Circular thin plates buckling analysis with HRPIM method

Sokayna BAID[1]*, *Youssef* Hilali[1,2], *Saïd* Mesmoudi[3], and *Oussama* Bourihane[1]

[1]Mechanical Engineering Laboratory, Faculty of Science and Technology, Sidi Mohamed Ben Abdellah University, Fez, Morocco
[2]Faculty of Science and Technology, Hassan First University, Settat, Morocco
[3]LISA Laboratory, National School of Applied Sciences, Hassan First University, Berrechid, Morocco

Abstract This paper aims to introduce a new approach to simulate geometrically nonlinear problems that require shape functions with higher order continuity such as the buckling analysis of circular thin and thick plates based on two theories; Classical plate theory (CPT) and Third order Shear Deformation Theory (TSDT). The algorithm integrates high-order continuation (HOC) solver and the Hermite-type radial point interpolation method (HRPIM). The in-plane displacement is approximated with RPIM method, while the HRPIM approach is used to compute the transverse component and its derivatives. The governing partial differential equations are discretized using the Galerkin method and solved by combining a Taylor series expansion with a continuation procedure. The paper includes two numerical examples of clamped and simply supported circular plates subjected to radial compression. The critical buckling loads have been calculated for different values of h/R ratio and compared with Finite Elements Method to illustrate the efficiency, robustness and accuracy of this approach across various boundary conditions.

Keywords: *Buckling, circular Plate, RBF, RPIM, HRPIM, High Order Continuation, CPT, TSDT.*

1. Introduction

The Finite Element Method (FEM) is a popular numerical approach for solving problems involving complex and varied geometries. However, advanced numerical techniques like meshfree methods are now being developed to achieve more precise approximate solutions, even for highly complex systems. These methods aim to overcome the limitations of FEM, such as element distortion, the need of re-meshing during large deformations, and the issues associated to mesh generating step.

To analyze the linear and nonlinear behavior of plates, numerous theories have been developed such as the classical Love-Kirchhoff theory [3-5] that neglects the effects of transverse deformation, and it predict specially the behavior of rectangular thin plates.

However, for circular plates, the Classical Plate Theory (CPT) estimations of the critical buckling load do not align frequently with theoretical values. Therefore, other theories, such

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* Corresponding author: soukabaid@gmail.com

as Higher-order Shear Deformation Theories (HSDT) [7] and the First-order Shear Deformation Theory (FSDT) [6], have been formulated to take into account others parameters such as the material gradients, nonlinearities, and geometric complexities.

The variational principle is used to establish stability equations for the Kirchhoff-Love plate, which are then discretized using the Hermite-type Radial Point Interpolation Method [14]. The displacement field is obtained by solving these equations, the stress and strain fields are derived using Taylor series expansions. To compute the complete solution, a continuation technique is applied. This method converts the initial problem into a set of equations, and constructs the solution step-by-step through successive branches.

2. Hermite Radial Point Interpolation Method

The shape functions of radial point interpolation method (RPIM) are formulated using the radial basis functions (RBFs) in order to overcome the singularity issue faced with the polynomial method (PIM) [2,9,11]. When considering derivatives at specific points as additional unknowns, a Hermite-type interpolation technique is adopted using the same formulation as RPIM but the additional degrees of freedom associated with the derivatives are considered to ensure more precise representation of field variables and their derivatives at specified points.

The shape functions are constructed with the radial point interpolation method (RPIM) formulated with radial basis functions (RBFs) to address the singularity issue encountered in the polynomial interpolation method (PIM) [2, 9, 11]. When the displacement derivatives are considered as additional unknowns at specific points, a Hermite-type interpolation technique is used. This approach has the same formulation as RPIM method but includes additional degrees of freedom associated with derivatives, ensuring a more accurate representation of field variables and their derivatives at the interpolation points.

$$u^h(x) = \sum_{j=1}^{n} R_j(x) a_j + \sum_{j=1}^{n} \frac{\partial R_j(x)}{\partial_x} b_j + \sum_{j=1}^{n} \frac{\partial R_j(x)}{\partial_y} c_j + \sum_{K=1}^{m} p_k(x) d_k \tag{1}$$

