Modeling Seabed Response Under Ocean Waves: Research with Different Wave Properties

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Abstact: In order to study the seabed liquefaction under the wave effect and its key factors, this project will obtain the pore pressure in seabed soil, based on the Airy Wave theory and Biot's theory. Referenced with the proper liquefaction criterion, the liquefaction depth is calculated easily under different wave heights and fixed soil mechanical parameters. The results show that the wave height has a significant influence on the seabed liquefaction depth. Increasing the wave height value can enhance the pore pressure development and the increase of liquefaction depth at the corresponding position in the seabed. According to the results output in this study, predicting and choosing the appropriate wave height values can be proposed as one of the methods to prevent seabed liquefaction.

1 Introduction

In the past decades, with the growth of offshore activities, the cost and safety in the design of offshore pipelines or submarine structures attract more attention from people. Offshore wind turbine with mono-pile foundation is a very popular form for the shallow and middle oceans. However, mono-file foundations supporting offshore wind turbines will be damaged by the wave effect in the ocean. The seabed soil around mono-pile foundations might be liquefied under the wave loading, resulting in damage to offshore wind turbines[1].A great number of studies have been carried out about soil liquefaction around marine structures under the wave effect. For example, Li et al.(2011)construct a 3D FEM(finite element method)model to analyze embedded mono-pile foundations on the seabed response under wave effect. Wave has a significant influence on residual liquefaction distribution around the bottom of piles[3].Chang and Jeng(2014)develop the three-dimension porous model by simulating the Wave-Seabed-Structures Interaction(WSSI)around the foundation, based on Reynolds-Averaged Navier-Stokes equations and Biot's Poro-elastic theory[2].Lin et al.(2017) analyze the WSSI(wave-seabed-structure interaction)model and evaluate the soil liquefaction around the mono-foundation by using the OpenFOAM frame.[1] According to the outputs above, wave and seabed pore pressure are key factors in soil liquefaction research. Therefore, this report aims to build the wave-seabed interaction model without structure effect to improve the understanding of seabed response under wave effect.

Soil liquefaction is a professional term in earthquake engineering, which refers to the effective stress between soil particles changed to zero and the transformation from solid soil to liquid or a viscous fluid because of the loss of the shear strength, due to applied external quick forces or earthquake[4].soil liquefaction can be distinguished into two types depending on different time scales, residual liquefaction, and momentary liquefaction respectively, with a similar criterion, which is that pore water pressure difference surpasses the self-weight of soil. Soil liquefaction may cause further earthquakes, surface collapse, and the failure of existing structures[5].Therefore, this report will mainly understand and make a proper prediction for the changes in seabed soil pore pressure, and the conditions and mechanics of seabed soil liquefaction.

Wave-induced seabed dynamic responses have been extensively studied, which carries out multiple physical experiments to improve the understanding of the whole wave-induced seabed dynamic response. In this stage, there are three different experimental methods used in seabed response under the wave effect, including One-dimensional compression tests[6],geo-centrifuge tests[7],and wave flume tests[8].Although using natural materials is easy to capture the realistic mechanism and situation of seabed dynamic responses under wave-induced, it is relatively expensive and limited to scales. Therefore, numerical simulation has been widely used as a cost-effective method to study seabed dynamic responses induced by various wave conditions[Lin et al.(2017),Chang et al.(2014),and Lin et al.(2011)].WSSI numerical integrated model, developed by Lin et al.(2017),can be referenced in this project to build the numerical simulation model in wave-induced seabed dynamics response, which integrates the Navier-Stokes equation and QS(Quai-static)Biot's model. However, in this study, the wave will be resolved assuming it is a linear wave applied on the bottom of the sea by numerical simulation, to observe the seabed response and study the

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*Corresponding author email: xuanyiwang0430@163.com relation between the wave, pore pressure, and soil liquefaction condition.

