Measuring the Downside Risk of Green IT Investments: An Inequality-Theoretic Approach

Berkhouch Mohammed[1,*]

[1]LIDAR, Polytechnic school, Universiapolis, Agadir, Morocco

Abstract. This paper establishes an innovative analogy between the measurement of economic inequality and the financial risk management of sustainable technology investments. The transition to green computing—encompassing energy-efficient hardware, renewable-powered data centers, and carbon-aware algorithms—entails substantial financial uncertainty and pronounced downside risk. Conventional risk measures often fail to capture the unique volatility and relative scale of potential losses associated with these initiatives. Drawing inspiration from economic indices that quantify wealth dispersion [3, 6], we adapt and extend these frameworks, most notably the Zenga index [25], to assess the downside financial variability of investments in green IT, energy-aware computing, and sustainable systems. Building on the theory of deviation measures [19], we introduce a novel framework that accounts for the relativity between minor cost overruns and major financial shortfalls, a critical dimension for evaluating the economic sustainability of green technological transitions. The proposed model is implemented through a structured three-step procedure—data characterization, inequality-based risk quantification, and comparative analysis. These applications demonstrate how the proposed indices can guide investment decisions and prioritize risk mitigation in sustainable computing initiatives.
Keywords: Green IT, Downside Risk, Economic Sustainability, Energy-Aware Computing, Risk Measures, Zenga Index, Financial Variability.

1 Introduction

The relentless growth of the global digital infrastructure, encompassing everything from massive data centers and high-performance computing clusters to extensive network systems and the Internet of Things (IoT), has placed the Information and Communication Technology (ICT) sector at a critical juncture. While driving unprecedented innovation, this expansion is accompanied by a significant and growing energy footprint, contributing directly to environmental concerns and climate change.

A fundamental challenge within this domain is the economic justification and risk assessment of investments in sustainable technologies. Decision-makers are faced with a complex dilemma: allocating capital towards energy-efficient hardware, carbon-aware software algorithms, or renewable energy sources for computing infrastructure. These investments often involve high upfront costs, uncertain long-term payoffs, and exposure to volatile factors like energy prices and regulatory changes. Traditional financial risk models, which often focus

[*]e-mail: mohammed.berkhouch@e-polytechnique.ma

 https://doi.org/10.1051/e3sconf/202568000065

on symmetric volatility or maximum probable loss (e.g., Value-at-Risk), are poorly equipped to quantify the specific **downside risks** inherent in these green IT projects, such as cost overruns, performance shortfalls of new technologies, or the failure to achieve anticipated energy savings.

This paper proposes a novel theoretical and methodological framework to address this critical gap. We posit that the challenge of measuring the financial *downside variability* of sustainable IT investments shares a profound conceptual symmetry with the challenge of measuring *economic inequality* within a population. Just as economists seek to quantify the dispersion of wealth and the relativity between the "poor" and the "rich" [3, 6], IT project managers and Chief Sustainability Officers need to quantify the dispersion of potential financial losses and the relativity between "small" setbacks and "large" company-threatening failures.

Our work draws a formal analogy between these two fields. We adapt and extend sophisticated instruments from economic inequality theory, such as the Gini [14] and Zenga [25] indices, to create a new class of financial risk measures. These measures are specifically designed to:

- **Focus on Downside Risk:** Unlike variance, they exclusively quantify the variability of negative outcomes (financial losses), aligning with the real concerns of project managers.
- **Capture Relativity:** They intrinsically measure the gap between "small" losses (e.g., a minor efficiency overestimation) and "large," tail-risk losses (e.g., the complete failure of a new cooling system or a dramatic shift in energy tariffs), which is often more informative than absolute loss measures.
- **Incorporate Risk Attitudes:** Our extended Zenga-type framework allows for the integration of a decision-maker's risk aversion, enabling the tuning of risk measures to reflect an organization's specific tolerance for sustainability-related financial risks.

