Epidemic Driven by Border Routing in Complex Networks of Networks

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Abstract. Traffic-driven epidemic spreading on single or isolated complex networks has been extensively studied over the past decade. However, traffic-driven epidemic spreading in a network of networks has not received sufficient attention. Understanding the dynamics of traffic flow in such a network of networks is crucial for optimizing performance and minimizing congestion. Inspired by the variety of studies on routing algorithms in isolated networks, this paper shifts attention to the impact of routing schemes on traffic dynamics within a network of networks, and further demonstrates that the structure of a network of networks can significantly influence the spreading of epidemics driven by traffic dynamics. Simulation results show that our proposed method Border Administrative Authority (BAA) yields superior outcomes without extra cost and load of traffic to overcome the congestion problem in traffic dynamics. Moreover, we find that coupling through peripheral nodes delays epidemic outbreaks more effectively than hub-to-hub coupling between two networks.

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Keywords. Epidemic spreading, border routing schemes, traffic dynamics, Scale-free networks.

1. Introduction

Modern society relies heavily on the efficient operation of various networked infrastructures, including maritime networks, power grids, the Internet, transportation networks, and more [1,2]. Traffic management within these networked systems poses a significant challenge. Complex networks have garnered substantial attention due to the interactions among elements in such systems [3]. Additionally, the theory of complex networks offers a set of tools and diagnostics that prove useful in gaining insight into these systems.

In the late 1950s, Erdos and Renyi proposed a random graph model that opened up the systematic study of complex network structure in mathematics [4]. Indeed, numerous studies have focused on understanding the interplay between network structure and dynamics from the perspective of complex networks [5], since the discovery of small-world [6] and scale-free [7] topological features at the end of the last century have generally assumed that all of the nodes interact on a single network with a single degree distribution; however, this assumption implies that every node is part of a single network and is represented by a single underlying topology.

In reality, however, societies are composed of many interconnected networks. In particular maritime transport has led many researchers to investigate and describe the structures and dynamics that make transportation an interconnected complex network [8-10].

To improve traffic flow and alleviate congestion in rapidly expanding communication networks, many traffic models have focused on developing better packet routing strategies. These traffic models aim to describe real traffic systems by introducing concepts such as the packet generating rate R, with randomly selected sources and destinations for packets [11]. At the critical packet generating rate Rc, a continuous phase transition from a free-flow state to a congested state occurs. During the free-flow phase, the numbers of created and delivered packets remain balanced, resulting in a steady state. Conversely, in the congested phase, the number of accumulated packets increases over time, due to the limited delivering capacity or finite queue length of each node.

Numerous, studies have been conducted on traffic congestion within the framework of isolated complex networks over the past decade [12]. These studies have revealed that traffic congestion is closely related to the network structure [13–15]. For instance, in [16], the authors examined several representative topologies of complex networks and provided the corresponding theoretical estimates of the traffic capacity. In [17], the authors proposed, building on Zhao's work, two general types of scenarios to alleviate traffic congestion and improve traffic performance: modification of the network topology and development of more effective routing algorithms. Compared with the potential cost of changing the structure of well-established networked systems, proposals of clever routing

criteria seem to be more practical and thus have attracted much interest [18]. Among the various proposed routing protocols, the packets in these studies are primarily forwarded based on two broad classes of routing strategies. In the first class, packets are forwarded based only on local information, known as the local routing strategy [19]. In contrast, the second class involves utilizing information from the entire network, including strategies such as the shortest path strategy [20, 21], the efficient path strategy and the global routing strategy [2].

While the issue of traffic congestion has been extensively researched, most previous studies have concentrated on isolated networks. However, many real-world infrastructure networks are interconnected or interact with one another. In order to model the interactions between real-world networks, several coupled network models have been developed [22]. In [23], the authors proposed a theoretical model of interdependent networks. Another kind of coupling model involves interconnected networks that include physical links between networks and provide paths for traffic transmission [24]. In another study on traffic congestion, authors in [25] show the effect of interconnections on traffic congestion in interconnected Brabasi-Albert scale free networks. Furthermore, in [26], the authors analytically proved that the structure of multiplex networks can induce congestion for flows that otherwise would be decongested if the individual layers were not interconnected.

The importance of the study of complex networks is justified by the different phenomena that occur on these networks, such that epidemic spreading. Indeed, The Human society has suffered from various viruses that have to do with human nature; such as AIDS, H1N1 influenza and viruses that have to do with technological development such as computer viruses [27–30]. Much researches have been done to understand the dynamic process of epidemic spreading on complex networks [31,32].

Many works have been concentrated on the traffic driven and epidemic spreading. The first work on the traffic-driven epidemic spreading was achieved by Meloni et al. [33,34], who found that the behavior of agents influences the disease spreading. Meanwhile, Yang et al. in [35] treated the relation of outbreak time associated with traffic-driven epidemic on the complex network. O. Bamaarouf et al in [12] studied the effect of the different routing algorithm strategies on epidemic spreading in isolated complex networks.

The effect of routing algorithm strategies on epidemic spreading in complex networks of networks has been ignored and has not received an adequate attention. This reason motivated us to get along to study this topic. In this paper, we studied the effect of structure on epidemic spreading in complex networks.