This report constructs a numerical simulation model by using SPH(Smoothed Particle Hydrodynamics)method, based on linear wave theory and QS Biot's equation, to solve the wave seabed interaction and dynamic seabed response. This paper will research the liquefaction mechanics by observing the relationship between the pore pressure spatial distribution with time scales and wave height. The descriptions and theories of the wave and soil model are presented in. The resources and treatment approaches of the experimental data sets will be outlined in.In, the results output of the linear wave-induced seabed dynamics model is interpreted around the relation among the wave, pore pressure, and soil liquefaction. The main conclusions are listed in

2 Literature Review

2.1 Wave model

2.1.1 Linear wave theory used in this study

The governing equation of incompressible fluid dynamics is:

$$\nabla \cdot \mathrm{u} = 0$$

$$\frac{\partial \rho \mathrm{u}}{\partial \mathrm{t}} + \nabla \cdot (\rho \mathrm{u})\mathrm{u}^2 = \nabla \mathrm{p}^* - (\mathrm{g} \cdot \mathrm{x})\nabla \rho + \nabla(\mu \nabla \mathrm{u})$$

$$\frac{\partial \alpha}{\partial \mathrm{t}} + \nabla \cdot \mathrm{u}\alpha + \nabla \cdot \mathrm{u}_\mathrm{r}\alpha(1-\alpha) = 0$$

Where u is the velocity field; ρ is the fluid density; t is time; p^* is the modified pressure,related to static pressure and total pressure, $\mathrm{p}^* = \mathrm{p} - \rho \mathrm{g}\mathrm{x}$; g is gravity acceleration; x is Carsian coordinate vector; μ is the dynamic viscosity; α is the scalar field of volume fraction function,related to the types of incompressible flow, α equals to 1 when the simulated field is water, α equals to 0 when the simulated field is air, $0 < \alpha < 1$ when the field is the mixture of water and air.

When the linear wave is propagating along the horizontal direction upon the seabed, the water pressure will be created at the boundary between water and seabed, varying with time scale, which will promote the vertical distribution of pore pressure inside the seabed[9]. Figure 1 shows a sketch of the hypotony that the wave in this study propagates along the X-direction in the seabed.

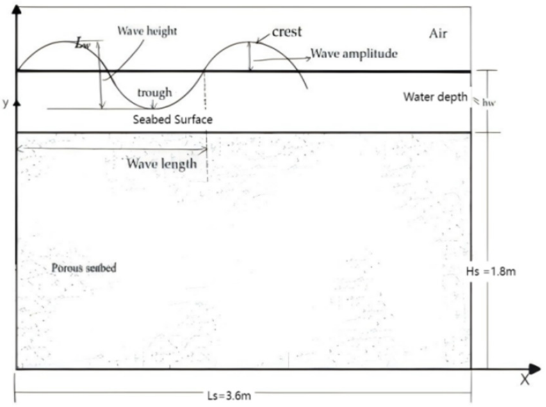

Fig 1. The sketch of the wave propagates along the horizontal direction in this study.

According to the linear wave theory (also called Airy Wave Theory), the water pressure on the seabed surface is:

$$\mathrm{P_b} = \rho \mathrm{g} \mathrm{a} \mathrm{e}^{\mathrm{kz}} \cos(\mathrm{kx} - \omega \mathrm{t})$$

$$\eta(\mathrm{x}, \mathrm{t}) = \mathrm{a} \cos(\mathrm{kx} - \omega \mathrm{t})$$

Where $\mathrm{P_b}$ is the water pressure on the seabed surface from the wave; $\eta(\mathrm{x}, \mathrm{t})$ is the free surface elevation related to position and time; ρ is the density of the water; g is the gravity acceleration; a is the wave amplitude,related to the water height H by $\mathrm{a} = \frac{\mathrm{H}}{2}$; k is the angular wavenumber,related to the wavelength λ by $\mathrm{k} = \frac{2\pi}{\lambda}$; z is the water depth; x is the horizontal position($\mathrm{x} = 1.8$ in this study); ω is the angular frequency,related to the period T and frequency f by $\omega = \frac{2\pi}{\mathrm{T}} = 2\pi \mathrm{f}$; t is the time.

The pore water pressure, the effective stress, and the shear strength of the soil in the seabed will change dynamically as the water pressure varies on the seabed surface. The assumptions made in developing the linear wave theory are as follows:

The fluid is homogeneous and incompressible, and the gravity cannot be neglected;

Ideal fluid is irrotational with the velocity field;

Wave is the linear wave(small amplitude wave),in which wave height is much smaller than the wavelength.

The concept will be influenced by these three assumptions in this study, which include establishing the wave model, calculating the boundary water pressure, and optimizing the available experimental data sets. The application of these assumptions will be stated in.

2.1.2 . Other common wave theories for wave-seabed interaction in the fluid field.

