilon_u), \tag{7}$$

where σ_u, ε_u- intensity of the stress and strain tensor [4], respectively,

$$\sigma_u=\sqrt{\frac{1}{2}S_{ij}S_{ij}},\quad \varepsilon_u=\sqrt{\frac{1}{2}e_{ij}e_{ij}}, \tag{8}$$

where e_{ij}, $S_{ij}-$ deviator of the strain and stress tensor in the following form [9]

$$\begin{aligned} e_{ij}&=\varepsilon_{ij}-\frac{\theta}{3}, \qquad \theta=\ \varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33},\\ S_{ij}&=\sigma_{ij}-\sigma\delta_{ij}, \quad\ \ \sigma=\ (\sigma_{11}+\sigma_{22}+\sigma_{33})/3. \end{aligned} \tag{9}$$

It is known that relation (7) can be represented in piecewise linear form [9]

$$\sigma_u=2\mu\varepsilon_u+2(\mu-\mu')(\varepsilon_u-\varepsilon_u^*)\quad for\quad \varepsilon_u\ge\varepsilon_u^*. \tag{10}$$

where $\mu'-$ tangent module, ε_u^*- elastic limit.

The defining relation (5), using relations (11) can be written in the following form at $\varepsilon_u\ge\varepsilon_u^*$ i.e.

$$\sigma_{ij}=\lambda\theta\delta_{ij}+2\mu\varepsilon_{ij}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})e_{ij}\quad for\quad \varepsilon_u\ge\varepsilon_u^*, \tag{11}$$

Thus, in the boundary value problem (1-4), instead of (6) considering (11), we obtain a nonlinear model equation that describes the process of nonlinear deformation of solids [4]:

$$\sum_{j=1}^{3}\sigma_{ij,j}+X_i=0,\quad x_i\in V,\quad i=1,2,3. \tag{12}$$

$$\sigma_{ij} = \begin{cases} \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \quad for \quad \varepsilon_u < \varepsilon_u^*, \\ \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} - 2(\mu - \mu')(1 - \frac{\varepsilon_u^*}{\varepsilon_u})e_{ij} \quad for \quad \varepsilon_u \ge \varepsilon_u^*, \end{cases} \tag{13}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{14}$$

$$u_i\big|_{\Sigma_1} = u_i^o, \quad \sum_{j=1}^{3}\sigma_{ij}n_j\big|_{\Sigma_2} = S_i^o, \quad x_i \in \Sigma_2. \tag{15}$$

The boundary value problem (12-15), in the two-dimensional case, can be written in the form of Lame equations for displacements u, v i.e.

$$\begin{aligned} &(\lambda + 2\mu)\frac{\partial^2 u}{\partial^2 x} + \mu\frac{\partial^2 u}{\partial^2 y} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} - f_1 = 0, \\ &(\lambda + 2\mu)\frac{\partial^2 v}{\partial^2 y} + \mu\frac{\partial^2 v}{\partial^2 x} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} - f_2 = 0, \end{aligned} \tag{16}$$

with boundary condition

$$\begin{aligned} &(\sigma_{11}n_1 + \sigma_{12}n_2)\big|_{\Gamma} = S_1, \\ &(\sigma_{21}n_1 + \sigma_{22}n_2)\big|_{\Gamma} = S_2. \end{aligned} \tag{17}$$

where σ_{ij} defined by relation (11) and (3), $f_i -$ nonlinear part of differential equations:

$$f_i = 2(\mu - \mu')(1 - \frac{\varepsilon_u^*}{\varepsilon_u})\sum_{j=1}^{2}\frac{\partial e_{ij}}{\partial x_j} \quad for \quad \varepsilon_u \ge \varepsilon_u^*.$$

To formulate an elastoplastic boundary value problem, we need to have conditions for compatibility of deformations for plastic theories. Why consider the conditions for compatibility of deformations [3]

$$\nabla^2\varepsilon_{ij} + \theta_{,ij} - \varepsilon_{ik,jk} - \varepsilon_{jk,ik} = 0, \quad \theta = \varepsilon_{kk}. \tag{18}$$

Differentiating relation (11) by x_j and substituting it into Hooke's law, and from it we can find that

$$\varepsilon_{ij,j} = -\frac{\lambda}{2\mu}\theta_{,i} + \frac{P}{2\mu}e_{ij,j} - \frac{1}{2\mu}X_i. \tag{19}$$

Substituting the last relation in (18) after some transformations, the compatibility condition for the deformation theory of plasticity can be found [13] i.e.

