Application of Stokes' higher order theory and JONSWAP spectrum in modelling extreme wave events validation with historical data

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Abstract. Accurately modelling wave elevations is critical for the design and safety of offshore structures. This study explores the application of Stokes' 5th Order Wave Theory and JONSWAP (Joint North Sea Wave Project) spectrum to simulate wave conditions similar to those experienced during the Draugen Ringing Event. Hindcasting on the dataset collected form Norwegian Meteorological Institute sensor system Draugen is performed. Through rigorous analysis performed by integrating these models, we aim to validate their accuracy and reliability against empirical data. The methodology includes generating the JONSWAP spectrum based on significant wave heights and peak periods, followed by computing wave surface elevations using Stokes' 5th Order Wave Theory and spectral analysis. The outcome of the analysis shows a strong correlation between the modelled irregular wave elevations and actual wave data, demonstrating the effectiveness of these models.

1 Introduction

In the field of offshore engineering, the accurate modelling of wave elevations is a fundamental aspect of ensuring the safety and stability. The offshore wind industry is an extension of offshore oil and gas industry in a sense that the structural forms, material, and loading conditions are similar. The past incidents of structural failure in oil and gas industry provide key insight in the development of offshore wind industry. These accidents highlighted the structure's need to withstanding extreme wave events, such as solitary rogue waves and wave overtopping. Historical disasters, including the Alexander L. Kielland, Ocean Ranger, Usumacinta, and Draugen platform incidents, underscore the severe impacts that oceanic forces can have on structural integrity.

The primary objective of this paper is to investigate a comprehensive model for mapping wave elevations of large nonlinear waves. Stokes' 5th Order Wave Theory and the JONSWAP spectrum will be applied to reconstruct the wave surface elevation. By simulating wave conditions similar to those experienced during the Draugen Ringing Event, we aim to

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validate the accuracy and reliability of these theoretical models. To compare the modelled wave elevations with actual values, data is collected from Norwegian Metrological Institute for a period between *01-01-1995* to *12-31-1995*. The data is imported from their sensor number *SN 76925* short named *Draugen* located at *64.35°N, 7.77°E* to assess the model's performance [1].

2 Historical background

In the offshore oil and gas industry, the interplay between engineering structures and the forces of nature often results in scenarios that highlight the importance of structural integrity and environmental readiness. Solitary rogue waves and wave overtopping events, unpredictable and overpowering, present a profound threat to the stability and safety of offshore platforms. The disasters of the Alexander L. Kielland, Ocean Ranger, Usumacinta, and Draugen platforms serve as testaments to the severe impacts these oceanic forces can wield, marking critical episodes in the evolution of offshore safety protocols and engineering standards.

The Draugen platform, designed for extreme conditions at a depth of 250 meters, faced a abnormal conditions that resembled a hundred-year wave that violently displaced its equipment, highlighting the continuous risk these structures face from potent maritime forces on 12th March 1995. The significant wave heights grew from 5 meters to 9.5 meters in just seven hours [2]. The historical accounts of severe wave-induced disasters in the offshore industry emphasize the need to model and predict extreme events as a fundamental design consideration.

3 Modelling of ocean waves

3.1 Classification of ocean waves

Ocean waves can be categorized using multiple methods. One of the approaches is to classify them based on wave period and corresponding wavelength. The ocean waves can be organized into seven distinct classifications based on their period bands, with each category characterized by specific generating and restoring forces. Capillary waves, with periods less than 0.1 seconds, are primarily generated by wind and restored through surface tension. Ultra-gravity waves, occurring within 0.1 to 1 second, are also wind-driven but are restored by both surface tension and gravity. Gravity waves, with periods ranging from 1 to 20 seconds, are generated by wind and restored by gravity alone. Infra-gravity waves have longer periods, ranging from 20 seconds to 5 minutes, and are influenced by both wind and atmospheric pressure gradients, with gravity serving as the restoring force. Long-period waves, which can last from 5 minutes up to 12 hours, are generated by atmospheric pressure gradients and earthquakes, with gravity as the restoring force. Ordinary tidal waves, with periods between 12 and 24 hours, are influenced by the gravitational pull of celestial bodies and are restored by both gravity and the Coriolis force. Lastly, trans-tidal waves, which exceed 24 hours in duration, are driven by storms and gravitational forces and restored by gravity and the Coriolis force. This classification helps in understanding the dynamics and interactions of oceanic wave motions [3].