Using the RPIM and HRPIM methods, the displacement field can be interpolated as:

$$\begin{cases} u(x) & = & \sum_{k=1}^{n} \phi_k(x)\, u_k \\ w(x) & = & \sum_{k=1}^{n} \Psi_k\,(x) w_k + \Psi_k^x(x) w_{,xk} + \Psi_k^y(x) w_{,yk} \end{cases} \tag{2}$$

where the first line is the classical Meshless approximation while the second is a Hermite-type one. The vector of degrees of freedom can be expressed in function of the vector of neighboring nodes as:

$$u = \Phi u_e \tag{3}$$

The gradient of displacement vector can be written as:

$$\theta = \Psi u_e \tag{4}$$

By integrating (3) and (4), the system of equilibrium equations can be expressed as:

$$\int_A B(\theta)^T N dA + \int_A \beta^T M dA - \lambda \int_A T \Phi^T f d\bar{\Gamma}_t = 0 \tag{5}$$

with,

$$\begin{cases} \beta & = & \Psi H_2 \\ B(\theta) & = & \Psi(H_1 + A(\theta)) \\ f & = & \langle f_x \quad f_y \quad f_z \quad M_x \quad M_y \rangle \end{cases} \tag{6}$$

where f_x, f_y are the in-plane distributed loads, f_z is the bending load, M_x, M_y are the distributed moments about (Ox), (Oy) axis respectively.

The equilibrium equations for the stability analysis of thin plates can be expressed as follows:

$$K_T u + F^{nl} = \lambda F \tag{7}$$

where, K_T is the global tangent stiffness matrix, u is global vector of degrees of freedom, (F^{nl}) represents the nonlinear forms vector and F is the external forces vector.

The nonlinear system (7) is solved using a higher-order solver [5,9,10,11], which employs a Taylor series expansion and a continuation technique.

3. Numerical examples and discussions

To demonstrate the efficiency and convergence of the proposed HRPIM-HOC approach, two numerical examples based on Kirchhoff–Love plate theory will be presented. The results obtained will be compared with analytical solutions and Abaqus results. The HOC algorithm used in our numerical study has the following parameters: truncation order of N= 13, a tolerance parameter $\varepsilon = 10^{-6}$. Additionally, a bi-linear polynomial (8) and a bi-cubic polynomial (9) are employed as basis functions to calculate the shape functions for HPIM and HRPIM, respectively.

$$P^T(x) = [1\ x\ y\ xy] \tag{8}$$

$$\boldsymbol{P^T}(\boldsymbol{x}) = [1 \quad x \quad y \quad x^2 \quad xy \quad y^2 \quad x^3 \quad x^2y \quad xy^2 \quad y^3 \quad x^3y \quad xy^3] \tag{9}$$

with $m = 3 \times n = 12$. For the Radial basis function, many tests with the several RBFs and with various values of the parameters q, c and n have been conducted and we have concluded that the multi-quadratic RBF (10) gives a good result with the following parameters: $q = 0.5$ and $\mathrm{c} = \frac{2}{\sqrt{nnodes}}$, [12] , n represents the number of neighboring nodes and it is determined with a maximal research distance $d\_max = 1.7$, $nnodes$ is the number of the nodes used to discretize the plate

$$\text{Multi} - \text{quatric: } \mathrm{R(x, y)} = (\mathrm{r}^2 + \mathrm{c}^2)^q \tag{10}$$

3.1. Radial compression of simply supported circular isotropic plate

In this example, we examine the behavior of a thin elastic circular plate under radial compression load ∂_r expressed as $\partial_{rx} = \frac{-2\pi * x_r}{4*nR}$ following the x-direction and $\partial_{ry} = \frac{-2\pi * y_r}{4*nR}$ in the y-direction ,where nR is the number of nodes in the force edge, x_r and y_r are the coordinates of theses nodes, the plate is simply-supported in force edge as illustrated in Fig. 1. Due to symmetry, only the upper right quadrant of the plate is studied, Hence, symmetry conditions are imposed on the left and bottom edges.

