

# Probabilistic-fuzzy programming model with chance-constrained to optimize wastewater treatment plants: a case study with the Bantul wastewater treatment plant layout

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**Abstract.** This study introduces a novel optimization approach, employing chance-constrained probabilistic-fuzzy uncertain programming, to enhance the efficiency of facultative ponds in wastewater management systems. Unlike traditional deterministic or stochastic models, this approach integrates both probabilistic and fuzzy uncertainties capturing real-world variations in Biological Oxygen Demand (BOD) degradation rates and wastewater loads. The model addresses the decision-making scenario where some uncertain parameters, like the rate of BOD reduction, are probabilistic with some probability density functions, and some other uncertain parameters, like wastewater load, are represented as fuzzy variables with membership functions determined by the decision-maker. Amid this uncertainty, the goal is to maximize the volume of wastewater treated while maintaining adequate safety margins via chance-based rules is implemented to the objective and the constraints. Using the layout of the Bantul residential wastewater treatment plant located in Yogyakarta, Indonesia, this research succeeded in determining optimal decisions regarding processing time and wastewater flow rate. Consequently, it is inferred that the developed model effectively resolves the problem at hand, rendering it applicable for decision-makers in similar contexts.

## 1 Introduction

Wastewater must be stabilized prior to disposal to ensure water conservation and environmental sustainability, which are linked. However, given the limited number of treatment facilities for wastewater underscores the necessity to optimize their efficiency for maximum wastewater processing. These facilities commonly utilize algae, bacteria, and zooplankton to reduce pollutants [1]. Various parameters, including Biological Oxygen Demand (BOD), are utilized to assess water quality post-treatment [2]. Domestic wastewater originating from residences, hotels, and general industries comprises the typical wastewater types [3]. Facultative ponds are preferred in wastewater treatment for their straightforward approach to breaking down pollutants to meet standard concentration levels, as measured by, e.g., the BOD parameter. Additionally, mathematical optimization models are commonly

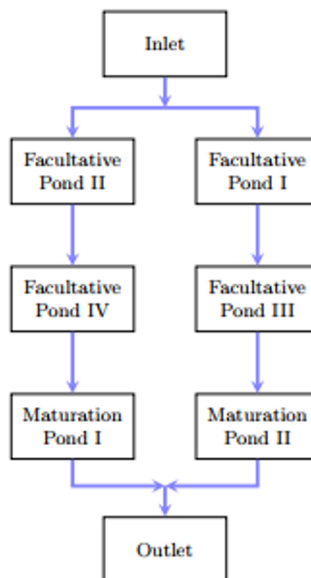
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used in management of wastewater treatment to increase facultative ponds' capacity and effectiveness.

Several wastewater management models have been developed, each designed to tackle the unique challenges encountered by decision makers [4-6]. For example, A straightforward model proposed in [7] controls pollutant concentrations through a quantitative prototype. Other straightforward models include the linear one with parameters that are deterministic [8], an analysis model for adsorbents used in wastewater treatment [9], a quantitative model for waste processing [10], and a model for estimating the cost of building wastewater treatment facilities [11]. In addition to optimizing wastewater treatment, various models have concentrated on different aspects, including waste management [12], energy assessment [13], analysis of effluent and sludge [14], removal of microplastics [15], and (bio)energy production from wastewater [16-21]. On the other hand, several previous studies have explored the application of chance-constrained programming across various fields, such as car rental management optimization [22], design of closed-loop manufacturing-remanufacturing system network [23], vessel scheduling problem [24], optimizing integrated networks for desalination and water treatment [25], and robust trajectory optimization during manipulation [26]. Nevertheless, none models have been formulated within a chance-constrained approach with both probabilistic and fuzzy parameters in one model for optimizing wastewater treatment plants, which would allow decision makers to incorporate uncertainty by specifying chance-based constraints; this study aims to address that gap.

