A Critical Review of Structural Topology Optimization Algorithms

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Abstract. In the past 30 years , the field of structural topology optimization has developed rapidly, and many representative algorithms such as homogenization algorithm, solid isotropic material with penalization algorithms and evolutionary structural optimization algorithms have emerged. In this paper, the above three representative algorithms' principles and development history are briefly introduced. Secondly, with the classic example of long cantilever beams, the three methods are compared in all aspects of the optimisation process, and their similarities and differences in terms of optimisation objectives, constraints, and characteristics of the results are analysed. Finally, the above algorithms are summarized, and their advantages, disadvantages and applicable scenarios are listed, which will provide reference for future designers to a certain extent.

Keywords: Topology optimisation; homogenisation; SIMP method; ESO method

1 Introduction

With continuous developments in the realm of architecture, the complexity and innovativeness of structures have reached an unprecedented level, and the requirements for material utilisation efficiency and structural safety are also increasing. In this context, the traditional experience-based optimisation methods have been difficult to meet the needs of today's structural optimisation, and many complex structures are difficult to be optimised using traditional methods. The emergence of topological optimisation methods is a good solution to this problem. Compared with traditional structural optimisation methods, *topological optimisation methods show higher optimisation efficiency and greater versatility, and thus have gained the attention of researchers.

In the past 30 years, the methods of topology optimisation have made great development, and many mature algorithms have appeared both domestically and internationally [1].Bendsoe et al proposed the homogenisation algorithm, which transforms the topology optimisation problem from a problem of considering existence or non-existence of the materials at each individual point in the domain to a continuum material distribution problem,

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and provides a computational framework for the topology optimisation of a continuous structure [2]. In order to reduce the complexity of the homogenisation method, Bendsoe proposed the solid isotropic material with penalization(SIMP) method on its basis, which takes the artificially assumed 0-1 density value as the design variable, and completes the material redistribution through the change of the density value, so as to realise the topology optimisation of the structure [3].Xie et al proposed and perfected the evolutionary structural optimization(ESO) algorithm, which can be optimised for a wide range of stress, stiffness, frequency and other constraints [4]. Currently, these topology optimisation algorithms have been extensively used in many realms such as building structure design, aircraft design, 3D printing, etc., showing high potential and practical value [5].

However, at present, domestic and international research on structural topology optimisation algorithms mainly focuses on the improvement and validation of the algorithms, while less attention is paid to the differences between the various algorithms and the applicable scenarios. Based on the above three algorithms, this paper analyses the advantages and disadvantages of each algorithm and the applicable scenarios with the example of long cantilever beam, which will help future researchers to choose the appropriate optimization algorithm and drive topology optimisation to more areas of application.

2 Concept of structural topology optimisation

2.1 Definition of structural topology optimisation

Structural topology optimisation refers to the design approach to achieves goal of using less material and better performance without affecting the design function of the structure by optimising the material distribution and structural form with the help of technical means such as mathematical algorithms and computer programs under the given material properties, boundary conditions and loads [6]. The three elements of topology optimisation include design variables, objective function and constraints.

Topology optimisation is significantly different from traditional size optimisation and shape optimisation. Traditional optimisation methods can be solved step-by-step using mathematical planning methods with parameters like the cross-sectional area of the beam and the slab thickness as design values. However, In the topology optimisation issue, due to the existence of an infinite number of potential topologies at each point within the structure, nodes or beams may exist at any position within the computational domain under the complex structure, and it is not possible to build up a system of equations with a finite number of parameters to accurately simulate the mechanical analysis of a real structure, but only to use a finite number of parameters and combine with the algorithm to approximate the expression, which is exactly the difficulty of the current structural optimisation of the topology [7].

2.2 Process of structural topology optimisation

As is shown in the Fig. 1, a systematic structural topology optimisation process should consist of three phases: modelling, analysis and optimisation. In the initial mathematical modelling stage, the researcher needs to construct a mathematical model that can approximately characterise the topological properties of the structure using finite parameters in the design domain and determine the optimisation objectives. The parameters used to construct the mathematical model can be the volume, displacement, solid frequency, etc. of the structure. Considering the practical needs of engineering, researchers often use the maximum structural stiffness as the optimisation target [8]. The parameters chosen for constructing the mathematical model greatly affect the efficiency and accuracy of structural topology optimisation, as well as being the fundamental difference between different topology optimisation algorithms.

