Use of statistics as a tool to evaluate the strength between interfaces with geomembranes and other materials obtained by Conventional Direct Shear (CDS) and Inclined plane (PI) testing

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Abstract. Understanding the behavior of interface resistance involves factors such as the friction angle of the interface, the asperity height of the geomembrane, type of soil in contact with the synthetic material, the applied stresses, among others. In this context, statistical techniques have the ability to establish correlations between variables for evaluating interface strength tests, such as conventional direct shear (CDC) and inclined plane (PI). In this study, the interfaces between sand and geomembrane (SG) were analyzed for the CDC and PI tests. The aim was to obtain a statistical understanding of the parameters that most influenced the interface friction angle (ϕ_{int}) using Pearson's correlation matrix. It was observed that the soil friction angle (ϕ_s) and the asperity height of the geomembrane (h_G) showed the highest correlations with ϕ_{int} for the tests carried out with the SG-CDC interfaces. The results also showed that for the SG-CDC interfaces, 7.15% of the correlations showed a strong or moderate correlation, while 35.70% showed weak correlations and 50% showed very weak correlations. On the other hand, for the SG-PI interface, only h_G showed a strong correlation, with 29%, 50% and 14% of the variables classified as moderate, weak and very weak, respectively.

1 Introduction

In recent decades, it's been evident that the use of geosynthetics in geotechnical projects has expanded. For the proper application of geomembranes, one of the most commonly used geosynthetics according to ([1]), it is important to have an understanding of the interaction between this material and the overlying soil. This understanding encompasses not only the interface friction angle but also comprehending the properties of geomembrane roughness, as well as the type of soil that will be placed over it. According to ([2]) the interface friction angle is directly linked to the asperity of the geosynthetic, and in addition to this parameter, there are other factors that can affect interface strength, such as grain size, the level of stress applied in the field, among others.

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The use of statistics, as well as programming languages, to evaluate the parameters that most influence the interface resistance appears to be applicable, in view of the growth of studies in this area ([3-6]). The aim of this article is to statistically evaluate the behavior of the interface friction angle (ϕ_{int}) in relation to other variables, namely normal stresses (σ_n), contact area (A_c), geomembrane roughness height (h_G), geomembrane thickness (t_{GM}), curvature (C_c) and non-uniformity (C_u) coefficients of the soil, average grain diameter (D_{50}), soil friction angle (ϕ_s), weight (M_A), among others. This statistical evaluation can possibly identify the main variables influencing interface resistance, as well as to define the correlation using the Pearson Matrix (MCP), which is widely used in statistical studies.

2 Materials and Methods

In this research, 445 interface friction angles were statistically evaluated against the influence of 11 numerical variables for geomembrane interfaces with sands (SG). It should be noted that some variables were considered as categorical, represented not by values obtained by the tests but as they were used or not used. This case occurred for types of geomembrane manufacturing material in each interface. In other words, for the SG-CDC and SG-PI interfaces, the categorical variables considered were HDPE, PVC and LDPE, HDPE, PVC and LDPE, respectively. It is important to note that the types of tests selected, as well as the boundary conditions considered, were adopted in this format in order to maintain randomness in the search for the variables that most influence the resistance between geosynthetic interfaces ([6]).

In this work, the preference for obtaining data was initially given to shear tests that were carried out for the sand-geomembrane interface and, for this reason, the variables considered were as follows: test speed (v), normal stresses (σ_n), contact area (A_c), geomembrane thickness (t_{GM}), geomembrane roughness height (h_G), density index (I_D), specific mass (ρ_s), curvature coefficient (C_c), coefficient of non-uniformity (C_u), average grain diameter (D_{50}), peak soil friction angle (ϕ_s), interface friction angle (ϕ_{int}), type of test (CDC or PI), type of geomembrane manufacturing material.

The selection of data consisted of a survey of national and international studies on resistance between interfaces with geosynthetics. The focus of the database was on conventional direct shear (CDC) and inclined plane (PI) tests, with a total of 270 and 175 interfaces evaluated, respectively.