Therefore, in this paper, a novel approach is devised to enhance the efficiency using facultative ponds for wastewater treatment, particularly considering uncertain factors such as pollutant concentrations at the entrance. This strategy enables decision makers to apply extra probabilistic restrictions to the model, like the likelihood of breaching uncertain thresholds at specified values. Fuzzy parameters are used to accommodate these uncertainties, and functions of membership are established grounded in the decision-maker's insights. Among the various parameters examined in this investigation, the BOD degradation rate parameter is scrutinized. The layout of the facultative ponds located in Bantul, Yogyakarta, Indonesia, was utilized as a case study to expand the model and facilitate optimal decision-making.



**Fig. 1.** The diagram of the Sewon waste water treatment facility in Yogyakarta

## 2 Fuzzy Programming

### 2.1 Formulation of the problem and assumptions

This study aims to investigate the BOD degradation within facultative ponds, with a focus on the layout of Bantul wastewater treatment plant, which is located near Yogyakarta, Indonesia. This facility collects domestic wastewater from houses, offices, industry, and hotels which is processed in stages, as seen in Figure 1. The basic objective is to optimize the volume of wastewater treated across every facultative ponds to meet quality standards regarding the BOD concentration under various uncertain parameters: probabilistic parameters and fuzzy parameters. The assumptions and specific specifications used in this research are described as follows:

1. Optimizations and observations were carried out for one day.
2. BOD is used as a parameter in monitoring wastewater quality.
3. Domestic wastewater processed at this facility only comes from Yogyakarta province.
4. The uncertainty of the inflow waste load is calculated in one day by adding fuzzy uncertainty. Using observations, past data, and intuition, decision makers assign membership functions to incoming waste loads.
5. The rate of BOD degradation was also considered a fuzzy parameter, and a membership function was developed by the decision-maker using both secondary historical data and observations. Observations regarding BOD levels were carried out on wastewater samples originating from several grid points for every facultative pond.
6. The BOD degradation process is used using index values, according to the mathematical model's description.
7. Discrete membership functions are assumed for all fuzzy parameters.
8. Normal distributions were assumed for all probabilistic parameters.

The method used in the current study is as follows. Initially, fuzzy parameter membership functions and determination of possibilities that do not violate lower probability-based constraints in the constraint function are carried out by the decision maker. The objective function is subsequently constructed to represent the wastewater inflow rate. Additionally, a control term for the BOD efficiency index is calculated as the quadratic difference between the actual effectiveness index and an initial point, determining reference points according to the intuition of decision maker's and expertise concerning the performance of the facultative ponds. Furthermore, functions of constraints are specified and formed within a mathematical model, taking into account the construction of the facility for wastewater treatment and the necessary circumstances. The formulated mathematical model is then solved computationally, initially by encoding it in a language for programming, LINGO 21.0 in this study, followed by solution using the embedded solver within the software. The optimal decision is computed using chance-constrained programming, as elaborated in [27]. Finally, the derived solution is implemented within the facilities.

### 2.2 Mathematical symbols

The mathematical symbols used to create the model for optimization are listed following;

Decision variables:

- |               |   |
|---------------|---|
| $R_0$         | : The inlet's inflow rate ( $m^3/day$ ),                      |
| $R_j^e$ [day] | : Flow rate entering the facultative ponds $i$ ( $m^3/day$ ), |
| $t$           | : Waste water retention time (day),                           |

Fuzzy parameter(s):

$W_0$  : Waste load entering through the inlet (kg/day),

$p_1$  : Waste load transferred from pond I,

$p_2$  : Waste load transferred from pond II,

Probabilistic parameter(s):

$k$  : The rate of BOD degradation for a one-day retention period,

Intermediate parameters:

$W_i$  : Waste load entering pond  $i$  (kg/day),

$W_i^e$  : Waste load treated in pond  $i$  (kg/day),

Certain parameters:

$E_i^r$  : Goal/reference point,

$SQ$  : Standard for wastewater levels,

Auxiliary parameters:

$C_i$  : Concentration of BOD in pond  $i$  (mg/L),

$E_i$  : efficiency index of BOD degradation in pond  $i$  (in percentage).