$$\nabla^2\varepsilon_{ij} + k\theta_{,ij} - (1 - \frac{\mu'}{\mu})(1 - \frac{\varepsilon_u^*}{\varepsilon_u})(e_{ik,kj} + e_{jk,ki}) = -\frac{1}{2\mu}(X_{i,j} + X_{j,i}), \quad k = 1 + \frac{\lambda}{\mu}. \tag{20}$$

which, in combination with the equilibrium equation (12) and the corresponding boundary conditions, constitute the boundary value problem of the deformation theory of plasticity in deformations.
Substituting (13) into (1), we can find the equilibrium equations expressed with respect to deformations

$$\begin{aligned} &(\lambda + 2\mu)\frac{\partial\varepsilon_{11}}{\partial x} + \lambda\frac{\partial\varepsilon_{22}}{\partial x} + 2\mu\frac{\partial\varepsilon_{12}}{\partial y} - 2(\mu - \mu')(1 - \frac{\varepsilon_u^*}{\varepsilon_u})(\frac{\partial e_{11}}{\partial x} + \frac{\partial e_{12}}{\partial y}) = 0, \\ &(\lambda + 2\mu)\frac{\partial\varepsilon_{22}}{\partial y} + \lambda\frac{\partial\varepsilon_{11}}{\partial y} + 2\mu\frac{\partial\varepsilon_{21}}{\partial x} - 2(\mu - \mu')(1 - \frac{\varepsilon_u^*}{\varepsilon_u})(\frac{\partial e_{22}}{\partial y} + \frac{\partial e_{12}}{\partial x}) = 0, \quad for \ \varepsilon_u \ge \varepsilon_u^*, \end{aligned} \tag{21}$$

which, in combination with the deformation compatibility condition (20) for plane deformation, i.e.

$$\mu\nabla^2\varepsilon_{12} + (\lambda + \mu)\theta_{,12} - 2(\mu - \mu')(1 - \frac{\varepsilon_u^*}{\varepsilon_u})(e_{1k,k2} + e_{2k,k1}) = 0, \tag{22}$$

border conditions

$$(\sigma_{11}n_1 + \sigma_{12}n_2)\big|_{\Gamma} = S_1, \quad (\sigma_{21}n_1 + \sigma_{22}n_2)\big|_{\Gamma} = S_2, \tag{23}$$

and, additional boundary conditions [6]

$$\left(\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}\right)\bigg|_{\Gamma}=0,\quad \left(\frac{\partial\sigma_{22}}{\partial y}+\frac{\partial\sigma_{21}}{\partial x}\right)\bigg|_{\Gamma}=0. \tag{24}$$

constitute a plane boundary value problem of the deformation theory of plasticity in deformations. In boundary conditions (23-24), voltage σ_{ij} is determined by relation (11).

In the boundary value problem (21-24), equilibrium equations (21) can be used in a differentiated form. Then the plane problem of plasticity theory takes the form

$$\begin{gathered}(\lambda+2\mu)\frac{\partial^2\varepsilon_{11}}{\partial x^2}+\lambda\frac{\partial^2\varepsilon_{22}}{\partial x^2}+2\mu\frac{\partial^2\varepsilon_{12}}{\partial x\partial y}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})(\frac{\partial^2 e_{11}}{\partial x^2}+\frac{\partial^2 e_{12}}{\partial y^2})=0,\\ (\lambda+2\mu)\frac{\partial^2\varepsilon_{22}}{\partial y^2}+\lambda\frac{\partial^2\varepsilon_{11}}{\partial y^2}+2\mu\frac{\partial^2\varepsilon_{21}}{\partial x\partial y}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})(\frac{\partial^2 e_{22}}{\partial y^2}+\frac{\partial^2 e_{12}}{\partial x^2})=0,\\ \mu\nabla^2\varepsilon_{12}+(\lambda+\mu)\theta_{,12}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})(e_{1k,k2}+e_{2k,k1})=0,\end{gathered} \tag{25}$$

with boundary conditions expressed relative to deformations

$$\begin{gathered}((\lambda+2\mu)\varepsilon_{11}+\lambda\varepsilon_{22}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})e_{11})n_1+2(\mu\varepsilon_{12}-(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})e_{11})n_2)|_{\Gamma}=S_1,\\ (2(\mu\varepsilon_{12}-(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})e_{12})n_1+(\lambda\varepsilon_{11}+(\lambda+2\mu)\varepsilon_{22}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})e_{22})n_2)|_{\Gamma}=S_2,\end{gathered} \tag{26}$$

and also, with two additional conditions obtained by considering the equilibrium equations on the boundary of a given area, i.e.