2.1 Statistics and Python language applied in the research

The focus of this article was to construct the Pearson correction matrix, which was obtained by correlating the numerical variables used. Obtaining a correlation matrix is based on the calculation of the correlation coefficient shown in Equation 1. To obtain the Pearson matrix, you need to know the Pearson correlation coefficient (r) for each pair of variables studied. This coefficient can be obtained using Equation 1, presented by ([7]):

$$r = \frac{cov(X,Y)}{\sqrt{var(X).var(Y)}} \tag{1}$$

In this article, the correlations were classified according to the limits established by ([8]), as shown in Table 1.

Table 1. Interpretative values of Pearson's correlation coefficient (Adapted of Pant & Ramana, 2021).

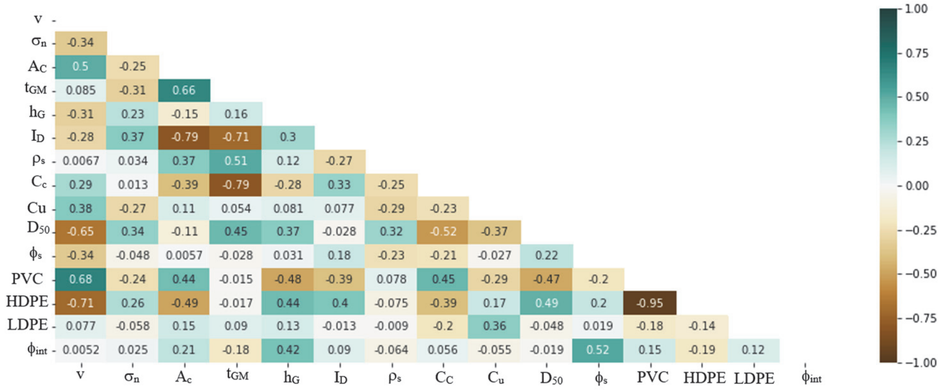
Pearson's correlation coefficient value (r)	Correlation Classification
0,0 a 0,10 (-0,0 a -0,10)	Positive (ou negative) very low
0,10 a 0,20 (-0,10 a -0,20)	Positive (ou negative) low
0,20 a 0,50 (-0,20 a -0,50)	Positive (ou negative) moderate
0,50 a 0,70 (-0,50 a -0,70)	Positive (ou negative) strong

3 Results

3.1 Sand-geomembrane interface – SG-CDC

As explained at the beginning of this paper, the Pearson Matrix was applied in order to assess the correlations between the variables studied. The result of this construction for the SG-CDC interface can be seen in Figure 1. Due to the limited space available to present the results, emphasis will be placed on the most important discussions.

Figure 1 shows that the highest correlation coefficients at the SG-CDC interface were for the variables soil friction angle (ϕ_s) and geomembrane roughness height (h_G), with correlation values of 0.52 and 0.42, respectively. These variables have been evaluated by several authors, who have concluded that the height of the geomembrane roughness, as well as the size of the soil particles, have a great influence on the interface resistance ([9-15]).



	v	σ_n	A_c	t_{GM}	h_G	I_D	ρ_s	C_c	C_u	D_{50}	ϕ_s	PVC	HDPE	LDPE	ϕ_{int}
v															
σ_n	-0.34														
A_C	0.5	-0.25													
t_{GM}	0.085	-0.31	0.66												
h_G	-0.31	0.23	-0.15	0.16											
I_D	-0.28	0.37	-0.79	-0.71	0.3										
ρ_s	0.0067	0.034	0.37	0.51	0.12	-0.27									
C_c	0.29	0.013	-0.39	-0.79	-0.28	0.33	-0.25								
Cu	0.38	-0.27	0.11	0.054	0.081	0.077	-0.29	-0.23							
D_{50}	-0.65	0.34	-0.11	0.45	0.37	-0.028	0.32	-0.52	-0.37						
ϕ_s	-0.34	-0.048	0.0057	-0.028	0.031	0.18	-0.23	-0.21	-0.027	0.22					
PVC	0.68	-0.24	0.44	-0.015	-0.48	-0.39	0.078	0.45	-0.29	-0.47	-0.2				
HDPE	-0.71	0.26	-0.49	-0.017	0.44	0.4	-0.075	-0.39	0.17	0.49	0.2	-0.95			
LDPE	0.077	-0.058	0.15	0.09	0.13	-0.013	-0.009	-0.2	0.36	-0.048	0.019	-0.18	-0.14		
ϕ_{int}	0.0052	0.025	0.21	-0.18	0.42	0.09	-0.064	0.056	-0.055	-0.019	0.52	0.15	-0.19	0.12	