The research on the widely used wave theory has undergone a transition from regular waves to random waves. The theoretical characteristic of regular waves is to regard wave motion as a deterministic function and to explore the dynamic properties and motion laws of waves through fluid mechanics. The research on regular wave theory has experienced a transition from linear waves to nonlinear waves in which common theories include the Airy Wave theory, Stokes wave theory, cosine wave theory, and solitary wave theory. At present, in the field of wave-induced dynamic seabed response, the Stokes theory is commonly used to model wave models and control wave motion[Zhang et al.(2022)[10]and Asumadu et al. (2019)[11]].High-order Stokes waves do not consider the effects of changes in water depth, which are suitable for deep water areas. However, in offshore shallow water areas, the accuracy of using the Stokes theory cannot meet the desired level, but the cosine wave theory, which can determine the main factors of wave properties, needs to be adopted to describe wave motion and obtain more satisfactory results. Therefore, for a research area with limited water depth and complex conditions, it is necessary to select the appropriate wave theory to describe wave motion according to the actual marine conditions.

2.2 Seabed model

2.2.1 QS Biot's equation used in this present seabed model.

Combined with the soil inertial effect and fluid compressibility, Biot established the classic Biot's consolidation theory in 1956,which can be used to analyze the wave-induced seabed dynamic response. And the combined continuity and motion equation for pore water and force equilibrium in poro-elastic seabed are:

$$\nabla^2 P_p - \frac{\gamma_w n_s \beta_s}{k_s}\frac{\partial P_p}{\partial t} = \frac{\gamma_w}{k_s}\frac{\partial \varepsilon_s}{\partial t}$$

$$G\nabla^2 v + \frac{G}{1-2v}\nabla\varepsilon_s = \nabla P_p$$

Where$P_p$is wave-induced pore pressure;$\gamma_w$is the unit weight of pore water;$n_s$is soil porosity;$k_s$is the Darcy's permeability;$\beta_s$is the compressibility of pore fluid,related to bulk modulus of pore pressurek_w,the soil saturation degreeS_r,absolute static water pressureP_{w0},by$\beta_s = \frac{1}{k_w} + \frac{1-S_r}{P_{w0}}$;$n_s$is the soil porosity;$\varepsilon_s$is the volume strain,related to soil displacement vector v ,by $\varepsilon_s = \nabla \cdot v = \frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} + \frac{\partial w_s}{\partial z}$; G is the shear modulus of soil,related Young's Modulus E ,and Poisson's ratio $\mathcal{L}$,by $G = \frac{E}{2(1+\mathcal{L})}$;Young's Modulus is related to confining pressure$\sigma_3'$and$\sigma_{3,ref}'$,and a constant soil parametera_s,by$E = E_{ref}(\frac{\sigma_3'}{\sigma_{3,ref}'})^{a_s}$.

2.2.2 Soil liquefaction

The liquefaction phenomenon of seabed sediments under wave loads can cause a series of changes in the geological properties of the seabed engineering, as well as induce marine geological disasters such as erosion and landslides, which seriously affect the safety of marine structural engineering and ecological environment. Therefore, the liquefaction of the seabed and marine foundation caused by wave loads is a key problem that urgently needs to be solved. Currently, teams are focusing on the main factors that affect seabed liquefaction, including wave period and soil saturation[12].Many scholars use soil indicators to evaluate the liquefaction of soil under dynamic loads, including clay content, liquid limit, plasticity index, and water content[9].And many methods have been proposed by domestic and foreign scholars to calculate the instantaneous liquefaction of soil, and several of these methods are explained below:

Okusa(1995)[13]believes that if the vertical effective stress is greater than the weight of the soil, liquefaction will occur in the seabed[13].

$$-(\gamma_s - \gamma_w)z - \sigma_z \le 0$$

Zen et al.(1990)[6]proposed a standard for identifying seabed liquefaction under advancing water waves[],

$$-(\gamma_s - \gamma_w)z \le P_f - P_b$$

Jeng(1997)[12]extended the liquefaction identification standard of Zen et al.(1995)to three-dimensional situations[].

$$-\frac{1}{3}(\gamma_s - \gamma_w)(1 + 2K_0)z \le P_f - P_b$$

Lin et al.(2021)introduced a liquefaction parameter(determined by the pore water pressure gradient/soil buoyancy)to identify liquefaction occurrence[16].

$$LF = \frac{-\frac{dp_f}{dz}\Big| z - 0}{\gamma_s - \gamma_w}$$

When$LF \ge 1$,the soil will be liquefied.