In the structural analysis stage, the model is analysed with the help of finite element method to determine the target function and sensitivity function of the structure. The target function is generally a flexibility function, which is used to determine when the structure converges. The sensitivity function is used to reflect the influence of parameter changes on the overall structure index.

Finally, in the topology optimisation stage, the researcher needs to make the parameters of the mathematical model gradually approach the optimal values through the optimisation algorithm. When the objective function converges to a certain accuracy under the manufacturing constraints , it can be considered that a topologically optimised structure that meets the requirements has been obtained [9].

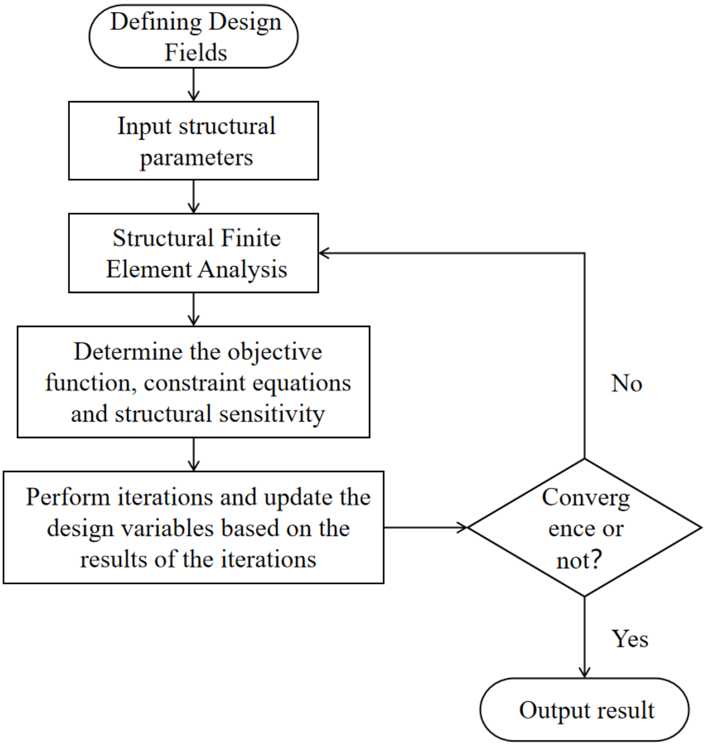

Fig. 1. Topology optimisation process. (Picture credit: Original)

2.3 Classification of structural topology optimisation

The two main categories of structural topology optimization are illustrated in Fig. 2, respectively continuous structure topology optimization and discrete structure topology optimization, and this classification is due to the characteristics of the original structure. Continuum structure topology optimisation usually assumes that the material distribution is continuous and the design variables are continuous values. This approach is suitable for most engineering applications, especially when the distribution of structural materials can vary arbitrarily. Therefore, most current topology optimisation algorithms use the continuum assumption. Discrete structure topology optimisation treats the design variables as discrete, discontinuous values, and the main representative methods include genetic algorithms and particle swarm optimisation algorithms. Discrete structure topology optimisation methods are difficult to describe more complex structures, with a narrow range of applications and poor engineering practicability, so current research mainly focuses on continuum topology optimisation.

Structural topology optimisation is also categorized into material-based and geometry-based according to the parameters used in building the mathematical model [5]. Material-based topology optimisation methods mainly include homogenisation and SIMP methods. These methods start from describing certain properties of the material, such as the density and elastic modulus of the material, and achieves structural optimisation by improving the materia distribution of the structure. Geometry based topology optimisation methods include ESO method, level set method. These methods focuses on defining the boundary condition of the structure and constrain the structure by describing it.