Fig. 1. Pearson correlation matrix for the SG-CDC interface.

Some studies have shown that it is possible to obtain a higher interface shear strength when the average particle size (D_{50}) of the sand coincides with the size of the asperities in the interface material, causing better interlocking at the interface due to the sand particles fitting into the asperities ([12], [14], [15]). However, Figure 1 shows a very weak correlation of -0.019 between ϕ_{int} and D_{50}, according to the classification defined by ([8]). Analyzing the database studied for the SG-CDC interface in order to compare it with the aforementioned conclusion regarding the size of the average diameter, it was observed that the D_{50} of this interface was 0.47 mm and the height of the asperities was 0.17 mm when the data was treated together. Therefore, average grain size was around 2.50 times greater than the average height of the asperities, and possibly this fact contributed to the fact that it was not possible to evaluate the conclusion reached by ([12]), given that the authors stated that there is an increase in interface resistance when the average diameter is similar to the size of the geomembrane roughness.

The very weak correlation obtained between ϕint and D_{50} in this study can also be explained by ([16]), who suggested that the average grain size was around 2 times greater than the average height of the asperities, a conclusion that agrees with the findings of this study and disagrees with the proposition put forward by ([14]). It is also important to note that the correlation matrix calculates a linear value between each pair of variables and does not consider multiple correlations for the same variable.

Regarding the contact area of the tests, it should be noted that for the study of geosynthetic interfaces the D5321 standard (ASTM, 2021) indicates the use of square or rectangular containers, which must have a minimum dimension of 300 mm, 15 times the d_{85} of the soil with the largest granulometry used in the test, or a minimum of 5 times the maximum opening size (in the plane) of the geosynthetic tested. In such cases, it is more common to apply the modified direct shear test ([17]; ([14-16]; [18]; [19]; [13]; [20]). In the database for the CDC test, a variation of 36 to 100 cm² was observed in the areas used and, therefore, it was noticed that this range is smaller than that recommended by ([21]).

However, it is worth pointing out that the contact area of the tests can normally follow the definitions of the research projects themselves, as well as the dimensions of the equipment developed during the studies. There are cases in which authors are interested in studying different contact area sizes, sometimes really different from those established in standards, to check the influence of this change on the contact area of the tests, which are known in the literature as modified shear tests ([17]; [22]; [14-15]; [18]; [19]; [13]; [23]; [20]).

In terms of the correlation with ϕ_{int} (interface friction angle), the contact area (A_c) in CDC tests demonstrated a moderate correlation according to the classification by ([8]) with a positive value of 0.21 for the SG-CDC interface, as shown in Figure 1. This means that an increase in the contact area in CDC tests tends to lead to an increase in the interface friction angle ([9]; [13]). ([13]) concluded, through the study of ITAC (Influence of Contact Area Size) that this value had an influence on the results of the interface tests conducted by the author. It is worth noting that in contrast to these conclusions, ([33]) found out that the contact area was inversely proportional to the interface friction angle.

As for the influence of soil specific mass (ρ_s) on interface strength, ([24]) asserts that there is a gradual increase in the maximum shear stress as the specific mass of the soil increases. If there is greater grain penetration into the geomembrane, the tendency is for interface strength to also increase. For the SG-CDC interface, a strong correlation between specific mass and the interface friction angle was not observed. This could be explained by the fact that the average density index of the database resulted in approximately 60%, indicating that the grains, during the tests, likely needed greater loading to rearrange themselves and penetrate the geomembrane material.