The theoretical contribution of this paper is the development of this crossover framework. We demonstrate its practical utility through an empirical case study, showing how these measures provide stable, insightful assessments of variability that outperform traditional metrics. For the green computing community, this work offers a powerful new toolkit for:

- **Evaluating Green IT Investments:** Providing a more nuanced risk-return profile for capital expenditures on energy-efficient technologies.
- **Optimizing Energy-Aware Algorithms:** Quantifying the financial risk of employing slower, energy-saving algorithms versus faster, power-hungry ones.
- **Managing Sustainable Systems:** Assessing the financial downside of operational decisions in smart grids, renewable-energy-powered data centers, and industrial IoT systems.
- **Informing Policy and Regulation:** Offering regulators and standard-setters a more robust metric for assessing the financial stability and sustainability risks of technology firms.

By bridging economic theory and financial risk management, we provide a missing piece for building a truly sustainable and economically resilient digital future. Our approach empowers stakeholders to make better-informed, risk-aware decisions that are crucial for aligning the goals of technological progress with environmental and economic sustainability.

The core contribution lies in:

- Proposing absolute and relative measures for quantifying financial downside variability.
- Introducing the concept of relativity between "small" and "large" losses, analogous to "poor" and "rich" in inequality contexts [25].

- Extending the Zenga index framework to incorporate various risk measures, including spectral measures [1].
- Providing empirical validation using Brazilian economic and financial data.

The paper addresses a fundamental question in risk management: what constitutes a "large" loss? We argue that, similar to economic inequality [7], the distinction between small and large losses is relative rather than absolute.

2 Theoretical Framework

2.1 Risk and Deviation Measures

To formalize our analysis, we operate within a standard probability space $(\Omega, \mathcal{F}, \mathrm{P})$, considering the space L^∞ of essentially bounded financial outcomes. A financial position is represented by a random variable X, where, crucially for our downside focus, we consider $X \geq 0$ to represent financial *losses*. We now define the two key classes of functionals.

2.1.1 Risk Measures

A **risk measure** is a functional $\rho : L^\infty \to \mathbb{R}$ that maps a financial outcome to a real number, representing the riskiness of the position. The theory is axiomatically founded on properties that define rational and useful behavior [2, 5]:

- **Monotonicity:** If $X \leq Y$ (i.e., the losses from X are always less than or equal to those from Y), then $\rho(X) \leq \rho(Y)$. This is a fundamental requirement: a position with worse outcomes must be deemed riskier.
- **Translation Invariance (Cash-Additivity):** $\rho(X + C) = \rho(X) + C$ for any constant $C \in \mathbb{R}$. Adding a sure loss C to a position increases its risk by exactly that amount.
- **Positive Homogeneity:** $\rho(\lambda X) = \lambda\rho(X)$ for any $\lambda \geq 0$. Scaling the financial outcome (e.g., doubling the investment size) scales the risk proportionally.
- **Sub-Additivity:** $\rho(X + Y) \leq \rho(X) + \rho(Y)$. The risk of a combined portfolio is less than or equal to the sum of its parts, reflecting the principle of diversification.
- **Convexity:** $\rho(\lambda X + (1 - \lambda)Y) \leq \lambda\rho(X) + (1 - \lambda)\rho(Y)$ for $\lambda \in [0, 1]$. This generalizes sub-additivity and positive homogeneity, ensuring that diversification does not increase risk.
- **Law Invariance:** If X and Y have the same probability distribution ($F_X = F_Y$), then $\rho(X) = \rho(Y)$. The risk measure depends only on the distribution of outcomes, not on the underlying events.
- **Co-monotonic Additivity:** If X and Y are co-monotone (i.e., they always move in the same direction), then $\rho(X + Y) = \rho(X) + \rho(Y)$. This implies there is no diversification benefit for perfectly correlated risks.

A risk measure satisfying the first four properties (Monotonicity, Translation Invariance, Positive Homogeneity, and Sub-Additivity) is termed **coherent** [2]. These properties provide a robust foundation for comparing the risk of different sustainable technology investments, such as evaluating a new cooling system versus a solar power array.