In this paper, the propagation of viruses in the network is modeled via the Susceptible–Infected (SI) model.

In real interconnected networks, there are different administrative routing policies [36]. However, these policies have some drawbacks; such as the difficulty of exchanging and updating information about the subnetworks for all routers, along with the high economic cost associated with information exchange. To address this drawback, the internet uses a well-known routing protocol

between different Autonomous Systems (AS). This routing protocol is called the Border Gate Way Protocol (BGP) [37]. In this paper, it is proposed to use of a simplified model version of this protocol, which separates the two administrative authorities and introduces rules for exchanging information between the two networks. The main results show that comparable performance can be achieved without the additional cost and load of a global routing protocol.

2. Model and methods

2.1. Network structure

Many real-world systems are modelled by the classical network topology "Scale Free Networks (SFN)". To build an SFN that was developed by Barabasi-Albert (BA) m_0 starts with fully connected nodes. At each time step, a new node is added, and the network continues to evolve over time through the growth concept. m new links are established with the existing nodes according to the preferential attachment at each time step. This process generates a scale-free network with a power law degree distribution characterized by the existence of high-degree nodes (called hubs) and low-degree nodes (called peripheral nodes).

In the present study, only two BA scale-free networks labelled A and B were focused. These two distinct networks are interconnected by adding links, facilitating traffic paths between them. This system is referred to as a global network. Here, N_{il} represents the number of links interconnecting sub-network A and sub-network B. The connection process involves selecting a node from network A and another from network B, and then connecting them. This process is repeated until N_{il} interconnected links are added. Nodes from sub-network A (respectively sub-net B) connected to sub-net B (respectively sub-net A) are called border nodes.

2.2. Traffic routing schemes

In a given Global network, at each time step, R new packets are generated with randomly chosen sources and destinations (within sub-net A or sub-net B). Each node can deliver at most C packets towards their destinations (we set C=1). It is assumed that the queue length of each node is unlimited. The first-in-first-out (FIFO) principle applies to the queue. Every newly generated packet is placed at the end of the queue at its source node. The packet is removed from the system when it reaches its destination. In this model, packets are forwarded according to different routing strategies:

- Unique Administrative Authority "UAA"

  In this case, there is a single policy strategy (PS) for routing across the entire global network. As there is a uniform policy strategy, there is no distinction between the packet belonging to sub-net A or to sub-net B. In this study, the shortest path routing strategy is selected as the PS for the global network.

- Multi Administrative Authority “MAA”

  In this case, there are two policy strategies PS_A and PS_B, and each network has its policy strategy; sub-net A adopts policy strategy A, while sub-net B adopts policy strategy B. Each packet belonging to sub-network A is forwarded using policy strategy A, and likewise, packets belonging to sub-network B are forwarded using policy strategy B. In this work, PS_A represents the shortest path routing strategy, while PS_B denotes an efficient path routing scheme.
  In both models mentioned above, the construct of an optimal shortest path between any source and destination across the two sub-nets necessitates that every node in the network possesses global information about the network structure. However, this approach is not cost-effective due to the additional traffic load required for exchanging and updating information about the network's status.

- Border Administrative authority “BAA”

  In this strategy, each node within a sub-net possesses global information only about its respective sub-net. Additionally, border nodes possess information about all the other border nodes. There is no exchange of information between sub-nets *A* and *B* (Figure 1). PSA (respectively PSB) is used as a routing algorithm within sub-net A (respectively B) independently. However, between sub-nets A and B, packets are forwarded randomly from one border node to another in the destination sub-net. If a packet is generated with both the source and destination, within sub-net A, it is forwarded exclusively to PSA. The identical procedure applies to packets generated within sub-network B, utilizing PSB. If a packet is generated with a source within sub-net A and the destination within sub-net B, it is initially forwarded to PSA. Upon reaching a border node of sub-net A, the packet is then randomly sent to a border node of sub-net B. Upon arrival in sub-network B, it adheres to PSB. The same procedure holds for packets generated with source within sub-net B and destination within sub-net A.

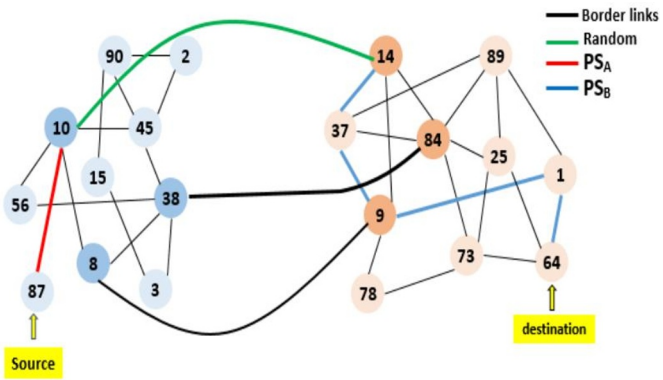


Figure 1. Schematic represents a border administrative authority (BAA).