Of the three types of geomembranes, it was observed that for PVC and LDPE the correlation was positive while for HDPE it was negative (Figure 1). It is important to note that in the database used, 40% of the geomembranes are smooth and made of HDPE. The influence of the geomembrane material was investigated by ([24]) who concluded that for HDPE geomembranes the interface friction angle tends to be lower than for PVC geomembranes because due to the fact that the mechanisms of grain sliding are different.

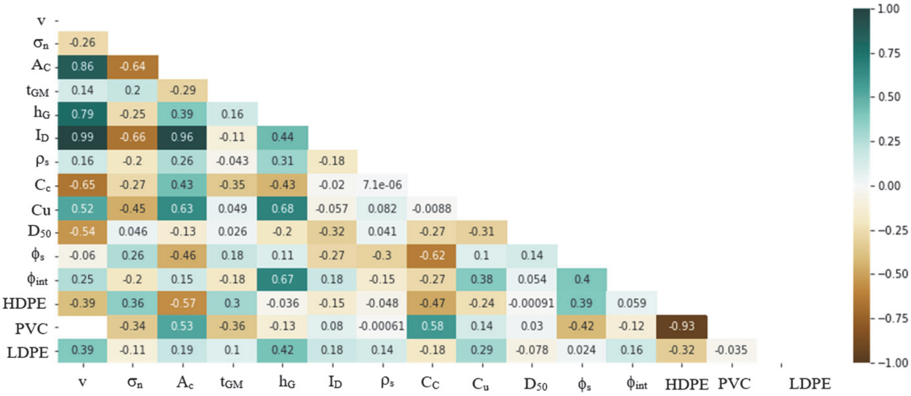
The studies by ([25]) also revealed a decrease in interface strength of approximately 15% when a smooth HDPE geomembrane is used in the soil-geomembrane interface. Therefore, the negative correlation found in this research between the interface friction angle and HDPE geomembranes aligns with these findings and is consistent with the idea that smooth HDPE geomembranes can result in reduced interface strength.

The very weak and positive correlation for the normal stress (σ_n) can be explained by a possible existence of a nonlinear correlation between these variables. The concept of nonlinearity was not addressed in this research, but it has been observed in the studies by

([6]). Additionally, very high stresses can damage the geomembrane asperities, causing the sliding mechanism to lose contact, thus not showing a high correlation between ϕ_{int} and σ_n.

3.2 Sand-geomembrane interface – SG-PI

To assess the correlations between all the variables observed in SG-PI interface, the Pearson Matrix was obtained, which can be seen in Figure 2.



	v	σ_n	A_c	t_{GM}	h_G	I_D	ρ_s	C_C	C_u	D_{50}	ϕ_s	ϕ_{int}	HDPE	PVC	LDPE
v															
σ_n	-0.26														
A_C	0.86	-0.64													
t_{GM}	0.14	0.2	-0.29												
h_G	0.79	-0.25	0.39	0.16											
I_D	0.99	-0.66	0.96	-0.11	0.44										
ρ_s	0.16	-0.2	0.26	-0.043	0.31	-0.18									
C_c	-0.65	-0.27	0.43	-0.35	-0.43	-0.02	7.1e-06								
Cu	0.52	-0.45	0.63	0.049	0.68	-0.057	0.082	-0.0088							
D_{50}	-0.54	0.046	-0.13	0.026	-0.2	-0.32	0.041	-0.27	-0.31						
ϕ_s	-0.06	0.26	-0.46	0.18	0.11	-0.27	-0.3	-0.62	0.1	0.14					
ϕ_{int}	0.25	-0.2	0.15	-0.18	0.67	0.18	-0.15	-0.27	0.38	0.054	0.4				
HDPE	-0.39	0.36	-0.57	0.3	-0.036	-0.15	-0.048	-0.47	-0.24	-0.00091	0.39	0.059			
PVC		-0.34	0.53	-0.36	-0.13	0.08	-0.00061	0.58	0.14	0.03	-0.42	-0.12	-0.93		
LDPE	0.39	-0.11	0.19	0.1	0.42	0.18	0.14	-0.18	0.29	-0.078	0.024	0.16	-0.32	-0.035	

Fig. 2. Pearson correlation matrix for the SG-PI interface.