$$\begin{gathered}\left[(\lambda+2\mu)\frac{\partial\varepsilon_{11}}{\partial x}+\lambda\frac{\partial\varepsilon_{22}}{\partial x}+2\mu\frac{\partial\varepsilon_{12}}{\partial y}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})e_{11}\right]_{|\Gamma}=0,\\ \left[(\lambda+2\mu)\frac{\partial\varepsilon_{22}}{\partial y}+\lambda\frac{\partial\varepsilon_{11}}{\partial y}+2\mu\frac{\partial\varepsilon_{21}}{\partial x}-2(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u})e_{22}\right]_{|\Gamma}=0.\end{gathered} \tag{27}$$

Thus, equations (25-27) represent a plane problem of the theory of plasticity in deformations.

3. Solution methods

To numerically solve plastic problems in displacements (16-17) and deformations (21-24), we use the finite-difference method. At the same time, we consider these boundary value problems in a rectangular region and replacing the derivatives with finite relations, we can find the following finite-difference equations in the case of a problem involving displacements:

$$\begin{gathered}(\lambda+2\mu)\frac{u_{i+1}^j-2u_i^j+u_{i-1}^j}{h_1^2}+(\lambda+\mu)\frac{v_{i+1}^{j+1}-v_{i-1}^{j+1}-v_{i+1}^{j-1}+v_{i-1}^{j-1}}{4h_1h_2}+\mu\frac{u_i^{j+1}-2u_i^j+u_i^{j-1}}{h_2^2}+f_1=0,\\ (\lambda+2\mu)\frac{v_i^{j+1}-2v_i^j+v_i^{j-1}}{h_2^2}+(\lambda+\mu)\frac{u_{i+1}^{j+1}-u_{i-1}^{j+1}-u_{i+1}^{j-1}+u_{i-1}^{j-1}}{4h_1h_2}+\mu\frac{v_{i+1}^j-2v_i^j+v_{i-1}^j}{h_1^2}+f_2=0.\end{gathered} \tag{28}$$

where

$$\begin{gathered}f_1=(-\frac{4}{3}\frac{\partial^2 u}{\partial x^2}-\frac{1}{3}\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2})(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u}),\\ f_2=(-\frac{4}{3}\frac{\partial^2 v}{\partial y^2}-\frac{1}{3}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial^2 v}{\partial x^2})(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u}).\end{gathered} \tag{29}$$

To solve difference equations (28), firstly, we resolve them with respect to u_i^j, v_i^j and apply the iterative Seidel method.

To solve the difference equations of the plastic problem in deformations (21-24), you can use the method of sweeping sequentially along the corresponding axes [11]. For this purpose, we write the finite-difference equations in the form of a system of equations for the components of the strain tensor with a three-diagonal matrix, i.e.

$$a_i\varepsilon_{i+1,j}^{11}+b_i\varepsilon_{i,j}^{11}+c_i\varepsilon_{i-1,j}^{11}=f_{ij}^x, \tag{30}$$

$$\begin{cases} \alpha_{01}\varepsilon^{11}_{0,j} + \beta_{01}\varepsilon^{11}_{1,j} = \gamma_{01}, \\ \alpha_{02}\varepsilon^{11}_{n-1,j} + \beta_{02}\varepsilon^{11}_{n,j} = \gamma_{02}. \end{cases} \quad (31)$$

where

$$a_i = \frac{\lambda + 2\mu}{h_1^2},\ b_i = \frac{-2(\lambda + 2\mu)}{h_1^2},\ c_i = \frac{\lambda + 2\mu}{h_1^2},$$

$$f_{ij}^x = -\lambda \frac{\varepsilon_{i+1,j}^{22} - 2\varepsilon_{i,j}^{22} + \varepsilon_{i-1,j}^{22}}{h_1^2} - 2\mu \frac{\varepsilon_{i+1,j+1}^{12} - \varepsilon_{i-1,j+1}^{12} - \varepsilon_{i+1,j-1}^{12} + \varepsilon_{i-1,j-1}^{12}}{4h_1h_2} + P^x,$$

$$P^x = 2(\mu - \mu')(1 - \frac{\varepsilon_u^*}{\varepsilon_u})(\frac{\partial^2 e_{11}}{\partial x^2} + \frac{\partial^2 e_{12}}{\partial y^2}) = 0.$$