2.3. Epidemic model

The interest of the previous studies on the spreading was mainly on how the network topology affects the epidemics [35]. Indeed, the typical approach is to assign a certain probability of infection to each link originating from an infected node. In many realistic situations, even when there is a link connecting two nodes, the infection will not propagate unless some kind of traffic happens between the nodes. Following this logic, we have implemented the Susceptible–Infected (SI) model in which the nodes can be divided into two states: either susceptible (S) or infected (I). Beginning with an initial proportion of infected nodes $i_0 = I_0/N$, the infection spreads in the network through packet exchange. The entire packets in an infected node are infected and similarly all the packets in a susceptible node are uninfected. Once a susceptible node receives a packet from an infected neighbor node, it has the probability β of being infected (Fig. 1).

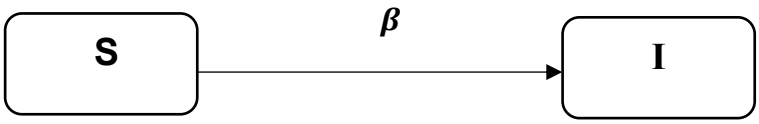


Figure 2. Schematic representation of an SI model with an infection probability $\boldsymbol{\beta}$

3. Results and discussion

In this section, we present our results by focusing on two main aspects: first, the impact of traffic dynamics within a network of networks, and second, the influence of the network-of-networks structure on epidemic spreading.

In this part of the results, we first investigate the effect of routing schemes on traffic dynamics within a network of networks. For the simulations, two networks are considered, denoted as A and B. It is assumed that these two networks have the same size ($N_A = N_B$) and a total network size of $N_{AB} = 500$. Additionally, they share the same average degree: $< k_A > = < k_B > = 4$, with 5 interconnecting links between the two subnets ($N_{int} = 5$). The selection of border nodes in both networks is done using preferential attachment, where the probability P_i of connecting to an existing node i is proportional to the degree k_i of the node as expressed by Eq. (1):

$$P_i = \frac{k_i}{\sum_j k_j} \quad (1)$$

To characterize the phase transition of traffic flow in the entire maritime network, density is employed as an order parameter, as it is described by Eq. (2) [30]:

$$\rho = \lim_{t\to\infty} \frac{C}{R} \frac{<\Delta N_p>}{\Delta t} \quad (2)$$

Where:

R are the new packets generated in the entire network.

C is the delivered node (here, we set $C = 1$ for each node).

$$\Delta N_p = N_p(t + \Delta t) - N_p(t) \quad (3)$$

ΔN_p : represents the change in the total number of packets within the network over a time window of width Δt.

$< \Delta N_p >$: indicates the average over time windows of width Δt.

The order parameter reflects the equilibrium between the inflow and outflow of packets. In the state of free flow, characterized by a balance between created and removed packets, the order parameter ρ approaches zero. As the packet generation rate R increases, a critical value R_c emerges, marking the phase transition from free flow to congestion. Once R exceeds R_c, packets begin to accumulate continuously throughout the entire network, resulting in a steady ρ value greater than zero. The network's capacity can be measured by the maximum generation rate R_c observed at the phase-transition point.

In Figure 3, the traffic density ρ, is plotted against the number of packets R, under various network administrative authorities. Specifically, ρ_A, ρ_B, and ρ_T represent the order parameters for sub-networks A, B, and the entire network AB, respectively.

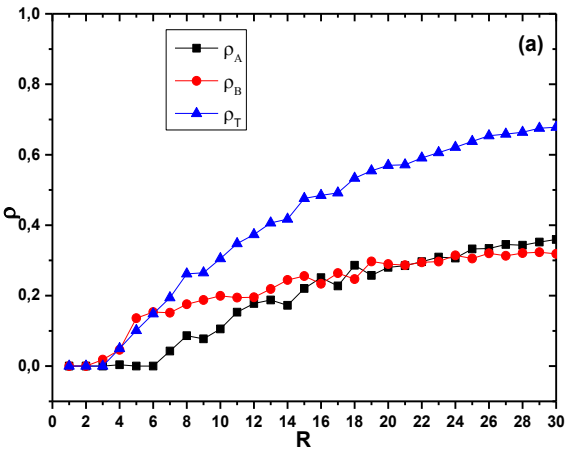
In Figure 3(a), a singular routing algorithm (shortest path: SP) is utilized across the entire network, while in Figure 3(b), multiple routing algorithms (SP for subnet A and efficient path: EP for subnet B) are employed. Both routing strategies require that every node within the subnet possesses global information regarding the network structure. In each network, there is a critical R value (Rc), below which the traffic density remains at zero (indicating free flow), while above which the traffic density increases with an increase in the packet number R (indicating congestion phase). Moreover, the UAA strategy (Figure 3.a) exhibits an asymmetry of Rc between networks A and B, where $R_{cA} = 6$, whereas $R_{cB} = 3$. However, in the MAA strategy (Figure 3.b), both networks exhibit almost the same critical value ($R_{cA} =$

$6\ and\ R_{cB} = 5$). Furthermore, at high R values, the UAA strategy shows that both networks A and B have almost the same traffic density. Nonetheless, adopting the MAA strategy reveals that network A experiences twice as much congestion as network B, indicating an imbalance between the two networks in terms of congestion.