It can be seen that the highest correlation coefficients at the SG-PI interface occurred for the asperity height of the geomembrane (h_G) and soil friction angle (ϕ_s), with positive correlation values of 0.67 and 0.40, respectively. These variables have been evaluated by several authors, who have concluded that the height of the geomembrane's roughness, as well as the characteristics of the particles have a great influence on interface resistance ([26]; [9]; [12]; [13]; [25]; [27-28]). Therefore, it can be concluded that the strong and moderate correlations for the variables h_G and ϕ_s were already expected and they converge to the conclusions drawn from the literature.

On the other hand, the Uniformity Coefficient (C_u) and displacement rate (v) also showed a moderate positive correlation when compared to the angle of interface friction, with values of 0.38 and 0.25, respectively. The Uniformity Coefficient indicates the granulometric variation of the soil, and the C_u presented by the database studied was 1.85, indicating a uniform granulometry ([29]).

For the displacement rate variable (v) there was a moderate positive correlation with the interface friction angle, with the average tilt rate obtained in this study being close to 2°/min. It should be noted that the faster the equipment tilts, the less time the soil particles have to readjust and this may influence the test results. For this reason, it is of the utmost importance to respect the normative recommendation for the speed of inclination of the ramp (2.50 to 3.50° per minute), as proposed by [30].

The variables that showed a very weak positive correlation with the interface friction angle were the density index (I_D) and the contact area (A_c). Regarding the I_D, this result was expected, as studies by ([12]) have shown that the morphology of particulate materials has a more significant impact on interface strength than the void ratio with which the interface is tested.

As for the variable A_c, the very weak positive correlation can be justified based on the conclusions reported by ([9]) who found that for contact areas greater than 900.00 cm², there was no noticeable influence on the equipment's tilt rate and, consequently, on the interface friction angle. In the database studied, more than 50% of the data had contact areas larger than the one mentioned by ([9]).

The normal stress (σ_n) showed a weak negative correlation of 0.20. This negative correlation was expected for the PI tests, as concluded by ([31-32], [13]). These studies found that the friction angle decreased with an increase in normal stress in the scenarios they examined.

Finally, a weak negative correlation was also found in this research for the variables soil density (ρ_s) and geomembrane thickness (t_{GM}). As the normal stresses typically applied in the inclined plane test are low, grain rearrangement and penetration into the geomembrane may not have occurred significantly, leading to a low influence on the interface friction angle. ([28]) concluded that for interfaces using sand, the size and density of the sand can interact differently depending on the surface characteristics of the geomembranes analyzed.

4 Conclusions

This paper assessed the use of the Pearson's matrix to evaluate the influence of different variables on the interface shear strength between sand and different types of geomembranes. Based on the results, the following conclusions can be drawn:

- It was observed that the parameters that most influences the interface resistance for SG-CDC tests were the soil friction angle (ϕ_s) and the asperity height of the geomembrane (h_G). It was also possible to conclude that for SG-CDC interfaces 7.15% of the correlations exhibited strong and moderate relationships, 35.70% were weak, and 50% of the correlations were very weak.
- On the other hand, for the inclined plane (PI) test, it was observed that the parameters that most influences the interface resistance for SG-PI were h_G and ϕ_s. For the SG-PI interface, only h_G showed a strong correlation, with 29%, 50% and 14% of the variables classified as moderate, weak and very weak, respectively.
- It's important to highlight that parameters showing very weak correlations may indicate that the behavior concerning ϕ_{int} is nonlinear. Further studies would be necessary to investigate this behavior, as nonlinearity was not the focus of this research.

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