Where$\gamma_s$and$\gamma_w$are the unit weight of soil and water respectively; σ_z is the effective stress in the z direction; K_0 is the horizontal soil pressure coefficient;$P_f$and$P_b$are the static pore water pressure and periodical wave pressure at the seabed surface.

Using the aforementioned liquefaction criterion, Equations⑧and⑨show that the seabed will be liquefied when the net excessive pore pressure surpasses the overburden soil pressure.And the wave dynamic effect is also considered in Equation⑨,which will be used in the following liquefaction analysis.

2.3 Numerical simulation method

The core of the numerical simulation is the establishment of the model and algorithm, in which the output result depends on computer graphics. With the development of computer science, the application of numerical simulation technology will be increasingly extensive and convenient. Compared with traditional experimental methods, the numerical simulation method has been extensively used in the wave-induced seabed response field. For example, some studies have used Finite Element Method[Li et al.(2016);Jeng and Lin(1999)[14]]and Boundary Element Method[Shivakumar,2021][15]to simulate seabed response. And Fluid-Structure Interaction numerical simulation has also been carried out to study dynamic response problems[Lin et al.(2017)[1];Chang et al.(2014)[2]].Combined with basic fluid mechanics theories, such as the Navier-Stokes equation, the numerical simulation method has a significant effect on studying fluid motion regulation, predicting fluid behaviors, and designing fluid control systems.

In this paper, Fortran software and the SPH(Smoothed Particle Hydrodynamics)method were used to establish a soil model based on Biot's consolidation theory. The current model can simulate the numerical model in wave-seabed interaction between wave and structure basically, and the cost is low, and the computational efficiency is fast. However, it also has some disadvantages, which are derived from the accuracy of model establishment. To obtain research results with high accuracy, it is necessary to improve the accuracy of the numerical model, and the accuracy of operational steps, or increase the number of mesh models to reduce errors, which will cause a large calculation amount, long time, excessive dependence on computer memory, and other consequences.

3 Experiment

3.1 Experimental theory

In this study, the soil domain is established by using Fortran software to build a numerical simulation model with a grid form of 55*14.And this numerical simulation model solves the wave and soil models in two steps at the same time. Firstly, according to the input wave parameters and the adjustable time step, in which the time interval is taken as 1 in this study, the wave model is solved by the algorithm of the Wave Theory. Then the dynamic wave pressure extracted from the wave model is applied to the surface of the seabed. Secondly, the soil model calculates the dynamic seabed response caused by waves by solving the QS Biot equation using the SPH method, including the effective stress, and the pore pressure in the seabed. After solving the two models in one time step, corresponding results will be output, and the solution of the next time step will be carried out until reaching the specific total time(the total time set in this study is 50).

In this study, published laboratory data were used to determine the variation of soil liquefaction. Table 1. lists the wave and soil properties used for numerical simulation to research the wave-induced soil response. The effect of wave characteristics on soil liquefaction was studied in this experiment.

3.2 Experimental cases

Table 1. Simulated wave parameters for cases.

Simulated wave parameters					
Experiment	**Wave height(m)**	**Wave frequency**	**Wave period(s)**	**Wave number**	**Water depth (m)**
1	2.5	0.7	9	0.098	5.2
2	3				
3	3.5				
4	4				

Table 2. Basic experimental soil parameters.

Basic soil parameters					
Permeability	**Porosity**	**Water bulk modulus**	**Soil saturation rate**	**Poisson's ratio**	**Shear modulus**
1.8×10^{-5}	0.4	2×10^{9}	0.996	0.3	9.62×10^{6}

The wave, shown as Table 1., and soil parameters, shown as Table 2., above are from the experiment suggested by Liu et al.(2015)[16].Due to the limited number of laboratory data and application of the linear wave theory, this study will enhance the simulated range through the special wave parameter, wave steepnessλ,which is determined by$\lambda = \frac{H}{L}$,H is the wave height and L is the wavelength.Because if$\lambda$is larger than$\frac{1}{7}$,the wave will break,the maximum wave height value in this study needs to be smaller than 9.13 m, as the water depth is 5.2m.

4 Presentation and interpretation of results

4.1 Vertical distribution of pore pressure in the seabed

Table 3. The observation points and corresponding depth data in the seabed.