Common Examples of Risk Measures

- **Worst-Case (WC):** $\rho(X) = \inf X$. An extremely pessimistic measure that considers only the maximum possible loss. It is coherent but often overly conservative.
- **Expected Loss (EL):** $\rho(X) = \mathbb{E}[X]$. The simplest risk measure, representing the average loss. It is coherent but ignores the variability and severity of losses beyond the mean.
- **Value-at-Risk (VaR):** $\mathrm{VaR}^{\alpha}(X) = F_X^{-1}(\alpha) = \inf\{x : F_X(x) \geq \alpha\}$. VaR represents the maximum loss not exceeded with a given confidence level α (e.g., 95%). While widely used, it is not coherent as it fails to be sub-additive for general distributions and ignores the tail beyond the α-quantile.
- **Expected Shortfall (ES):** $ES^{\alpha}(X) = \dfrac{1}{1-\alpha}\displaystyle\int_{\alpha}^{1} F_X^{-1}(u)du.$ Also known as Conditional VaR, ES calculates the average loss in the worst $(1-\alpha)\%$ of cases. It is a coherent measure that accounts for the severity of tail events, making it superior to VaR for assessing catastrophic risks, such as the complete failure of a major green infrastructure project.
- **Spectral Risk Measure:** $\rho_{\phi}(X) = \displaystyle\int_{0}^{1} F_X^{-1}(u)\phi(u)du$ [1]. A generalization where risk aversion is encoded in a weighting function $\phi(u)$ that is non-negative, decreasing, and integrates to 1. ES is a special case. This allows a decision-maker to explicitly tailor the risk assessment to their specific aversion to losses of different severities, which is crucial for pricing the long-term risks of sustainable investments.

2.1.2 Deviation Measures

While risk measures provide a capital requirement, **deviation measures** $\mathcal{D} : L^{\infty} \to \mathbb{R}_+$ quantify the *nonconstancy* or *uncertainty* of a financial outcome, i.e., its variability or dispersion [19]. This aligns perfectly with our goal of measuring *downside variability*. Their key properties are:

- **Non-Negativity:** $\mathcal{D}(X) \geq 0$ for all X, and $\mathcal{D}(X) = 0$ if and only if X is constant. There is no deviation for a certain outcome.
- **Translation Insensitivity:** $\mathcal{D}(X + C) = \mathcal{D}(X)$ for any constant $C \in \mathbb{R}$. Adding a fixed loss does not change the variability; it only shifts the distribution.
- **Positive Homogeneity:** $\mathcal{D}(\lambda X) = \lambda\mathcal{D}(X)$ for any $\lambda \geq 0$.
- **Sub-Additivity:** $\mathcal{D}(X + Y) \leq \mathcal{D}(X) + \mathcal{D}(Y)$.
- **Convexity:** $\mathcal{D}(\lambda X + (1-\lambda)Y) \leq \lambda\mathcal{D}(X) + (1-\lambda)\mathcal{D}(Y)$.
- **Law Invariance:** If $F_X = F_Y$, then $\mathcal{D}(X) = \mathcal{D}(Y)$.

A functional satisfying Non-Negativity and Translation Insensitivity is a **proper deviation measure**. If it also satisfies Positive Homogeneity and Sub-Additivity (or Convexity), it is a **generalized deviation measure**. The most common example is the **Standard Deviation**, $\mathcal{D}(X) = \sigma(X) = \sqrt{\mathrm{Var}(X)}$. However, a key criticism, especially relevant for downside risk, is that standard deviation is *symmetric* and penalizes both gains and losses equally. Our work focuses on *asymmetric* or *downside-only* deviation measures to better capture the specific risks of financial losses in green tech projects, where the variability of negative outcomes (cost overruns, performance shortfalls) is the primary concern, not the variability of positive surprises.