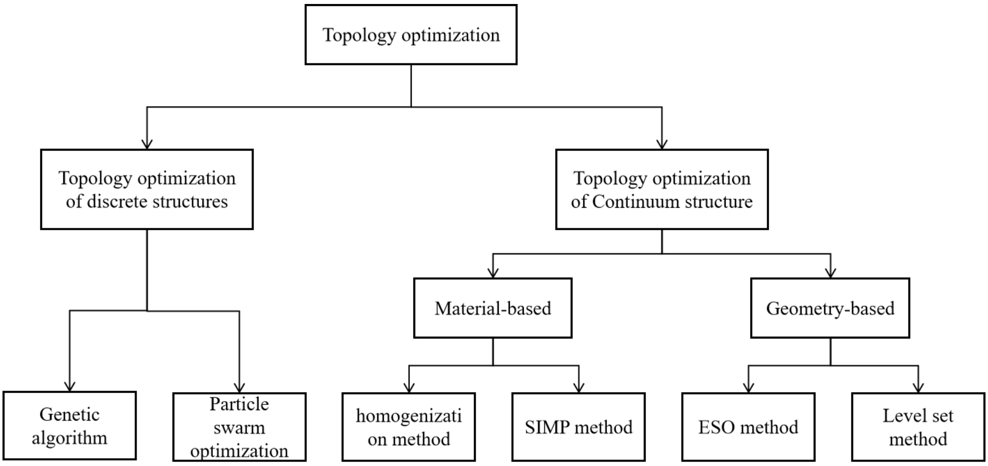

Fig. 2. Topology optimisation classification. (Picture credit: Original)

3 Representative algorithms for structural topology optimisation

3.1 Homogenisation method

In 1988, Bendsoe et al proposed the homogenisation theory, which establishes a new computational framework for topological optimisation of continuum structures [2]. As is shown in the Fig. 3, The principle of homogenisation is to define the structure by the material distribution in the design region, and the conventional method is to discretise the design region into a rectangular or square grid cell of equal size, and in the optimisation process, the material density coefficient of each grid cell is adjusted according to the stress magnitude in every grid cell by increasing the material density coefficient in the area with high stress and decreasing the material density coefficient in the area with low stress. With the goal of minimising the flexibility of the structure, after several iterations of calculation, a reasonable material density value is selected, and the cells below this value are deleted to obtain the final structure after topology optimisation. Compared with the original structure, the more inefficient parts are discarded, and the efficiency of material use is improved.Groen introduced a projection method for topology optimisation of orthogonal heterogeneous filler materials at larger sizes, which allows the materials to be modelled on coarser meshes and reduces the computational cost considerably [10].

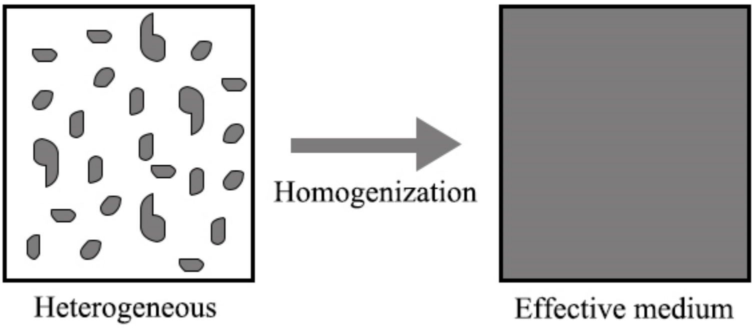

Fig. 3. The principle of homogenisation. (Picture credit: Original)

3.2 Solid isotropic material with penalization method

The solid isotropic material with penalization method introduces the concept of penalty factor based on the homogenisation method, where instead of deleting the material directly during the iterative calculations, the low-density region of the material is penalised by a penalty factor, which reduces the effective elastic modulus of the region. In the SIMP method, Bendsoe gives the relationship between the density design variables and the material properties by a power function [3]:

$$\mathrm{E}(\rho_{\mathrm{i}}) = \mathrm{g}(\rho_{\mathrm{i}})\mathrm{E}_0 = \rho_{\mathrm{i}}^{\mathrm{p}}\mathrm{E}_0 \tag{1}$$

$$\mathrm{g}(\rho_{\mathrm{i}}) = \rho_{\mathrm{i}}^{\mathrm{p}} \tag{2}$$

In the equation (1), p is the penalty parameter, the value range is p>1, commonly used value is p=3; E_0 is the Young's modulus of the solid material; $\square_{\mathrm{i}}$ is the i-th unit density.

This method significantly reduces the complexity of the homogenisation method, but at the same time introduces a new problem, i.e., it is often difficult to make a decision on whether to remove structural units at intermediate densities or not [5]. Yarlagadda improved the SIMP method by proposing the solid isotropic material with thickness penalization (SIMTP) method, which uses 2.5D planar units (2D for the state of stress and 3D for the state of volume) instead of the 2D units in the SIMP method [11]. The SIMTP method is more direct, cost-effective and efficient in generating high-resolution images of structural boundaries.