### 2.3 Fuzzy programming model with chance-constrained schemes

This optimization issue has two goals: 1) to maximize wastewater intake and 2) to minimize the difference of a quadratic function between the efficiency index of BOD degradation and the decision maker's reference values. The optimization problem resulting from the combination of the two objectives is as follows:

$$\min Z = -L_0 + \sum_{i=1}^4 [E_i - E_i^r]^2. \quad (1)$$

Note that the objective function has the aim of maximizing so it is necessary to multiply by negative one. This optimization problem has a constraint function which is formulated as follows:

- The calculation of waste load based on organic material and inflow rate:

$$L_i^e = \frac{Q_i^e \cdot C_i}{1000}, i = 1,2,3,4; \quad (2)$$

- Concentration of BOD remains below the upper limit:

$$E_i C_i \leq SQ, i = 1,2,3,4; \quad (3)$$

- The total of waste load entering ponds I and ponds II matched the waste load at the inlet, and:

$$W_1 + W_2 = W_0; \quad (4)$$

- The waste load transferred from ponds I into ponds III:

$$Cr\{L_3^e = (1 - p_1)W_1\} \geq \gamma_1; \quad (5)$$

where  $\gamma_1$  represents the credibility lower bound for the equality  $L_3^e = (1 - p_1)W_1$ ;

- The waste load transferred from pond II into pond IV:

$$Cr\{L_4^e = (1 - p_2)W_2\} \geq \gamma_2; \quad (6)$$

where  $\gamma_2$  represents the credibility lower bound for the equality  $L_4^e = (1 - p_2)W_2$ ;

- The pre-treatment levels limited the processed wastewater load in the facultative pond, as described by the following inequalities:

$$Cr\{W_1 + W_2 = W_0\} \geq \beta_i, i = 1,2,3,4; \quad (7)$$

where  $\beta_i$  represents the credibility lower bound for the equality  $W_1 + W_2 = W_0$ ;

- The inlet rate matched the influent rate to ponds I and ponds II, whereas the inflow rates to ponds III were half of those to ponds I and inflow rates to ponds IV were half of those II, respectively:

$$R_0 = R_1^e + R_2^e. \quad (8)$$

$$R_3^e = 0.5R_1^e \text{ and } R_4^e = 0.5R_2^e. \quad (9)$$

- The formula for determining efficiency index of the required BOD was established as follows:

$$Cr \left\{ E_i = \frac{m.t}{1+m.t} \right\} \geq \alpha_i, i = 1,2,3,4; \quad (10)$$

where  $m$  is the degradation rates of BOD, and the length of detention is set to one day and where  $\alpha_i$  represents the credibility lower bound for the equality  $E_i = \frac{m.t}{1+m.t}$ .

Gathering both objective function and constraint function results in a chance-based optimization dilemma. Optimal decision calculations require a chance constraint programming algorithm to overcome this challenge. Furthermore, all constraints are characterized by closure and boundedness, suggesting that this optimization problems consistently offers an optimal solution, as long as there is not an empty feasible region.

### 3 Material and method

Chance-constrained programming involves modeling systems with uncertain decisions under the assumption that those uncertainty constraints will be satisfied with a specified probability, known as the confidence level, which represents a safety buffer set by the decision maker [28]. Since the uncertainty parameters appear just on the function of models constraint (2)-(10), the functions of chance-based are required solely for these constraints. This is handled in the following manner. Let  $x$  be the vectors corresponding to the decision variables and  $\xi$  be the vectors that correspond to the decision variable. Regarding the generic form of the constraint function  $g_i(x, \xi) \leq 0$ , the formula for chance-based constraints is  $Cr\{g_i(x, \xi) \leq 0\} \geq \alpha$  for every  $i$ , where  $Cr$  denotes the credibility rate and  $\alpha$  denotes its lower bound. Descent point is one that meets the credibility rate for the event  $\{g_i(x, \xi) \leq 0, \text{ for every } i\}$  is at least  $\alpha$  i.e. The constraints may be breached in no more than  $(1 - \alpha)$  of all possible outcomes. Considering that the uncertainty parameter in this research are fuzzy, we initially assign weight values to these parameters. Remember the functions of membership is discrete. Now, let

$$\eta_\xi = \begin{cases} \eta_\xi^{(i)} & \text{if } \xi = \xi^{(i)}, i = 1,2,\dots,I, \\ 0, & \text{others;} \end{cases} \quad (11)$$

defining the discrete function of membership for the fuzzy parameter  $\xi$ . The expected value is then determined by