Figure 3(c) illustrates the outcomes of the BAA strategy. Here, distinct routing algorithms are applied to the two subnets, with SP utilized for subnet A and EP for subnet B independently. Packet transmission between the two subnets occurs randomly. In this network administrative authority, each node possesses information solely about the network to which it belongs. This figure indicates that using this strategy, the traffic density exhibits the same qualitative behaviour as the aforementioned strategies. However, the transition from free flow to congestion phase occurs at the same critical value of the number of packets, R_c ($R_{cA} = R_{cB} = 7$), for both networks A and B. Additionally, both networks exhibit asymmetry in terms of traffic density, with network A showing lower density compared to network B.

For a better comparison between the three strategies: UAA, MAA and BAA, the total density of the system, including both networks A and B, as an order parameter as a function of packets number for the three routing strategies is illustrated in Figure 4.

Significant differences are observed among the three strategies, with varying transition points from free flow to congestion phase: $Rc1 = 3, Rc2 = 6, and\ Rc3 = 7$ for UAA, MAA, and BAA respectively. Consequently, the proposed strategy (BAA) can sustain the system in free flow mode for a larger number of packets compared to UAA and MAA, facilitating congestion-free information transmission. Another distinction lies in the transition nature: UAA and MAA exhibit a second-order transition from free flow to congestion phase, whereas BAA demonstrates a first-order transition. (economic cost….).


(a)
ρA
ρB
ρT
ρ
R
1,0
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0,2
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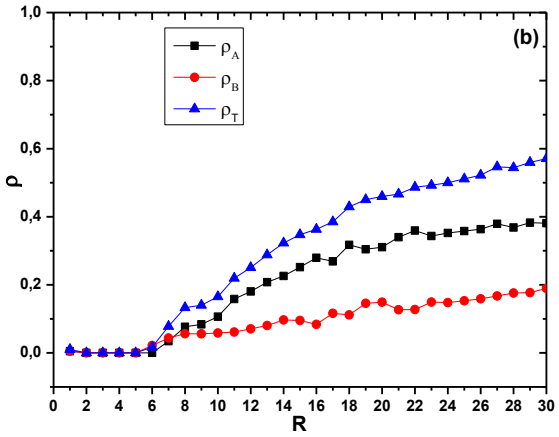
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30


(b)
ρA
ρB
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R
1,0
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0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

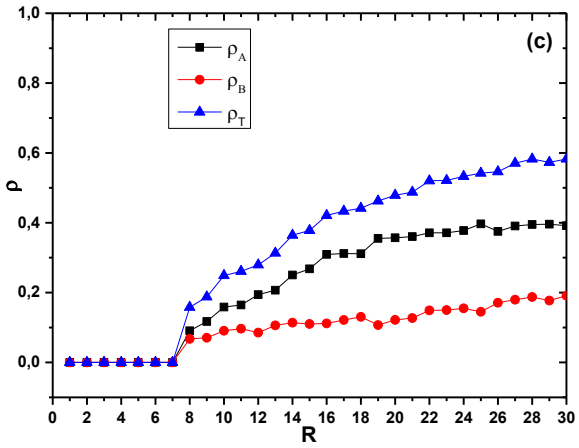


Figure 3. Evolution the order parameter ρ as a function of packets number R under different network administrative authorities. UAA (a), MAA (b), BAA (c).

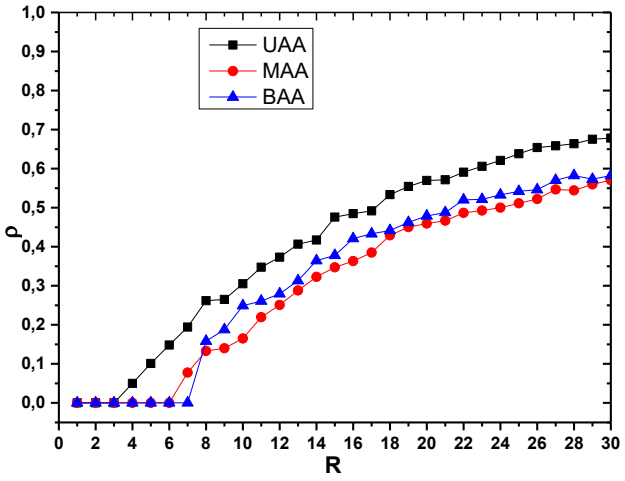


Figure 4. Comparison the capacity of traffic as a function of packets number, under different network administrative authorities. UAA, MAA and BAA.

For a deeper analysis of the impact of the proposed strategy, the travel time is analyzed using the three strategies.

The travel time, or transport time, of a packet is defined as the total duration it spends in the network from its creation until it reaches its destination [19]. Another approach to defining it involves adding the path length (total number of nodes along the path) to the total waiting time at the node queues along the packet trajectory. The average travel time is determined by summing and averaging the travel time of all packets that successfully reach their destinations.

Figure 4 demonstrates the traveling time as a function of the number of packets (R) for three routing schemes UAA, MAA, and BAA. Initially, as it is obvious that the first lowest number of packets ($R < 12$) are somehow similar when it comes to the three routing schemes, which means they are identical in their traveling time. However, the difference appears after the highest number of packets ($R > 12$), where it is noticeable that UAA and MAA have an increase in the average traveling time for the number of packets to reach their final destinations. Contrasting with BAA, it is noticed that it has a proficient traveling time when it comes to the number of packets toward its goal destination.