3.3 Evolutionary structural optimization method

Xie introduced the evolutionary structural optimization(ESO) method in 1993, the basic principle of which is to remove parts of low inefficiency from the structure gradually and eventually obtain an optimised structure with better material utilisation and performance [12]. In Fig. 4, the structural shaping process of the column by the ESO method is shown when the column is subjected to a vertical compressive force on its upper surface. The ESO method aims at minimising the weight of the structure, and finite element analysis is carried out on the model at each step of the iteration to eliminate structural elements that are in a low-stress state.The definition of the low stress state unit is given by the following equation:

$$\frac{\sigma_{\mathrm{ij}}}{\sigma_{\mathrm{maxj}}} \leq \mathrm{RR}_{\mathrm{n}} \tag{3}$$

In the equation (3), σ_{maxj} is the maximum stress of the unit; $\mathrm{RR_n}$ is the deletion rate of each iteration step, the value range is generally 0.05-0.2; σ_{ij} is the *Von Mises* stress of the unit, calculated as follows:

$$\sigma_{ij} = \sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]} \qquad (4)$$

In the equation (4), σ_1, σ_2 and σ_3 are the first, second and third principal stresses, respectively.

When the maximum stress in a structural cell reaches the permissible stress at a certain operating condition, the iteration is stopped and the optimised structure is obtained. This method was refined by Huang as a bi-directional evolutionary structural optimization (BESO) method [13]. It allows the removal of cells in the part of the structure where the material is utilised less efficiently while adding cells in the part of the structure where it is utilised more efficiently. On this basis, Rut proposed an automated image-based SBFE-BESO method using a quadtree/octree decomposition algorithm within the SBFE framework, which greatly reduces the number of degrees of freedom and the memory required for computation, while the topology results maintain a good accuracy compared to traditional grid methods [14].

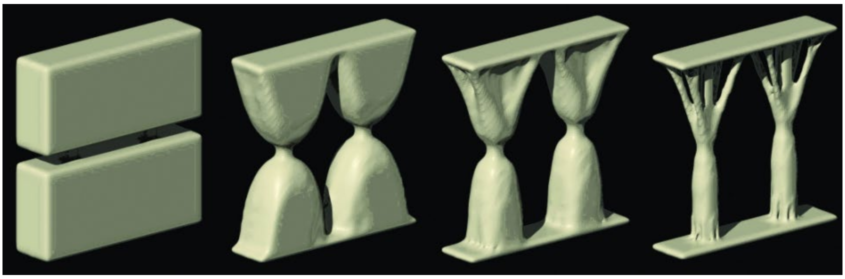

Fig. 4. Structural Formation Process of ESO method [4].

4 Comparative analysis of various optimisation algorithms

4.1 Introduction to long cantilever case.

In the field of topological optimisation, cantilever whose length to height ratio is greater than 1 (L>H) are called long cantilever. The topology optimisation of long cantilever is a classical example in the realm of structural topology optimisation. In this paper, a long cantilever with L=2H is selected as an example to analyse the similarities and differences among the topology optimization methods. This cantilever is subjected to a concentrated force in the vertical direction at the midpoint of the right end face, as shown in Fig. 5. The modulus of elasticity of the cantilever E=1,Poisson's ratio ν=0.33 and the optimised material occupancy are all 50%. The topology optimisation results of long cantilever for different algorithms are shown in Fig. 6.The number of iterations for the three methods are 50, 150 and 50 respectively.

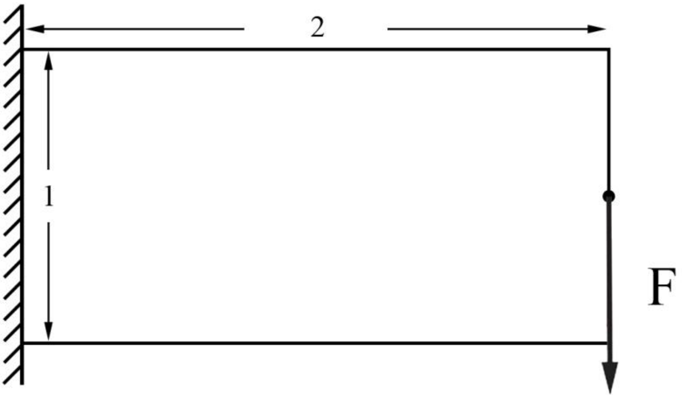

Fig. 5. long cantilever case. (Picture credit: Original)

(a) Homogenisation method [15] (b) SIMTP method [11] (c) BESO method [16]

Fig. 6. Optimisation results for long cantilever [11,15,16].