Position number	**Seabed depth(m)**	**Position number**	**Seabed depth(m)**	**Position number**	**Seabed depth(m)**	**Position number**	**Seabed depth(m)**
1	1.728	4	1.296	6	0.864	9	0.216
2	1.584	5	1.08	7	0.648	10	0
3	1.44			8	0.432		

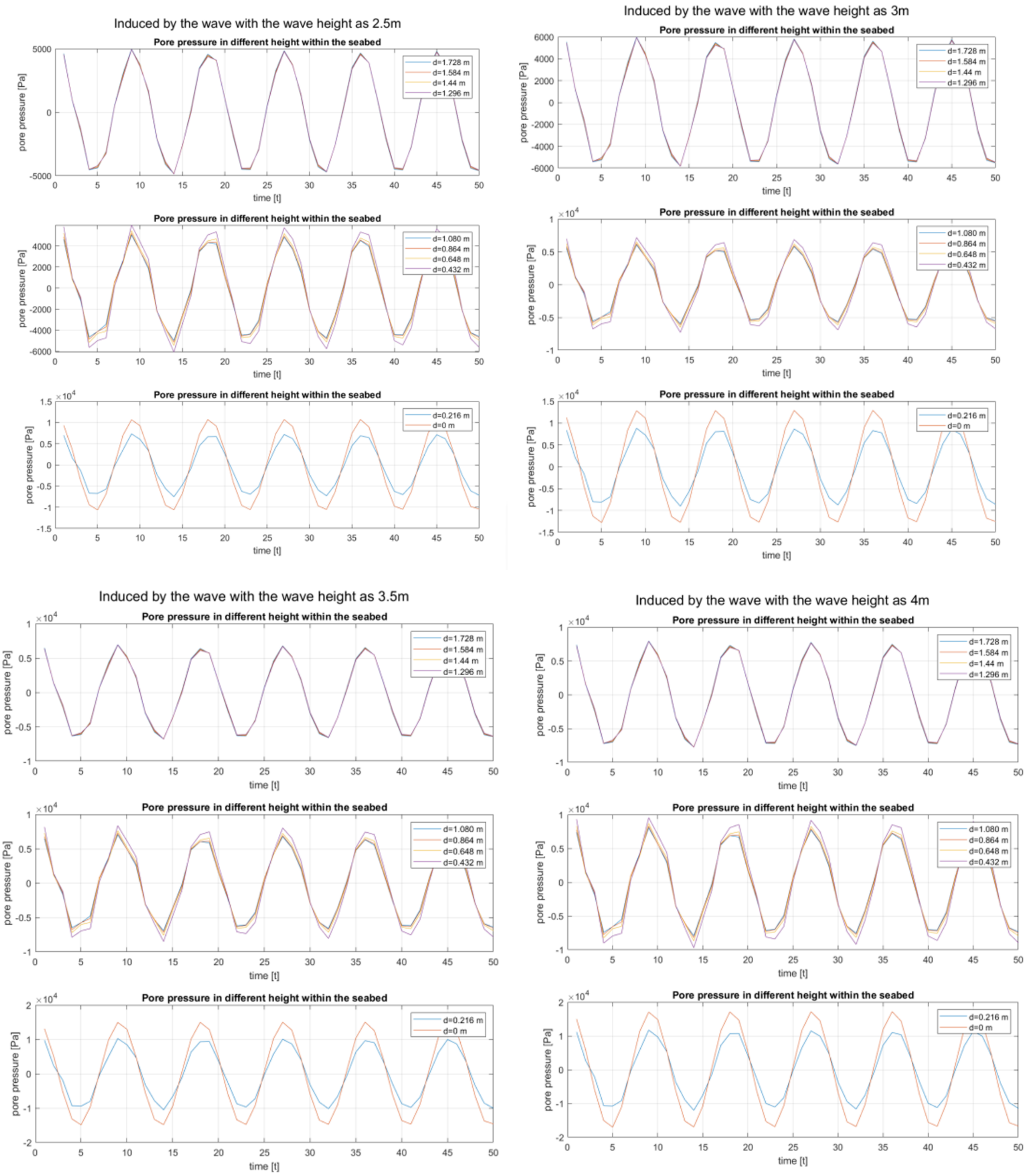

Fig 2. Time series of pore water pressure at different depths.