Another parameter to evaluate the performance of the network is the path length. it specifically pertains to the packets that have been successfully removed from the system. It is calculated by summing the path lengths of all removed packets and dividing this total by the number of removed packets. Here, the path length refers to the number of hops that each packet makes from its source to its destination .

Figure 5 illustrates the relationship between path length and the number of packets for the three routing schemes: UAA, MAA, and BAA.

UAA demonstrates the poorest option with the lowest path length compared to MAA and BAA. This indicates that UAA is not cost-effective and leads to additional traffic load. Conversely, BAA shows a significantly improved path length compared to UAA and slightly higher than MAA. Additionally, BAA demonstrates good performance without adding extra costs or traffic load compared to the other strategies. In the SP algorithm, packets try to navigate from source to destination following the shortest paths by traversing a minimum number of intermediate nodes. In contrast, when either the EP and GD algorithms are used, packets try to avoid congested nodes by using more intermediate no congested nodes following longer paths.

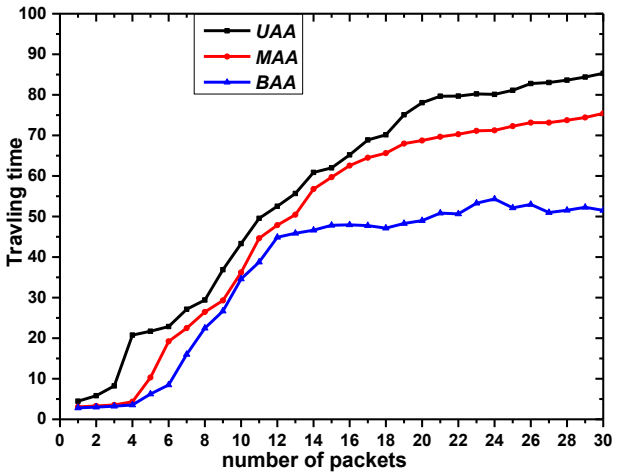


Figure 5. The average traveling time as a function of packets number, under different network administrative authorities. UAA, MAA and BAA.

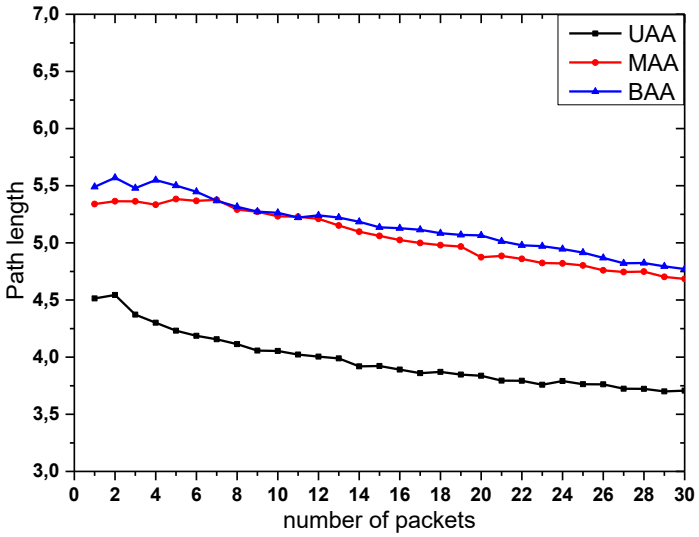
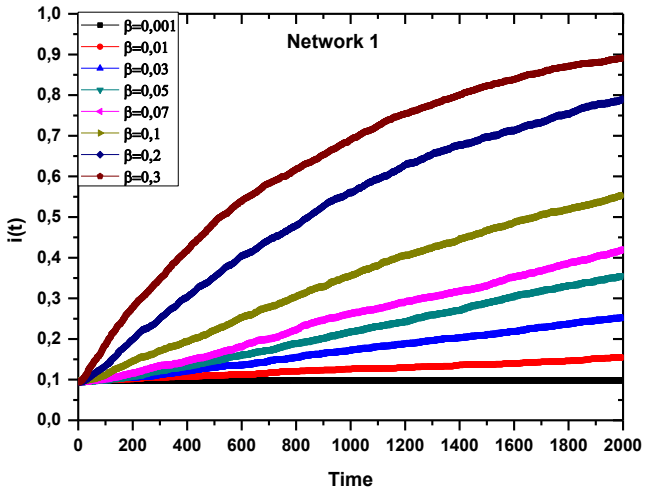


Figure 6. The average path length as a function of packets number, under different network administrative authorities. UAA, MAA and BAA.

Building on the analysis of traffic dynamics, we now turn to the second aspect of our study; the effects of epidemic spreading driven by border routing in a network of networks. In this simulation, we consider two networks, BA, with N1 = N2 = 500 nodes, interconnected by10 links. Two interconnection strategies are examined; Border Hubs (hub-to-hub) and Border Peripherals (peripheral-to-peripheral). We study the effects of epidemic driven by border routing in interconnected networks. When the epidemic occurs in subnet 1, who the epidemic spread to the interconnected subnets 2. We used the SI model. Begining with an initial proportion of infected nodes (I_0=50) in subnets A and any infection nodes in subnets 2.