From the boundary conditions it can be found[11] that

$$\alpha_{01} = 1,\ \beta_{01} = 0,\ \gamma_{01} = 0,\ \alpha_{02} = 0,\ \beta_{02} = 1,\ \gamma_{02} = 0. \quad (32)$$

For the remaining components of the deformation tensor, the following systems of equations with tridiagonal matrices can be found similarly:

$$\begin{aligned} a_i'\varepsilon^{22}_{i,j+1} + b_i'\varepsilon^{22}_{i,j} + c_i'\varepsilon^{22}_{i,j-1} &= f_{ij}^y, \\ \dot{a}_i\varepsilon^{12}_{i+1,j} + \dot{b}_i\varepsilon^{12}_{i,j} + \dot{c}_i\varepsilon^{12}_{i-1,j} &= f_{ij}^{xx}, \\ \tilde{a}_i\varepsilon^{12}_{i,j+1} + \tilde{b}_i\varepsilon^{12}_{i,j} + \tilde{c}_i\varepsilon^{12}_{i,j-1} &= f_{ij}^{yy}. \end{aligned} \quad (33)$$

From equations (30) and (33) it follows that the equations can be solved by four consecutive applications of the alternative method. This solution method, according to [11], is called the method of alternating directions.

4. Comparison of numerical results

This section is devoted to the numerical solution of plastic problems formulated regarding deformations and displacements, and comparison of the results obtained using the alternating direction method and the sweep method, respectively.

Let a rectangular plate of size (2a,2b) be under the action of a uniaxial load of a parabolic shape applied on opposite sides perpendicular to the *OX* axis [8]. The remaining grains are free from loads, i.e.

$$for\ \ x = \pm a:\ \ \sigma_{11} = S_0(1 - \frac{y^2}{a^2}), \quad \sigma_{12} = 0, \quad (34)$$

$$for\ \ y = \pm b:\ \ \sigma_{22} = 0, \quad \sigma_{21} = 0. \quad (35)$$

For the problem under consideration, in the work of Timoschenko-Goodyear [12], based on the condition for minimizing the strain energy using the Airy stress function, the following expressions were found for the components of the stress tensor

$$\begin{aligned} \sigma_{11} &= S(1 - \frac{y^2}{a^2}) - 0.1702S(1 - \frac{3y^2}{a^2})(1 - \frac{x^2}{a^2})^2, \\ \sigma_{22} &= -0.1702S(1 - \frac{3x^2}{a^2})(1 - \frac{y^2}{a^2})^2, \\ \sigma_{12} &= -0.6805S\frac{xy}{a^2}(1 - \frac{x^2}{a^2})(1 - \frac{y^2}{a^2}). \end{aligned} \quad (36)$$

Table 1. Comparison of stress tensor σ_{11} at *y=0*

Tasks	*x =-1*	*x=-0.8*	*x=-0.6*	*x=-0.4*	*x=-0.2*	*x=0*
Timoshenko-Goodyear	1.0000	0.9779	0.9303	0.8788	0.8431	0.8298
In displacements	0.9925	0.8791	0.8538	0.8214	0.7963	0.8214
In Strains	1.0000	0.9818	0.9818	0.9818	0.9818	0.9818

The initial data has the following dimensionless values:

$$\lambda = 0.8,\ \mu = 0.5,\ l_1 = 2a,\ l_2 = 2b,\ a = b = 1,\ N_1 = N_2 = 10,\ \mu' = 0.3,\ \varepsilon_u^* = 0.32,\ S = 1.$$

Table 1 compares the values of stress σ_{11}, in the middle of the plate, obtained by solving plastic problems formulated regarding displacements and deformations, as well as with the results of Timoshenko-Goodyear [12] at $\varepsilon_u < \varepsilon_u^*$ Comparisons of stresses σ_{11} show that the numerical results obtained by the iterative method with respect to displacements and the sweep method with respect to deformations are very close, which ensures the validity of the formulated plastic problem in deformations, as well as the reliability of the obtained numerical results. According to Fig. 1 you can compare the graphs of stress distribution σ_{11} in the middle *x=0* of a flat plate according to the results of Timoschenko-Goodyear (II) and the problem in displacements (III) and also the problem in deformations (I). In Fig. 1 yellow curve shows part of the parabolic load applied along the opposite faces of the plate along the *OX* axis.