3.2 Ocean wave theories

Ocean waves usually transmit the energy and forces following a circular motion. A great deal of research has been conducted to model the wave profile of the ocean waves. As discussed earlier it is prudent to maintain sufficient levels of structural integrity satisfying the requirements of serviceability throughout the lifespan of the structure. The stochastic wave forms the basis of the structural design load for which the structure is to be analysed. Modelling of these waves can be done in one of the two ways: deterministic and statistical. In deterministic models the wave can be modelled in one of the two basic ways, linear or non-linear, and categorized as regular and irregular. Dean and Le Mehaute (1976) have categorized the applicable region of wave theories based on non-dimensional parameters as we shall see later. In any case these methods fall short in describing the randomness of the sea, to account for these researchers have developed spectral methods which use the practice of sea state statistics, describing the wave profile by at least two of the parameters from wind speed, wave period, peak period, wave height, or wave length [4][5].

Here it is noteworthy to mention that the development of the spectral methods owing to model the randomness and the identifying the one extreme event is still significantly computationally intensive. This is because of several reasons including very small value of the probability distribution function (PDF) at extremes, absence of shape of the extreme event, nonlinearity in timeseries [6]. These issues are currently addressed by researchers via variety of methods that include analysing large dataset, addressing nonlinearities through modal decomposition, multifractal detrended fluctuation analysis, and machine learning. However, here we limit ourself to the discussion of finding the best solutions to model stochastic sea conditions using spectral and deterministic methods.

3.3 Selection of wave theories and their application in wave modelling

The classical nonlinear wave theories include Stokes (1847) and Dean (1965) and cnoidal theory for shallow waters Kortewg and de Vries (1985). It is important to select the right model for reconstruction of wave surface elevations. Similar to Dean and Le Mehaute (1976) IEC 61400-3 annexure C.1 provided curves to select the applicable wave theory for the prediction of kinematics of 2 dimensional waves. The curves are based on dimensionless parameters H/gT^2 representing wave steepness and d/gT^2 representing the relative depth of water in comparison to natural scales set by gravity and wave period in which H is the wave height L is the wave length and d is the depth of the ocean. When waves are too steep or water is too shallow the linear theory does not work [5]. For our data we have computed these values and created a scatter plot as shown in Figure. 1. The results clearly suggest the use of Stokes higher order theory. For similar values IEC 61400-3 (2009) suggest the use of Stokes' 5th order or stream function 3 [7]. The surface elevation η generated by Stokes' 5th order theory takes the form as in equation 1. Where k is the wave number, ω is angular frequency, x is the spatial coordinate, and t is time.

$$k\eta = \sum_{n=1}^{5} \epsilon^n b_n \cos(n\theta) \tag{1}$$

$$\theta = (kx - \omega t) \tag{2}$$

$$\epsilon = \frac{kH}{2} \tag{3}$$

The wave surface profile is computed by summing the series from 1st to 5th order terms the coefficients bn have been computed by Fenton (1985) are not shown here, can be found in [8].

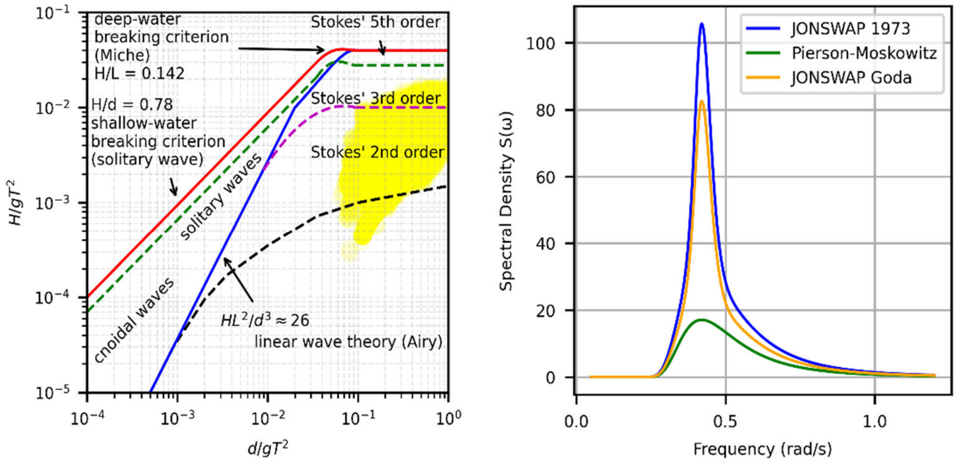

Fig. 1. Range of applicability of various wave theories [5] (left), wave spectra (right).