The vertical distribution of pore water pressure inside the seabed was recorded on horizontal position as 1.8m in the seabed model. Ten points are selected in the vertical direction from the seabed surface to the position with a depth of 1.728m,as shown in Table 3. The pore pressure values are shown in Figure 2.,varied with time under different wave heights. The pore pressure at each observation position varies periodically, with a period of about 9s.

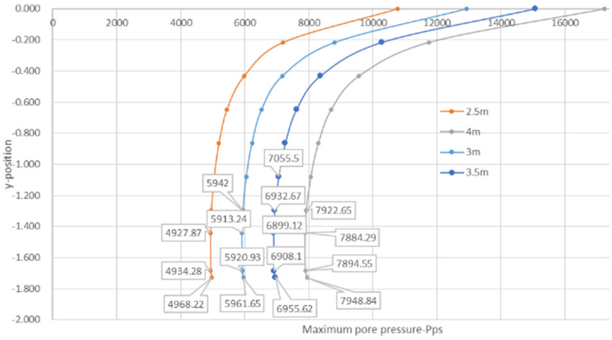

Fig 3. The maximum pore water pressure at different depths.

Although the wave height value changes in different cases,it is obviously seen that the variation curves of pore pressure within the seabed are similar at the same position. In addition, at the same position, the maximum pore water pressure value rises sharply with the increase in wave height. Under the same wave height effect, the pore pressure in vertical distribution will gradually decrease and then slightly increase near the bottom of the seabed as the soil depth increases from the max pore pressure value at three positions, d=1.44, d=1.584, d=1.728, respectively, shown as Figure 3.It is possible that the soil condition near the bottom of the seabed is different from the upper layer of the soil.

4.2 Soil liquefaction criterion in the seabed

The mechanics of soil liquefaction within the porous seabed is the focus of this study, based on liquefaction criteria presented suggested by Zen et al.[6], under wave-induced, liquefaction zone can be determined by:

$$-(\gamma_s-\gamma_w)*y \le (P_{ps}-P_p)$$

Where$\gamma_s$is the unit weight of seabed soil,$\gamma_w$is the unit weight of water,yis the depth of position;$P_{ps}$is the pore pressure within porous seabed;$P_p$is the water pressure on the seabed surface.