In figure 7 we have plotted the evolution of the fraction of infected nodes (i) in the two subnets under the Shortest Path (SP) routing strategy during the free-flow regime (R=3), for varying values of β. Across all three panels, the infection propagates rapidly to subnet 2 for both interconnection schemes (border hubs and border peripherals). Notably, when comparing the two strategies, the epidemic spread in subnet 2 is delayed for small β under the border-peripheral coupling, whereas for larger β values, the infection dynamics become comparable for both coupling approaches.


Network 1
β=0,001
β=0,01
β=0,03
β=0,05
β=0,07
β=0,1
β=0,2
β=0,3
i(t)
Time
0,0
0,1
0,2
0,3
0,4
0,5
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200
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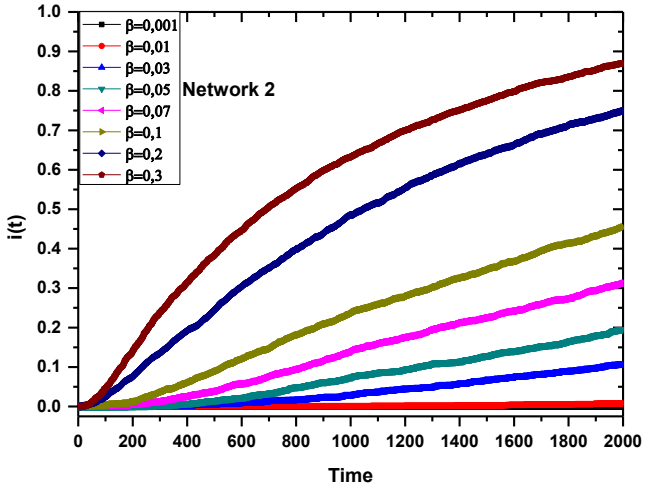
Network 2
β=0,001
β=0,01
β=0,03
β=0,05
β=0,07
β=0,1
β=0,2
β=0,3
i(t)
Time
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
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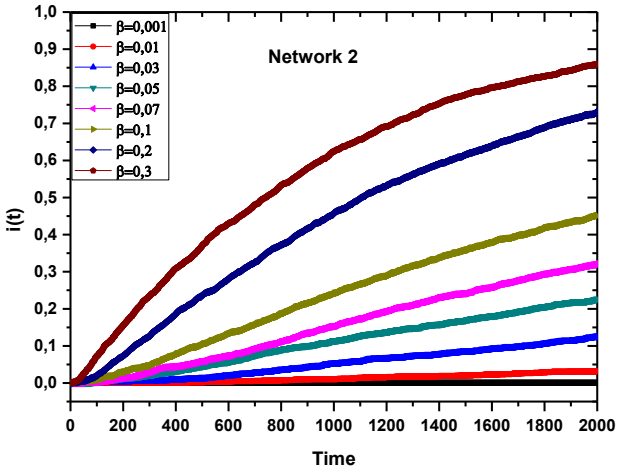
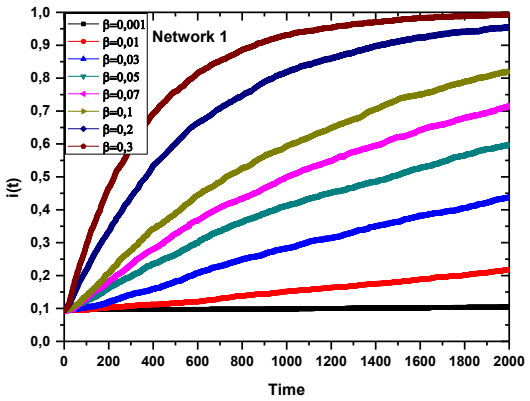


Figure 7. Proportion of infected nodes (i) over time in the free-flow phase, with an initial infection proportion i0=0.1 under the SI model. Results are shown for: (a) Network 1, (b) Network 2 with peripheral-to-peripheral border coupling, and (c) Network 2 with hub-to-hub border coupling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β=0,001
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β=0,03
β=0,05
β=0,07
β=0,1
β=0,2
β=0,3
i(t)
Time
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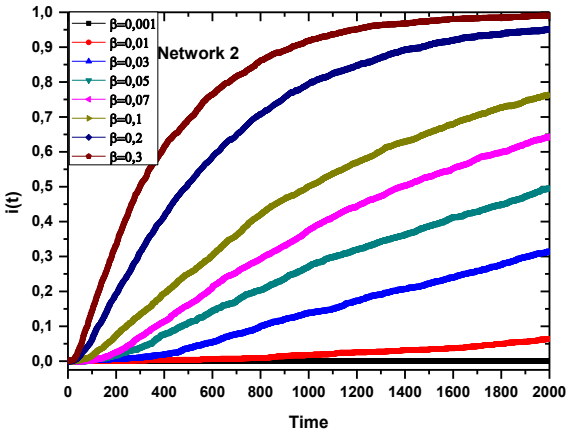
Network 2
β=0,001
β=0,01
β=0,03
β=0,05
β=0,07
β=0,1
β=0,2
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i(t)
Time
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0,1
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0,4
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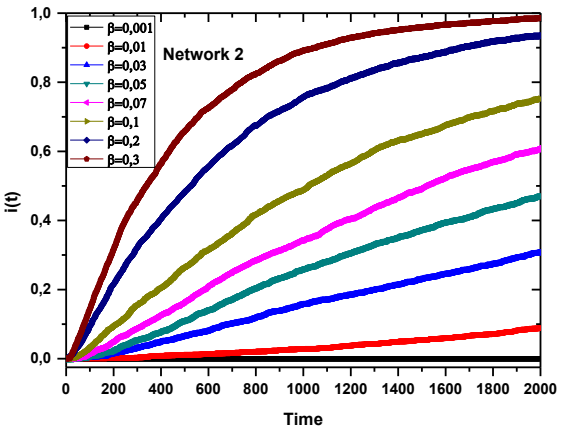
Network 2
β=0,001
β=0,01
β=0,03
β=0,05
β=0,07
β=0,1
β=0,2
β=0,3
i(t)
Time
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0,4
0,6
0,8
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1800
2000