The cyclical; force and energy transmission mechanism of the ocean waves, permits the use of spectral method for the reconstruction of the stochastic wave surface η. This property of the ocean wave allows the wave record to be transformed to spectral domain from temporal domain. This transformation is the basis of creation of several energy spectra S(ω) created for the modelling of irregular waves. Most notable of these are Pierson-Moskowitz (PM) and JONSWAP. The former describes the fully developed sea state, while the latter is used for the fetch limited conditions where sea is still developing. Hasselmann et al., (1973) formulated the JONSWAP spectrum from 2500 spectra collected in the North Sea, the spectrum is a modification of the PM spectrum with a spectral peak elevation factor γ. Non-dimensional fetch $\overline{X}$ which is a function depending on U_{10} the mean wind speed at 10 m above the sea level, and X the fetch is used to calculate α the other parameter which significantly governs the shape of the spectrum is ω_p which is the frequency corresponding to spectral peak. The shape of the legs of the spectra is governed by the factor σ. The mean values for these parameters along with the formulation of JONSWAP spectrum and its modifications are presented in Table 1.

Table. 1. JONSWAP formulation Hasselmann, and Goda.

Hasselmann et al., 1973	**Goda, 1999**
$S(\omega) = \alpha_p g^2 \omega^{-5} \exp\left[-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right]\gamma^{\delta}$ (4)	$S(\omega) = \alpha H_S^2 \frac{\omega^{-5}}{\omega_p^{-4}} \exp\left[-1.25\left(\frac{\omega}{\omega_p}\right)^{-4}\right]\gamma^{\delta}$ (9)
$\delta = \exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma_0^2\omega_p^2}\right]$ (5)	$\delta = \exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma_0^2\omega_p^2}\right]$ (5)
$\sigma_0 = 0.09$ for $\omega > \omega_p$ (6)	$\sigma_0 = 0.09$ for $\omega > \omega_p$ (6)
$\sigma_0 = 0.07$ for $\omega \le \omega_p$ (7)	$\sigma_0 = 0.07$ for $\omega \le \omega_p$ (7)
$\gamma = 3.3$ (8)	$\alpha = \frac{0.0624}{0.230+0.0336\gamma-0.185(1.9+\gamma)^{-1}}$ (10)

Creating JONSWAP spectrum requires several input parameters that have to be measured, to overcome this an improved version of the JONSWAP spectrum was presented by Goda (1985). This method is notable in its simplicity in determining the shape of the spectrum, as once the γ is selected remaining quantities are easily calculated. Furthermore, in practice γ is varied to achieve the best fit, the remaining key parameters significant wave height H_s and peak frequency ω_p can be directly accessed from buoy's data [9]. Other notable modifications of the JONSWAP spectrum include Hasselmann et al., (1976), Mitsuyasu etal., (1980), Lewis and Allos (1990), Battjes et al. (1987), Donelan et al. (1985) and more [3].

The wave spectra are calculated for H_s of 13.4 meters and ω_p 0.42 rad/sec corresponding to the timestamp of 10-12-1995 in our dataset is depicted on right hand side of the Figure. 1. There is marked difference is peak spectral energy between PM and JONSWAP formulation due to γ. For the two variations of JONSWAP spectrum γ is selected as 3.3, Hasselmann's curve require additional parameter, the mean wind speed U_{10}. Shows a higher peak spectral energy in comparison to the Goda's modified curve.

4 Modelling wave elevations

To reconstruct the ocean waves and simulate the extreme events as occurred at our location of interest as described in section 1 we collected the data from the Norwegian Metrological Institute for a period between *01-01-1995* to *12-31-1995*. The selection of the query period was based on a specific event discussed earlier in section 2. As a first step the data is cleaned and processed. A closer inspection of the data revealed that the reported incident was not the most extreme, on 12th of October 1995 the sensor recorded even higher significant waves reaching 4 to 13.4 meters in 7 hours. After cleaning the data irregular wave profiles were generated against H_s and ω_p. The values for γ were iterated between 1.0 and 5.5 and for each created wave train maximum wave height is computed which is then compared with the observed H_{max} the closed fit is retained. The analysis is performed using two different temporal resolution and γ increments, for the whole year $\Delta\gamma$ is set as 0.1 for the analysis of select months of March and October $\Delta\gamma$ is set as 0.01. Finally, RMSE, and coefficient of correlation are calculated to determine fitness and error of the model. Figure 2 shows modelled nonlinear waves on actual data. Figure. 3 presents flowchart of the proposed modelling technique.

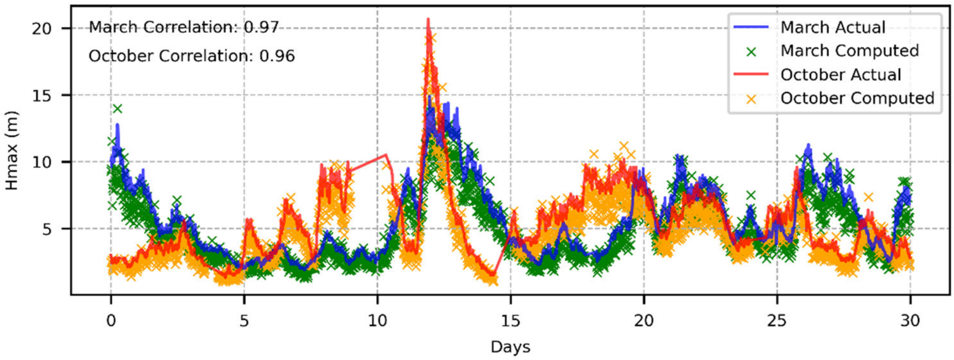

Fig. 2. Modelled nonlinear waves overlayed on actual data.