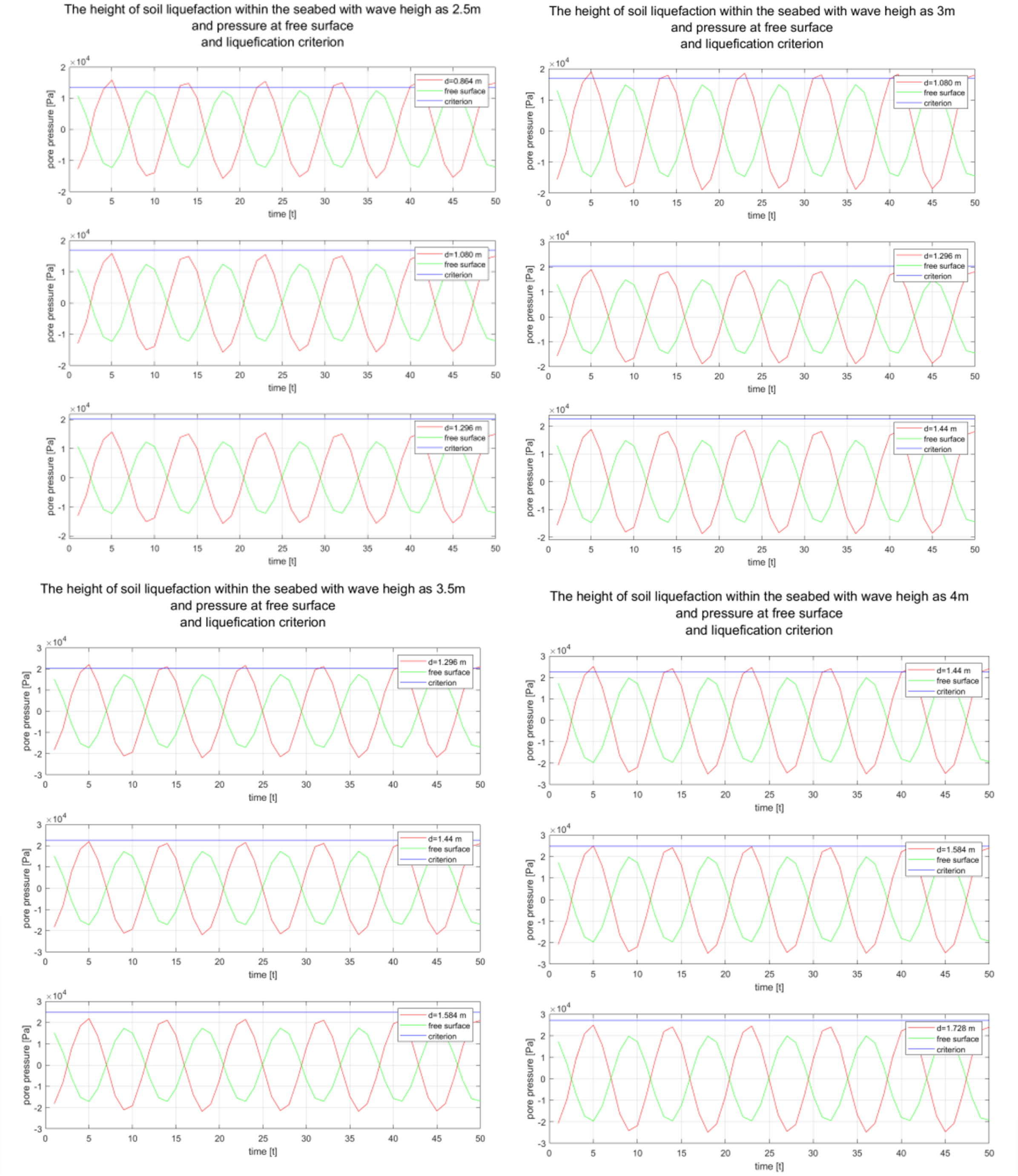

Fig 4. Time series of soil liquefaction depth varies within different wave height.

Liquefaction of the porous seabed may occur when the difference between the pore pressure in the seabed and the water pressure at the seabed surface is greater than the overburdened soil pressure. Using the above liquefaction criteria, the liquefaction zones can be evaluated in different cases. Figure 4. shows the momentary liquefaction depth when the wave height varies. The liquefaction conditions are mainly investigated at the seabed soil depth from 0.826m to 1.44m. When the wave height is 2.5m,the soil liquefaction stops as the soil depth reaches between 0.826m and 1.080m.The seabed depth reaches between 1.080m and 1.296m,the seabed stops liquefying with 3m-wave height. When the wave height is 3.5m,the seabed liquefaction depth rises to between 1.296m and 1.44m.When the wave height is 4m,the seabed liquefaction depth reaches between 1.44m and 1.582m.With the increase of wave height, the liquefaction depth of seabed soil increases gradually. In addition, compared with the free surface elevation equation, seabed soil usually occurs liquefied as the wave is at the trough state during every period.