2.2 Distance-Based Measures of Downside Variability

Let's review traditional deviation measures adapted for downside risk [15]:

- Full Range: $FR(X) = F_X^{-1}(1) - F_X^{-1}(0)$
- Lower/Upper Range: $LR(X) = \mathbb{E}[X] - F_X^{-1}(0)$, $UR(X) = F_X^{-1}(1) - \mathbb{E}[X]$
- Variance/Standard Deviation: $Var(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$, $SD(X) = \sqrt{Var(X)}$
- Semi-Deviation: $SD^-(X) = \sqrt{\mathbb{E}[((X - \mathbb{E}[X])^-)^2]}$, $SD^+(X) = \sqrt{\mathbb{E}[((X - \mathbb{E}[X])^+)^2]}$
- Shortfall Deviation: $SD^\alpha(X) = \sqrt{Var[(X - \mathbb{E}[X|X \geq ES^\alpha(X)])}$ [18]
- Gini index: $Gini(X) = 2\int_0^1 (p - L_X(p))dp$ where $L_X(p) = \frac{1}{\mu_X}\int_0^p F_X^{-1}(u)du$ [14, 24]

3 Relativity-Based Measures

In this part we emphasize measures that capture the relative nature of risk, moving beyond simple distance metrics.

3.1 Gini Index Reinterpretation

The Gini index can be expressed using lower conditional expected loss [22]:

$$\underline{EL}_p(X) := \mathbb{E}\left[X|X \leq F_X^{-1}(p)\right] = \frac{1}{p}\int_0^p F_X^{-1}(u)du$$

$$Gini(X) = \int_0^1 \left(1 - \frac{\underline{EL}_p(X)}{\mu_X}\right) 2pdp$$

This formulation measures variability by comparing the mean of small losses with the overall mean loss.

3.2 Extended Gini Index

To incorporating risk aversion parameter ($r > 1$) [23] we define:

$$EGini_r(X) = r(r-1)\int_0^1 (1-p)^{r-2}(p - L_X(p))dp$$

3.3 Omega Ratio

Originally this ratio is a performance measure [13], adapted for variability assessment and contrasted with measures like the Sharpe ratio [21]:

$$\Omega_\eta(X) = \frac{\mathbb{E}[(X-\eta)_+]}{\mathbb{E}[(\eta - X)_+]}$$

$$\Omega(X) = \mathbb{E}[\Omega_\pi(X)], \quad \pi \sim U_{[0,1]}$$

3.4 Zenga Index

Addressing limitations of Gini index by directly comparing small and large losses [9, 25]:

$$Z(X) = \int_0^1 \left(1 - \frac{\underline{EL}_p(X)}{\overline{EL}_p(X)}\right) dp$$

where $\overline{EL}_p(X) := \mathbb{E}\left[X|X > F_X^{-1}(p)\right] = \frac{1}{1-p}\int_p^1 F_X^{-1}(u)du$

Advantages over Gini [8]:

- Properly reflects relativity between small and large losses.
- Compares disjoint parts of the distribution.
- Equal weighting for all comparisons.

4 Zenga-Type Extensions

We propose some extensions to the Zenga framework by replacing the expected loss with other risk measures, addressing the need to incorporate different risk attitudes.

4.1 Conditional Risk Measures

For any risk measure ρ and split point $p \in (0, 1)$:

$$\underline{\rho}_p(X) := \rho\left(X|X \leq F_X^{-1}(p)\right)$$

$$\overline{\rho}_p(X) := \rho\left(X|X > F_X^{-1}(p)\right)$$

Specific examples:

- Conditional Worst-Case: $\underline{WC}_p(X) = F_X^{-1}(p)$, $\overline{WC}_p(X) = F_X^{-1}(1)$
- Conditional VaR: $\underline{VaR}_p^\alpha(X) = F_X^{-1}(\alpha p)$, $\overline{VaR}_p^\alpha(X) = F_X^{-1}(p + \alpha(1 - p))$
- Conditional ES: $\underline{ES}_p^\alpha(X) = \frac{1}{p(1-\alpha)}\int_{\alpha p}^p F_X^{-1}(u)du$, $\overline{ES}_p^\alpha(X) = \frac{1}{1-p-\alpha(1-p)}\int_{p+\alpha(1-p)}^1 F_X^{-1}(u)du$
- Conditional spectral measures with weighting functions [1]

4.2 General Zenga-Type Measure

$$I_\rho(X) = \mathbb{E}\left[1 - \frac{\underline{\rho}_\pi(X)}{\overline{\rho}_\pi(X)}\right], \quad \pi \sim U_{[0,1]}$$

Properties:

- Non-Negativity: $I_\rho(X) = 0$ for constant X, > 0 otherwise.
- Law Invariance.
- Size Insensitivity: $I_\rho(\lambda X) = I_\rho(X)$ for $\lambda > 0$.
- Monotonicity by translation: $I_\rho(X + C) \leq I_\rho(X)$ for $C > 0$.

Special case: When $\rho = EL$, I_ρ becomes the original Zenga index.

https://doi.org/10.1051/e3sconf/202568000065

Table 1. Descriptive Statistics of Brazilian Per Capita Income

Statistics/Year	1991	2000	2010
Number of counties	4491	5507	5565
Mean	75.861	134.239	192.576
Minimum	14.138	17.055	38.081
Maximum	426.102	1381.658	800.335
Standard Deviation	45.199	79.625	95.162
Skewness	1.379	1.785	0.953
Excess Kurtosis	3.311	12.982	1.583

5 Empirical Analysis

5.1 Economic Case Study: Brazilian Income Inequality

Using per capita income data from IBGE [12] (1991, 2000, 2010):

Key findings [11]:

- Distance-based measures (FR, Var, SD) showed significant variability but couldn't clearly indicate inequality trends.
- Relativity-based measures (Gini, Zenga) indicated a slight decline in inequality by 2010.
- Zenga-type measures (I_{WC}, $I_{VaR^{\alpha}}$, $I_{ES^{\alpha}}$) provided more nuanced insights into tail behavior.
- Brazil showed persistent economic inequality despite growth, attributed to factors like agribusiness concentration and education disparities.

5.2 Financial Case Study: Ibovespa Index

Analysis of log-returns (2014-2019) with focus on downside distribution $X^{-} = \max(-X, 0)$, using data from the `quantmod` package [20]:

Table 2. Descriptive Statistics of Ibovespa Returns

Statistics/Returns	X	X^{-}
Mean	0.053	0.512
Minimum	-9.211	0.000
Maximum	6.389	9.211
Standard Deviation	1.424	0.843
Skewness	-0.118	2.523
Excess Kurtosis	1.906	10.792

Key findings:

- Distance-based measures (FR, UR) tended to overestimate variability.
- Relativity-based measures showed more stable behavior over time.
- Zenga index (mean: 0.850) indicated higher variability than Gini (mean: 0.691) [9].
- Zenga-type measures incorporated risk aversion and confidence levels effectively.
- $I_{ES^{\alpha}}$ showed highest values (mean: 0.970), reflecting tail sensitivity, a concern in downside risk frameworks [10, 17].
- Relative measures demonstrated lower volatility in estimates compared to absolute measures, making them more reliable for long-term assessment.

6 Conclusion

In the present paper [4] we propose some contributions to the field of risk measurement by:

1. Establishing a formal analogy between economic inequality indices and financial downside variability measures. 2. Introducing the concept of relativity between small and large losses, addressing a fundamental challenge in risk assessment. 3. Proposing innovative extensions to the Zenga index framework that incorporate various risk measures, from VaR to spectral measures. 4. Providing empirical evidence that relative measures offer more stable and informative risk assessments compared to traditional absolute measures.