$$\mathcal{E}[\xi] = \sum_{i=1}^I [w_\xi^{(i)} \xi^{(i)}] \quad (12)$$

where  $w_\xi^{(i)}$  is referred to as the  $i$ -th weight value and is defined as [29]

$$w_{\xi^{(j)}} = \frac{1}{2} \left( \max_{1 \leq i \leq j} \eta_\xi^{(i)} - \max_{1 \leq i < j} \eta_\xi^{(i)} + \max_{j \leq i \leq I} \eta_\xi^{(i)} - \max_{1 < i \leq I} \eta_\xi^{(i)} \right). \quad (13)$$

The solution procedure for this type fuzzy programming may be represented as follows: first, using the weight value, chance-constrained programming is turned into its deterministic

representation. This involves transforming the chance-based constraint into a deterministic form, i.e.,  $Cr\{g_i(x, \xi) \leq 0\} \geq \alpha$  transforms to  $\mathcal{E}\{g_i(x, \xi) \leq 0\} \geq \alpha$  which is equal to

$$\sum_{j=1}^{N-N_\alpha} w_j g_i(x, \xi_j) \Big|_{Cr\{g_i(x, \xi_j) \leq 0\} \geq \alpha} \leq 0$$

with  $N$  represents the number of sampled value for the fuzzy parameter used in the calculation, and  $N_\alpha$  represents the number of sample that have  $Cr\{g_i(x, \xi_j) \leq 0\} < \alpha$ . Next, deterministic equations are solved using deterministic optimization techniques. In this study, the generally decreased gradient approach from LINGO 21.0 was applied. Finally, the ideal option determined by the calculations is executed.

Recent studies on fuzzy programming as well as chance-based programming have explored its applications, such as optimizing the integration of energy resources [30,31], optimizing management systems of water resource [32], fuzzy aggregate production planning [33], and transportation problem in tourism sector switching [34] with satisfactory results, showing its superiority; this is the main reason of using this approach in this study.

## 4 Case study

The research was done at the wastewater treatment plants in Bantul, with the Figure 1 was illustrated the treatment flow. The following subsections provide details on the parameter and output of the probabilistic-fuzzy programming model with chance-constrained inequalities.

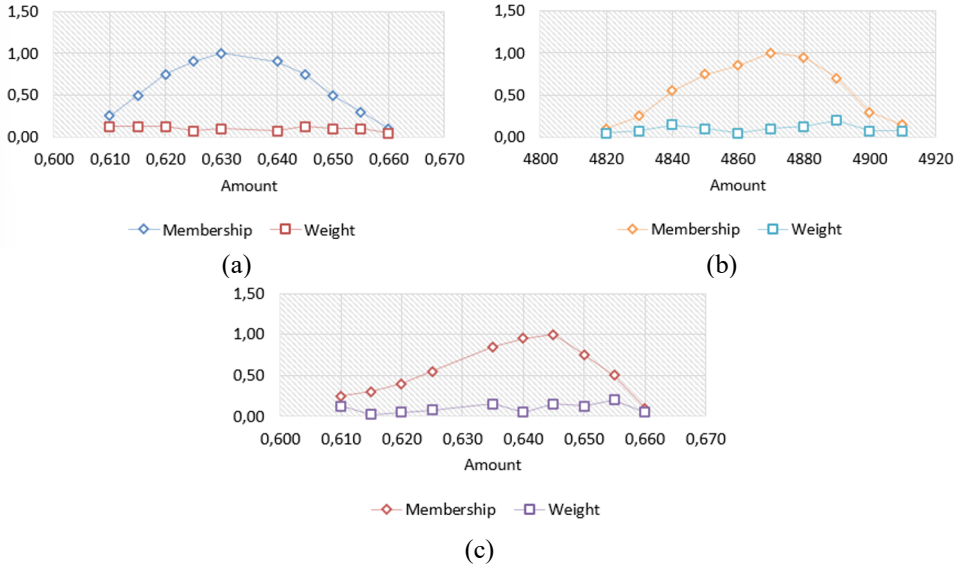
### 4.1 Parameter setting

The fuzzy parameters' membership functions are generated at random, with values centered on the average of the data from the observations in [8]. Figure 2 shows their relative weights and membership values. According to Yogyakarta Province regional governmental policy, the concentration of BOD in the treated wastewater cannot exceed 50 mg/L. Furthermore, the decision makers desired an efficiency rate of 0.5 for every ponds. The parameter  $m$  has a normal distribution, characterized by a mean of 1.1 and variance of 0.01. The computations of optimization were done using LINGO 21.0 software with stochastic optimization environment and the reduced gradient generalization technique [23, 24].