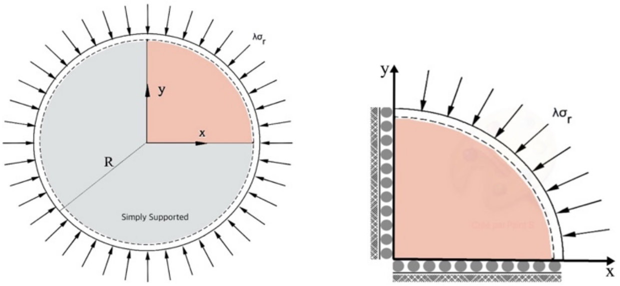

Fig. 1 Thin elastic circular plate with simply supported edges under a Radial compressive loading $\lambda\sigma_r$

The radius of the plate is R=10 mm, h is its thickness, it is made of Aluminum with Young's modulus E =70 GPa and Poison's ratio ν=0,3. The structure is discretized using 293 nodes and 260 integration cells, each integration cell contains 2 × 2 Gauss quadrature nodes.

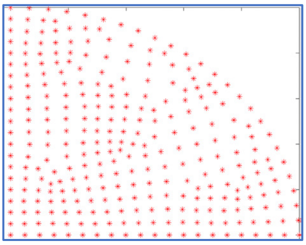

Fig. 2 Nodal discretization of the circular plate

The table 1 represents the critical buckling load versus the h/R ratio. It is evident that the critical buckling loads obtained using the proposed method align closely with both analytical and Abaqus results for the two theories.

We can notice also that the critical buckling load increases as the h/R increases and the approach results are in good agreement with analytic and Abaqus ones when the h/R value does not exceed 0.05.

The analytic critical buckling for a clamped and simply supported circular plate are calculated according to the following equation as proposed in [11]:

$$P_{cr} = \frac{\beta^2 D}{R^2} \tag{11}$$

with

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{12}$$

For the first root, $\beta = 2.0488$ in the case of a simply supported plate and $\beta = 3.8317$ for the clamped one.

h/R	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Analytic	0,2690	2,1526	7,26504	17,2208	33,6344	58,12035	92,2929	137,7667	196,1562	269,076
HRPIM-CPT	0.2425	1.9400	6.54757	15.5201	30.3128	52.3806	83.1785	124.1615	176.7846	242.503
HRPIM-TSDT	0.2642	2.0797	6.9019	16.0842	30.8821	52.4591	81.8935	120 .185	168.2659	227
Abaqus	0,269	2,1517	7,2574	17,188	33,532	57,865	91,739	136,69	194,21	265,78

Table. 1 Critical buckling load $P_{cr}[N/mm]$ of simply supported plate subjected to radial compressive loading in function of h/R ratio

3.2. Radial compression of clamped circular isotropic plate

In the second example, a circular plate with the same geometrical and mechanical characteristics as the first example is considered, it´s clamped on the force edge. The table 2 shows that, based on both theories, the critical buckling loading values obtained with the proposed approach are in strong agreement with those of the FEM ones for different h/R ratios.

It can be observed that the critical buckling load increases with the increase of h/R ratio and the approach results align strongly with analytic and Abaqus ones for the thin plates when the h/R value does not exceed 0.04

h/R	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Analytic	0,9411	7,5291	25.4111	60.2335	117.6436	203.2881	322.8141	481.8683	686.0976	941.1490
HRPIM-CPT	0.8678	6.9431	23.4330	55.5450	108.4864	187.464	297.686	444.3606	632.6932	867.8919
HRPIM-TSDT	0.9051	7.0860	23.3819	54.1706	103.4036	174.6437	271.0964	395.6359	550.8259	738.9368
Abaqus	0.9408	7.5168	25.313	59.817	116.37	200.12	316	468.65	662.43	901.37

Table. 2 Critical buckling load $P_{cr}[N/mm]$ of clamped plate subjected to radial compressive loading versus h/R ratio

4. Concluding remarks

The majority of meshless methods such as PIM, HPIM, RBF… give a good estimation of the thin rectangular plate critical buckling loads. Therefore, HRPIM approach is used to predict correctly the buckling behavior of circular plates, to avoid the singularity of the global stiffness matrix and to obtain consequently a good estimation of critical buckling loads.

This paper presents a buckling behavior analysis of isotropic circular plate using the Classical Plate Theory (CPT) and the Third-order Shear Deformation Theory (TSDT). The numerical solution is obtained by combining the HRPIM method with higher-order solver. The results show that the bucking load increases when the h/R ratio increases also and the proposed method provides sufficiently accurate solutions, and allows a good prediction of critical buckling loads for thin isotropic circular plate under two various boundary conditions, especially when the h/b values are less than 0.05 for simply supported plate and 0.04 for the clamped one.

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