Figure 8. Proportion of infected nodes (i) over time in congested phase, with an initial infection proportion i0=0.1 under the SI model. Results are shown for: (a) Network 1, (b) Network 2 with peripheral-to-peripheral border coupling, and (c) Network 2 with hub-to-hub border coupling

In the congested phase, Figure .8 illustrates the evolution of the fraction of infected nodes i(t) in both subnets for varying values of the infection rate β. The panels correspond to the two interconnection schemes; border hubs (figure 8.B) and border peripherals (figure 8.C), In contrast to the free-flow regime, the propagation dynamics here are markedly slower, particularly in subnet 2. For small values of β, the infection spreads very gradually with i(t) remaining below 0.2 throughout the simulation period. This slow growth highlights the inhibiting effect of congestion on the network ability to sustain efficient epidemic transmission. At higher values of β (β≥0.05), the infection still manages to propagate, but reaches saturation levels more slowly. This indicates that even under congested conditions, a sufficiently virulent epidemic can overcome the transmission limitations imposed by traffic saturation particularly within subnet 1, which serves as the initial infection source.

An important distinction emerges between the two interconnection strategies. For small β, the border-peripheral scheme results in a more pronounced delay in the infection's arrival and growth within subnet 2, compared to the border-hub configuration. However, as β increases, the infection dynamics under both coupling strategies become increasingly similar.

In the congestion regime, limitations due to traffic overload not only affect routing efficiency but also have a direct impact on epidemic dynamics by constraining the effective contact rate between susceptible and infected nodes. This interplay between network traffic conditions and disease spread emphasizes the critical role of routing-induced congestion in shaping epidemic outcomes in interconnected networks.

4. Conclusion

In summary, this paper explores the interplay between traffic dynamics and epidemic spreading in interconnected networks. Introducing a novel border routing scheme called Border Administrative Authority (BAA), the proposal relies on each node in the network having exclusive access to information pertaining only to its respective network. This stands in contrast to the global routing strategies of Unique Administrative Authority (UAA) and Multi Administrative Authority (MAA), where every node possesses information about the entire network. To assess transportation capacity in interconnected networks, metrics such as the order parameter, average traveling time, and average path length are employed. Importantly, our results extend

to traffic-driven epidemic spreading, demonstrating that routing strategies and interconnection structures significantly impact outbreak dynamics. In particular, peripheral node coupling delays epidemic spread more effectively than hub-to-hub connections, offering a promising approach for epidemic mitigation.