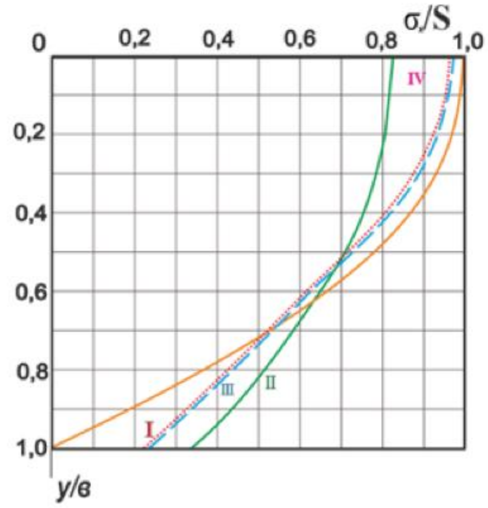

Fig.1. Stress distribution σ_{11} based on the results of the problem of stretching a plate under a parabolic load

Table 2. Comparison of stress tensor σ_{11} at *y=0*

Plastic tasks	*x=-1*	*x=-0.8*	*x=-0.6*	*x=-0.4*	*x=-0.2*	*x=0*
In displacements	0.9919	0.8339	0.7827	0.7417	0.7145	0.7051
In Strains	1.0000	0.9457	0.9275	0.9229	0.9220	0.9221

Table 2 shows the stress values obtained from the results of a numerical solution of plastic problems about stretching a rectangular plate formulated with respect to deformations and displacements under the condition $\varepsilon_u \geq \varepsilon_u^*$. Note that the stress values calculated from the results of the boundary value problem in displacements are underestimated, which apparently occurs as a result of an error in the approximation of the boundary conditions. In the case of boundary value problems in deformations, the boundary conditions are satisfied exactly.

5. Conclusion

A plane problem of the theory of plasticity with respect to deformations is formulated within the framework of Saint-Venant compatibility conditions. To compare the numerical results, the plastic problem of stretching a rectangular plate under displacement is also considered. Grid equations for deformations and displacements for a rectangular plate are compiled using the finite-difference method. Difference equations for displacements and deformations were solved, respectively, by the iterative method and the method of alternating directions. A comparison of numerical results shows the validity of the formulated plastic problem with respect to deformations. Numerical algorithms and corresponding software have been developed for solving elastoplastic boundary value problems regarding displacements and deformations.

References

1. A.A. Ilyushin. Plastic. Part 1. “Elastoplastic deformations,” Gostekhizdat, Moscow (1948)
2. L.M. Kachanov, Fundamentals of the theory of plasticity, Nauka, Moscow (1969)
3. V. Novatski, Theory of elasticity, Mir, Moscow (1975)

4. V. Novatsky, Dynamic problems of thermoelasticity, Mir, Moscow (1970)
5. S. Li, A. Gupta, X. Markenscoff, Conservation laws of linear elasticity in stress formulations, Proceedings of the Royal Society. *Mathematical, Physical and Engineering Sciences* **461**(2053), 99–116 (2005)
6. A.A., Khaldjigitov, U.Z. Djumayozov, Numerical Solution of the Two-Dimensional Elasticity Problem in Strains. *Mathematics and Statistics* **10**, 1081-1088 (2022)
7. N.M. Borodachev, Three-dimensional problem of the theory of elasticity in strains. *Strength of Materials* **27**(5-6), 296–299 (1995)
8. D. Turimov, A. Khaldjigitov, U.Djumayozov, W. Kim, Formulation and Numerical Solution of Plane Problems of the Theory of Elasticity in Strains. *Mathematics* **12**, 71 (2024)
9. B.E. Pobedrya, Numerical methods in the theory of elasticity and plasticity. Publishing House of Moscow State University, Moscow (1996)
10. B.E. Pobedrya. Deformation theory of plasticity of anisotropic media. *PMM* **48**, 29-37 (1984)
11. A.A. Samarski, E.S. Nikolaev. Methods for solving grid equations, Science, Moscow (1978)
12. S. P. Timoshenko, J.N. Goodier, Theory of Elasticity. McGraw-Hill (1970)
13. A. Khaldjigitov, U. Djumayozov, Model Equations of the Theory of Elasticity in Strains: Classical and New Formulations. *E3S Web of Conferences* **497**, 02015 (2024)
14. A.A. Khaldjigitov, U.Z. Djumayozov, D.A. Sagdullayeva, Numerical Solution of Coupled Thermo-Elastic-Plastic Dynamic Problems. *Mathematical Modelling of Engineering Problems* **8**(4), 510-518 (2012)
15. A.A. Khaldjigitov, A.A. Kalandarov, U.Z. Djumayozov, Numerical modeling of coupled problems of thermo-plasticity on non-uniform meshes. *AIP Conference Proceedings* **2686**, 020007 (2022)