As is shown in Fig. 6, optimisation results obtained by various optimisation methods are largely the same, although the optimisation results of the homogenisation method show a more severe checkerboard phenomenon. There is an obvious jagged phenomenon on the boundary of the optimisation results of homogenisation algorithms and the BESO algorithms. This is a numerical instability problem due to the discrete nature of the finite cell lattice [3].The optimisation results of the SIMTP method have very smooth boundaries, which is due to the fact that the planar cell of 2.5D avoids the instability problem of the boundaries in the SIMP method, and stable, high-resolution structure boundaries can be obtained without the need for a smoothing algorithm [6]. Comparison with the optimisation results of the homogenisation method shows that the BESO method converges slower, which is due to the nature of its evolutionary method.

4.2 Analysis and discussion

In this paper, we summarise the four aspects of each topology optimisation algorithm, namely the basic idea, optimisation objective, constraints and result characteristics, and the results are presented in Table 1.

Table 1. Summary of different optimisation methods. (Picture credit: Original)

Methods	Basic idea	Optimisation targets	Constraints	Result characteristics

Homogenisation	Uniform meshing of the structure	Minimum structural flexibility	Volume	Large number of holes in the result
SIMTP	Cells with densities below a certain threshold are deleted	Minimum structural flexibility	Volume	Resulting boundaries are smooth, stable and high resolution.
BESO	Gradual removal of inefficient materials from the structure	Multiple objectives	Stress , displacement	Severe jaggedness in the optimised boundary is evident

From the above table, it can be seen that the three representative methods have their own characteristics and each has its own advantages and disadvantages when performing structural optimisation. The homogenisation method can handle materials with complex microstructures, especially composites and non-homogeneous materials, because it is based on the principle of homogenising the structure into a uniform mesh, but its topology optimisation results contain a large number of holes and the phenomenon of tessellated lattice. SIMTP method is more accurate in analysing the structure under the influence of static loads, but it is not applicable to the optimisation of the structure under dynamic loads, and the BESO method is suitable for structures affected by dynamic loads and stability, especially when considering the limit states and failure modes of structures.

The homogenisation method and SIMTP method are both based on the optimisation objective of minimising structural flexibility, which is inconsistent with the objective of achieving the minimum amount of material usage in construction projects, and thus are often impractical.The BESO method is capable of being adapted to a wide variety of optimisation objectives, and has a wider range of applications. However, the BESO method is an evolutionary method that requires a higher number of iterations, leading to its lower computational efficiency. The homogenisation method and SIMTP method are only applicable to volumetric constraints, while the BESO method is able to apply to a wide variety of constraints like stress constraints and displacement constraints.

In addition, in order to satisfy the manufacturability of the structure, equal-thickness uniform isotropic structures with holes are often used as the optimisation objective in engineering. This isotropic material of equal thickness can also be interpreted as a constraint on the optimised structure, which should be taken into account when selecting the optimisation method.

5 Conclusions

This paper compares three representative topology optimisation algorithms, namely, the homogenisation method, SIMTP method and BESO method, describes the characteristics of each method in terms of optimisation results, optimisation efficiency and applicability, and compares the differences between the three methods in terms of optimisation objectives, constraints and optimisation results in conjunction with the example of a long cantilever beam, and obtains the following conclusions:

(1) The homogenisation method performs well in dealing with complex boundary conditions and composite material problems, and can effectively optimise the material distribution. However, its calculation is relatively complex, and the optimisation efficiency is relatively low.

(2) The SIMTP method has a simple optimisation process, fast convergence speed, and performs well in terms of the accuracy of the optimisation results, but it is unable to deal with non-uniform and composite materials, and has certain limitations.

(3) The BESO method, although inferior to the above two methods in terms of optimisation efficiency, has wide applicability as it is capable of being adapted to a wide variety of optimisation objectives and constraints.

Although this study provides a comprehensive comparison of the homogenisation method, SIMTP method and BESO method, there are still some shortcomings. This paper only analyses the simplified cantilever beam example, which is not universally representative and cannot characterise the variation of different algorithms under more complex structures and loads. In future research, more complex structural forms and diversified loading conditions should be considered, and combined with actual engineering cases, the various algorithms should be studied in greater depth.

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