References

1. Ducruet C. The geography of maritime networks: A critical review. Journal of Transport Geography, 2020, https://doi.org/10.1016/j.jtrangeo.2020.102824
2. Pablo-Martí F, Sánchez A. Improving transportation networks: Effects of population structure and decision making policies. Scientific Reports, (2017), 7(1), 4498, https://doi.org/10.1038/s41598-017-04892-2
3. Newman, Mark. Networks: An Introduction. 1st edn (Oxford, 2010; online edn, Oxford Academic, 1 Sept. 2010), https://doi.org/10.1093/acprof:oso/9780199206650.001.0001
4. ERD O, Renyi A. On random graphs. Publ. Math, 1959, 6, 290-297, https://www.bibsonomy.org /bibtex /99061fc859ba 540d4485abfbce44f298
5. Xia Y, Hill D J. Attack vulnerability of complex communication networks. IEEE Transactions on Circuits and Systems II: Express Briefs, 2008, 55(1), 65-69, doi: 10.1109/TCSII.2007.908954
6. Watts D J, Strogatz S H. Collective dynamics of 'small-world' networks. Nature, 1998, 393(6684), 440-442, https://doi.org/10.1038/30918
7. Barabási A L, Albert R. Emergence of scaling in random networks. Science, 1999, 286(5439), 509-512, doi: 10.1126/science.286.5439.509
8. Álvarez N G, Adenso-Díaz B, Calzada-Infante L. Maritime traffic as a complex network: A systematic review. Networks and Spatial Economics, 2021, 21(2), 387-417, https://doi.org/10.1016/j.aap.2016.04.030
9. Wu J, Zhong J, Chen Z, Chen B. Optimal coupling patterns in interconnected communication networks. IEEE Transactions on Circuits and Systems II: Express Briefs, 2018, 65(8), 1109-1113, doi: 10.1109/TCSII.2018.2808297
10. D'Agostino G, Scala A. Networks of networks: the last frontier of complexity. Berlin: Springer 2014, Vol. 340, https://link.springer.com/book/10.1007/978-3-319-03518-5
11. Zhang J, Ma J, Li H J. An efficient link closing strategy for improving traffic capacity on scale-free networks. Physica A: Statistical Mechanics and its Applications, 2022, 604, 127887, https://doi.org/10.1016/j.physa.2022.127887
12. Bamaarouf O, Alweimine A O B, Rachadi A, Ez-Zahraouy H. Selective epidemic vaccination under the performant routing algorithms. Physica A: Statistical Mechanics and its Applications, 2018, 496, 209-219, https://doi.org/10.1016/j.physa.2017.12.148
13. Boccaletti S, Ivanchenko M, Latora V, Pluchino A, Rapisarda A. Detecting complex network modularity by dynamical clustering. Physical Review E, 2007, 75(4), 045102, https://doi.org/10.1103/PhysRevE.75.045102
14. Guimera R, Arenas A, Díaz-Guilera A, Giralt F. Dynamical properties of model communication networks. Physical Review E, 2002, 66(2), 026704, https://doi.org/10.1103/PhysRevE.66.026704
15. Guimerà R, Díaz-Guilera A, Vega-Redondo F, Cabrales A, Arenas A. Optimal network topologies for local search with congestion. Physical review letters, 2002, 89(24), 248701, https://doi.org/10.1103/PhysRevLett.89.248701
16. Zhao L, Lai Y C, Park K, Ye N. Onset of traffic congestion in complex networks. Physical Review E, 2005, 71(2), 026125, https://doi.org/10.1103/PhysRevE.71.026125
17. Chen S, Huang W, Cattani C, Altieri G. Traffic dynamics on complex networks: a survey. mathematical Problems in engineering, 2012, doi:10.1155/2012/732698
18. Ling X, Hu M B, Jiang R, Wu Q S. Global dynamic routing for scale-free networks. Physical Review E, 2010, 81(1), 016113, https://doi.org/10.1103/PhysRevE.81.016113
19. Wang W X, Yin C Y, Yan G, Wang B H. Integrating local static and dynamic information for routing traffic. Physical Review E, 2006, 74(1), 016101, https://doi.org/10.1103/PhysRevE.74.016101

20. Newman M E. Scientific collaboration networks. II. Shortest paths, weighted networks, and centrality.Physical review E, 2001, 64(1), 016132, https://doi.org/10.1103/PhysRevE.64.016132
21. LEE E J, GOH K-I, KAHNG B, et al. Robustness of the avalanche dynamics in data-packet transport on scale-free networks. Physical Review E, 2005, vol. 71, no 5, p. 056108, https://doi.org/10.1103/PhysRevE.71.056108
22. Sole-Ribalta A, Gomez S, Arenas A. Congestion induced by the structure of multiplex networks. Phys. Rev. Lett, 2016, vol. 116, no. 10, p. 108701, https://doi.org/10.1103/PhysRevLett.116.108701
23. Buldyrev S V, Parshani R, Paul G, Stanley H E, Havlin S. Catastrophic cascade of failures in interdependent networks. Nature, 2010, vol. 464, no. 7291, pp. 1025–1028, https://doi.org/10.1038/nature08932
24. Hu Y, Ksherim B, Cohen R, Havlin S. Percolation in interdependent and interconnected networks: Abrupt change from second-to first-order transitions. Phys. Rev. E, 2011, vol. 84, no. 6, p. 066116, doi: https://doi.org/10.1103/PhysRevE.84.066116
25. Tan, F, Wu J, Xia Y, & Tse C K. Traffic congestion in interconnected complex networks. Physical Review E, 2014, 89(6), 062813, https://doi.org/10.1103/PhysRevE.89.062813
26. J.O. Kephart, S.R. White, Proc. IEEE Comp. Soc. Symp., Vol. 343, 1991.
27. P.V. Mieghem, J. Omic, R. Kooij, IEEE ACM Trans. Net. 17 (2009).
28. J. Amador, Appl. Math. Comput. 232 (2014) 1112.
29. X. Han, Q. Tan, Appl. Math. Comput. 217 (2010) 2520.
30. C. Gan, X. Yang, Q. Zhu, Nonlinear Dyn. 73 (2013) 1615.
31. R. Pastor-Satorras, A. Vespignani, Phys. Rev. Lett. 86 (2001) 3200.
32. M. Barthélemy, A. Barrat, R. Pastor-Satorras, A. Vespignani, Phys. Rev. Lett. 92 (2004) 178701.
33. S. Meloni, A. Arena, Y. Moreno, Proc. Natl. Acad. Sci. USA 106 (2009) 16897.
34. S. Meloni, N. Perra, A. Arenas, S. Gomez, Y. Moreno, A. Vespignani, Sci. Rep. 1 (2011) 62.
35. Y. Han-Xin, W. Wen-Xu, L. Ying-Cheng, Chaos 22 (2012) 043146.
36. Zhang B, Liu R, Massey D, Zhang L. Collecting the Internet AS-level topology. ACM SIGCOMM Computer Communication Review, 2005, 35(1), 53-61, doi: 10.1145/1052812.1052825
37. Rekhter Y, Li T, Hares S. A border gateway protocol 4 (BGP-4), 2016, (No. rfc4271), https://www.rfc-editor.org/rfc/rfc4271