4.3 The spatial distribution of pore pressure in the seabed

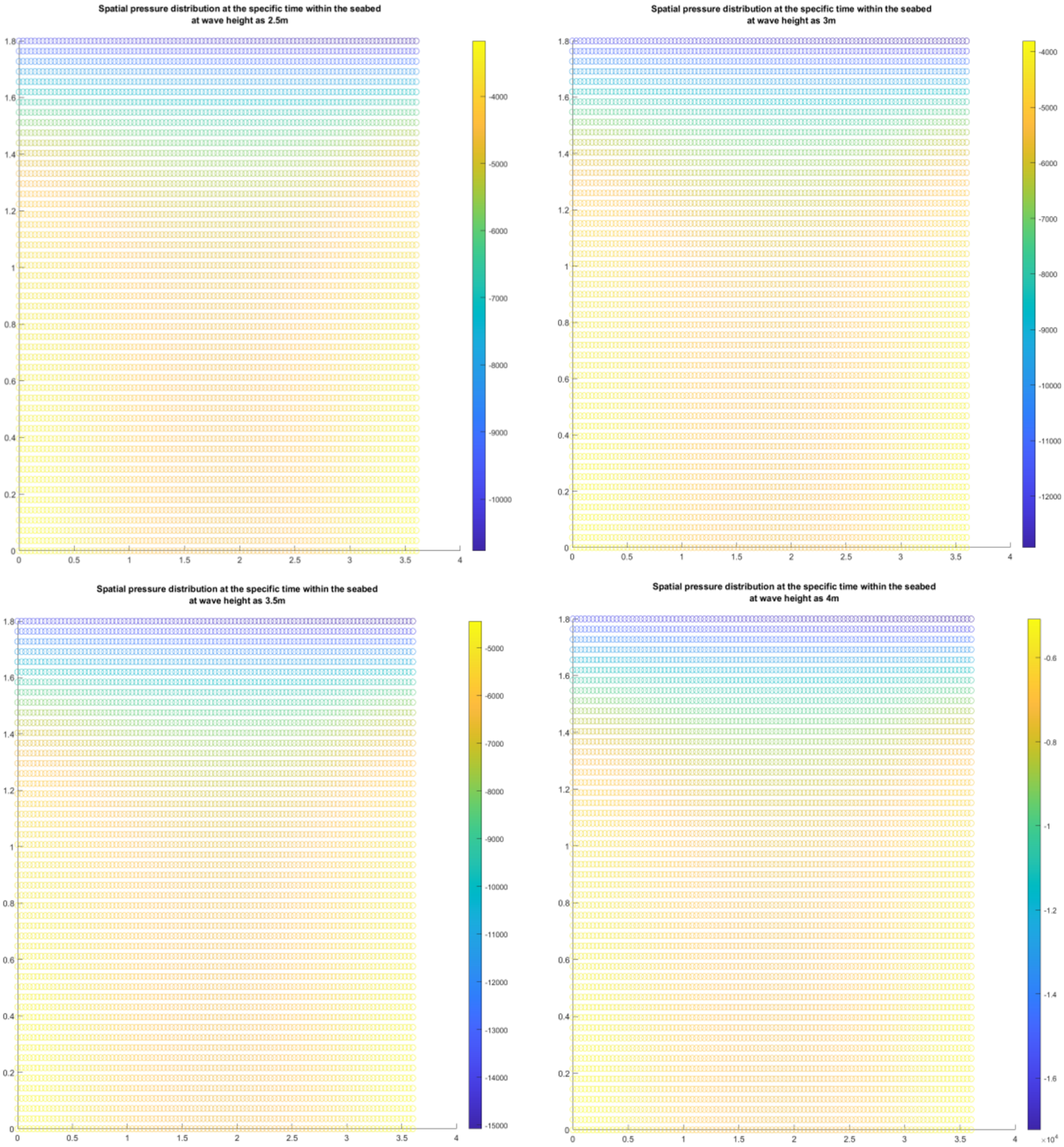

Fig 5. Spatial distribution of pore pressure in the seabed at t=50s.

Figure 5. shows the spatial distribution of pore pressure in the seabed at t=50s.The spatial distribution of the pore pressure near the seabed surface presents periodic changes. And the characteristic of periodic changing will be ignored with the increase of the seabed soil depth. In addition, the pore pressure in the middle position of the seabed is significantly greater than the pressure values on both sides. The horizontal distribution of pore pressure in the seabed model is not uniform, and the liquefaction situation corresponding to its position will be different from that at the fixed position x=1.8m. Therefore, it is necessary for improving the data analysis accuracy to take further simulated research in lateral distribution of pore

pressure the dynamic seabed response, and judge liquefaction conditions.

5 Conclusions

This study draws the following conclusion:

1)The wave-induced pore pressure is weakened as the soil depth increases under the fixed wave height effect at the vertical distribution at x=1.8m. And when the position is close to the bottom of the seabed, the change ratio of the pore pressure gradually decreases. The vertical distribution of the pore pressure in the porous seabed is related to the soil depth, at the same time, the pore pressure declines sharply from d=0 to d=0.432m,and then reduces slowly from d=1.44m to d=1.782m.

2) Compared with the wave-induced water pressure, the pore pressure within the porous seabed which declines with the soil depth increase, generates the upward pressure difference, while surpassing the unit weight of soil. Causing the seabed soil liquefaction. Based on the momentary liquefaction criterion, the soil liquefaction depth will increase with wave height increases. And in the study process, soil liquefaction will never happen if the wave height is too small to generate the small pore pressure within the seabed. And uniform spatial distribution of the pore pressure, it is recommended to make more lateral comparisons of seabed locations to fully understand the potential liquefaction of the whole space of the seabed.

3)Increasing wave height value has a significant effect on the soil liquefaction depth at the same position. The great wave height will influence the water pressure on the seabed surface pore pressure within the seabed value, and promote the momentary liquefaction occurred.

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