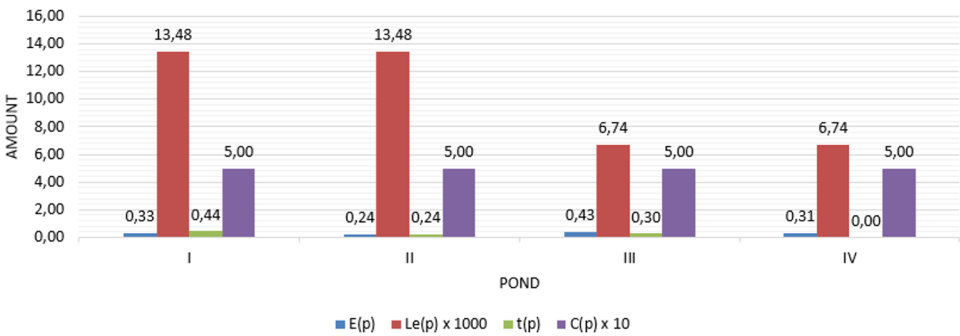
### 4.2 Results and discussion

Figure 3 shows the suggested model's best solution, with a daily input rate of 26.95 m<sup>3</sup> at the entrance. Optimal distribution of wastewater treatment over facultative ponds I and II per day was 13,48 m<sup>3</sup>, and the optimal distribution per day for pond III and IV was 6,74 m<sup>3</sup>. Based on calculation, the concentration of BOD after handling is anticipated to be 50 mg/L, and it doesn't take a full day to hold it. If the decision-maker decides to employ the entire day of detention time, the concentration of BOD is estimated to be less than 50 mg/L.

The efficiency index levels varied throughout the four ponds as a result of parameter changes, with an average of 33%. This suggests that it is necessary to improve the overall performance of the facultative pond, possibly by sludge removal. In particular, pool II must continue to maintain its performance level because it has the highest efficiency index, while pool I needs to make management improvements to increase its performance because it has the lowest efficiency index. Adding an aerator to Pond I could help address this issue.



**Fig. 2.** Membership function graph for fuzzy parameters are shown as follows: (a) waste load of pond I and pond II, (b) waste load of pond III and pond IV, and (c) rate of BOD degradation of ponds I and ponds II



**Fig. 3.** The optimal solution

Based on the output, several managerial recommendations for managing the facultative ponds were identified, including: (1) the decision-maker may choose different levels of confidence for every constraint functions in a mathematical model; this allows for adjustments to the level of trust for any aspect depending on the decision maker's experience and intuition, and (2) certain parameters' real values were unknown during the computation, therefore judgments were made with uncertainty; this implies that the results may deviate from the expected values predicted by the mathematical model. As a result, with different probability values, the actual results might differ, perhaps for the better or worse.

Additionally, the optimization can be repeated numerous times with other value of parameter, such as different membership functions, up to the decision maker is sure in his

conclusion. However, decision makers must consider computational time, especially if they have time and want better results.

## 5 Conclusion

This study introduced a new model chance-constrained probabilistic-fuzzy programming designed to optimize the performance of facultative wastewater ponds by managing uncertainties in biological oxygen demand degradation rates and wastewater inflows. The results from the case study at the Bantul wastewater treatment plant demonstrated that this approach effectively enhances treatment capacity while ensuring compliance with environmental standards. By incorporating both probabilistic and fuzzy uncertainties, the model offers a more realistic and flexible solution compared to traditional deterministic and singularly focused probabilistic or fuzzy models.

Future challenges involve developing more complex models to address intricate scenarios, such as degradation processes of pollutant occur in maturation pond. Additionally, investigating analysis of sludge will be valuable for understanding its impact on pond performance.

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