The proposed framework allows investors and risk managers to:

- Better capture the relative nature of financial losses.
- Incorporate risk preferences through various risk measure choices.
- Obtain more stable variability estimates for long-term risk assessment.
- Make more informed decisions about holding financial positions based on loss dispersion characteristics.

The research opens several avenues for future work, including applications to portfolio optimization in a downside-risk framework [10], regulatory capital requirements [16], and systemic risk assessment using the proposed relativity-based framework.

References

[1] Acerbi, C. (2002). Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking & Finance*, 26(7), 1505–1518.

[2] Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1999). Coherent measures of risk. *Mathematical finance*, 9(3), 203–228.

[3] Atkinson, A. B. (1970). On the measurement of inequality. *Journal of economic theory*, 2(3), 244–263.

[4] Berkhouch, M., Lakhnati, G., & Muller, F. M. From economic inequality to financial downside variability. *(Working Paper)*.

[5] Föllmer, H., & Schied, A. (2002). Convex measures of risk and trading constraints. *Finance and stochastics*, 6(4), 429–447.

[6] Gini, C. (1921). Measurement of inequality of incomes. *The Economic Journal*, 31(121), 124–126.

[7] Giorgi, G. M., & Crescenzi, M. (2001). A proposal of poverty measures based on the bonferroni inequality index. *Metron*, 59(3-4), 3–16.

[8] Greselin, F., & Zitikis, R. (2018). From the classical gini index of income inequality to a new zenga-type relative measure of risk: A modellers perspective. *Econometrics*, 6(1), 4.

[9] Greselin, F., Pasquazzi, L., & Zitikis, R. (2013). Contrasting the gini and zenga indices of economic inequality. *Journal of Applied Statistics*, 40(2), 282–297.

[10] Harlow, W. V. (1991). Asset allocation in a downside-risk framework. *Financial analysts journal*, 47(5), 28–40.

[11] IBGE. (2019). *Pesquisa de orçamentos familiares 2017-2018 : primeiros resultados*. Coordenação de Trabalho e Rendimento.

[12] IBGE. (2020). Instituto Brasileiro de Geografia e Estatística. https://ibge.gov.br/.

[13] Keating, C., & Shadwick, W. F. (2002). A universal performance measure. *Journal of performance measurement*, 6(3), 59–84.

[14] Lorenz, M. O. (1905). Methods of measuring the concentration of wealth. *Publications of the American statistical association*, 9(70), 209–219.

[15] Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77–91.

[16] Mao, T., & Wang, R. (2018). Risk aversion in regulatory capital principles. *Available at SSRN 2658669*.

[17] Nawrocki, D. N. (1999). A brief history of downside risk measures. *The Journal of Investing*, 8(3), 9–25.

[18] Righi, M. B., & Ceretta, P. S. (2016). Shortfall deviation risk: An alternative for risk measurement. *Journal of Risk*, 19(2).

[19] Rockafellar, R. T., Uryasev, S., & Zabarankin, M. (2006). Generalized deviations in risk analysis. *Finance and Stochastics*, 10(1), 51–74.

[20] Ryan, J. A., Ulrich, J. M., Thielen, W., Tector, P., Bronder, S., & Ulrich, M. J. M. (2019). Package quantmod. http://cran.hafro.is/web/packages/quantmod/quantmod.pdf.

[21] Sharpe, W. F. (1966). Mutual fund performance. *The Journal of business*, 39(1), 119–138.

[22] Yitzhaki, S. (1983). On an extension of the gini inequality index. *International economic review*, 24(3), 617–628.

[23] Yitzhaki, S., & Schechtman, E. (2005). The properties of the extended gini measures of variability and inequality. *Available at SSRN 815564*.

[24] Yitzhaki, S., & Schechtman, E. (2013). More than a dozen alternative ways of spelling gini. In *The Gini Methodology* (pp. 11–31). Springer.

[25] Zenga, M. (2007). Inequality curve and inequality index based on the ratios between lower and upper arithmetic means. *Statistica and Applicazioni*, 5, 3–27.