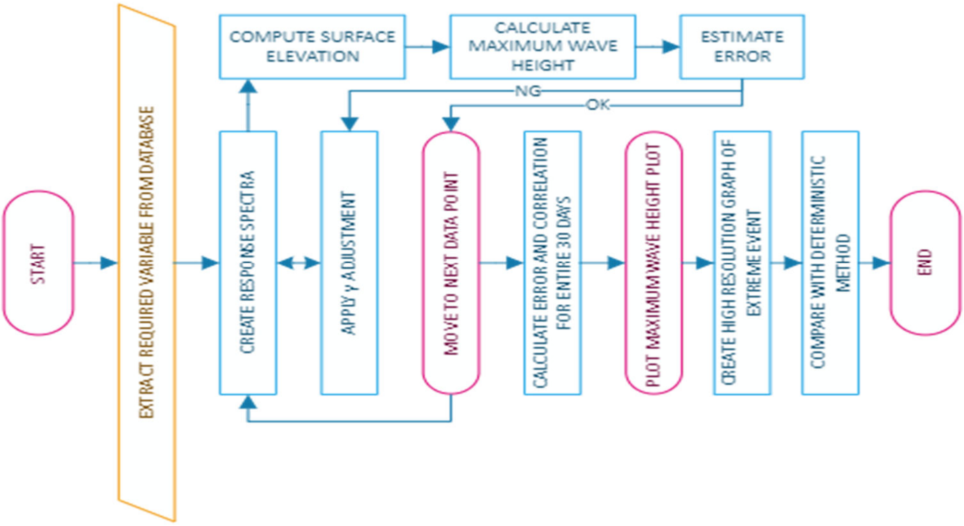

Fig. 3. Flow Chart for the proposed modelling scheme.

Finally, we have constructed surface elevation profile using Stoke's 5th order theory for peak period of 14.9 seconds and maximum wave height observed during the whole year of 20.7 meters. We also plotted a high temporal resolution wavelet using the spectral method adopted earlier as shown in Figure. 4.

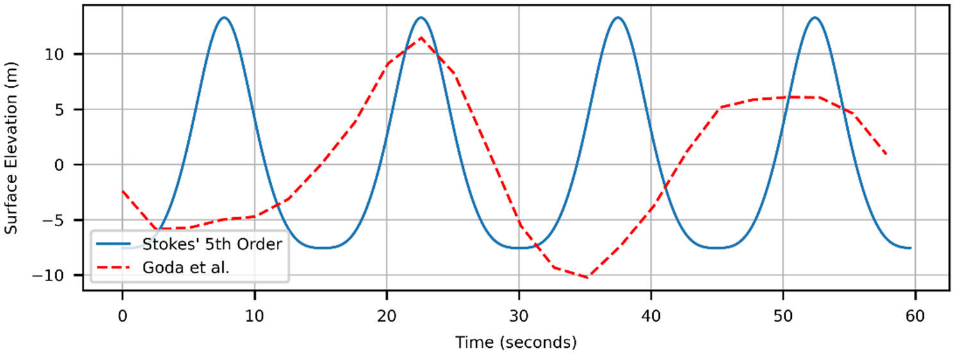

Fig. 4. High resolution plot of H_{max} modelled using Stokes 5th Order theory, and Goda's Spectrum.

5 Conclusion and discussion

A rigorous analysis has been performed to recreate the ocean surface elevation in extreme weather. The mean RMSE error between modelled and observed data is found to be around 15%, it is not unusually high considering the wave growth was highly nonlinear, a mark of nonstationary time series. However, the coefficient of correlation showed remarkable agreement with the measured data, Figure. 3. A comparative plot of deterministic vs spectral methods constructed in Figure. 4, highlighted that for the same wave height extensive computational prowess is required to model the stochastic nature of the ocean waves. However, it must not be neglected that deterministic methods can model a fix set of frequencies, and a potential of missing out a particular event still remains. The field of spectral analysis is still growing at rapid pace by incorporating elements of artificial intelligence leading to faster computing. The forecasting techniques were not discussed at the moment, however through literature review and reconstruction process it is notes that the ability to forecast an extreme event on the tails of PDF is computationally challenging. Next, the work will be further expanded in creation of 3-Dimensional wave field, and forecasting of extreme event coupled with Physics informed Machine Learning approach.

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