Numerical Study of the Nonlinear Viscoelastic Creep Behavior of a Mirror Epoxy Resin

Mohsen Dardouri[1*], *Kamel* Bousnina[2], *Jamel* Mars[3], *Ali* Fellah[4], *and Abdessattar* Aloui[1]

[1]Applied Mechanics, Innovative Processes and Environment Research Unit (UR22ES04), ISSATG, University of Gabes, Gabes, Tunisia.
[2]LMPE Laboratory, National School of Engineering of Tunis, University of Tunis, Tunisia.
[3]Laboratory of Mechanical Modeling, Energy & Materials (LM2EM), ENIG, University of Gabes, Gabes, Tunisia.
[4]Laboratory of Applied Thermodynamics (18ES33), ENIG, University of Gabes, Gabes, Tunisia

Abstract. This work presents a numerical validation of the nonlinear viscoelastic creep behavior of a mirror epoxy resin using the Schapery model. The material parameters were identified from experimental creep tests conducted under several constant stress levels. The Schapery creep formulation was implemented into the Abaqus finite element framework through a user-defined material subroutine (UMAT) and applied to a representative numerical model to reproduce the experimental loading conditions. The simulated creep strain evolutions show good agreement with the experimental results over the entire loading duration. Quantitative comparisons reveal low average errors (approximately 1.5–3%), high coefficients of determination ($R^2 > 0.90$), and moderate dispersion, confirming the robustness of the identified parameters. These results demonstrate the capability of the UMAT-based Schapery creep model to accurately capture the nonlinear time-dependent response of the epoxy resin under sustained loading. The proposed approach provides a reliable basis for predicting long-term viscoelastic creep behavior in polymer-based structural applications.

1 Introduction:

Epoxy resins are thermosetting polymers [1] widely used in many industrial sectors due to their mechanical performance, chemical stability, and excellent adhesion. They are commonly employed as anti-corrosion coatings and structural materials owing to their ease of processing, low curing shrinkage, and good resistance to aggressive environments [2]. Epoxy systems are also extensively used in the manufacture of industrial tooling and as structural adhesives, particularly in the aerospace industry [3].

In flooring applications, mirror epoxy resins have gained increasing interest because of their durability, waterproof properties, and high mechanical and chemical resistance. These two-component resins, typically formulated with a hardener-to-resin ratio of 50:100, exhibit nonlinear viscoelastic behavior dominated by creep, in which deformation depends

* Corresponding author: dardouri.mohsen@yahoo.fr

simultaneously on time and the applied stress level. Under such conditions, the linear superposition principle is no longer valid, making the use of appropriate nonlinear constitutive models essential.

Modeling nonlinear viscoelastic creep is commonly achieved by extending Boltzmann's superposition principle through the introduction of nonlinear functions. Early formulations were proposed by Leaderman [4], followed by further developments by Bernstein et al. [5] and Green and Rivlin [6]. Among these approaches, the Schapery model [7] has become one of the most widely used frameworks for describing nonlinear creep in polymeric materials, particularly for long-term time-dependent behavior.

Several studies have proposed finite element implementations of Schapery's model for nonlinear viscoelastic creep analysis. Henriksen [8] developed an algorithm based on the integral formulation of the model, while Lai and Bakker [9] and Haj-Ali and Muliana[10] proposed three-dimensional formulations and numerical integration schemes, implemented in particular through the UMAT user subroutine of the ABAQUS finite element code. However, these studies mainly address generic polymeric materials and do not account for the specific behavior of mirror epoxy resins, whose creep response is strongly nonlinear and highly dependent on the applied stress level.

In this context, the present study proposes a numerical modeling approach for the nonlinear viscoelastic creep behavior of mirror epoxy resin based on Schapery's model. The nonlinear parameters and Prony series were identified experimentally by Dardouri et al. [11] and implemented in a user-defined material subroutine (UMAT) within ABAQUS. A comparison between numerical predictions and experimental creep data is conducted to assess the predictive capability of the model and its relevance for engineering applications.

Thus, the remainder of this work is structured to guide the reader through the different steps of this approach: the formulation of the Schapery model for the creep case presents the constitutive equations and the incremental formulation adapted to creep; the implementation of the UMAT subroutine details the numerical implementation in ABAQUS; finally, the results and their discussion present a comparative analysis between simulations and experiments, allowing for the evaluation of the relevance and reliability of the model in predicting the viscoelastic behavior of the studied resin.

2 Finite Element Modeling of Nonlinear Viscoelastic Creep Behavior:

2.1 Formulation of the Schapery Model for the Creep Case:

Creep corresponds to the time-dependent deformation of a material subjected instantaneously to a constant applied stress. Among the existing approaches, the nonlinear viscoelastic formulation proposed by Schapery is widely used to model the behavior of polymeric materials. Developed on the basis of the principles of irreversible thermodynamics, this constitutive law can, in the case of uniaxial loading and isothermal deformation, be expressed in the following form [12][13]:

$$\varepsilon(\sigma,t) = \sigma\left[g_0 D_0 + g\sum_{i=1}^{m} D_i\left(1 - e^{\left(\frac{-t}{\tau_i}\right)}\right)\right] \tag{1}$$

Where:

- ✓ D_i and τ_i denote the transient compliance coefficients and the relaxation times, respectively, associated with different viscous mechanisms [11].
- ✓ D_0 denotes the instantaneous uniaxial elastic compliance

- ✓ g_0 represents the nonlinear instantaneous elastic compliance and reflects the change in stiffness of the material.

2.2 Implementation of the UMAT Subroutine

Within the framework of nonlinear viscoelastic creep modeling using Schapery's model, it is common to express the strain in an incremental form in order to facilitate time discretization and numerical implementation. At a given time t, the strain increment is defined as the difference between the strain at the current time and the strain at the previous time step:

$$\Delta\boldsymbol{\varepsilon}^{t} = \boldsymbol{\varepsilon}^{t} - \boldsymbol{\varepsilon}^{t-\Delta t} \tag{2}$$

Similarly, the stress increment is defined as:

$$\Delta\boldsymbol{\sigma}^{t} = \boldsymbol{\sigma}^{t} - \boldsymbol{\sigma}^{t-\Delta t} \tag{3}$$

By applying this definition to the one-dimensional Schapery model, the following expression for the strain increment is obtained:

$$\boldsymbol{\varepsilon}^{t} = \boldsymbol{\sigma}^{t}\left[g_0 D_0 + g\sum_{i=1}^{m} D_i\left(1 - e^{\left(\frac{-t}{\tau_i}\right)}\right)\right] \tag{4}$$

$$\boldsymbol{\varepsilon}^{t-\Delta t} = \boldsymbol{\sigma}^{t-\Delta t}\left[g_0 D_0 + g\sum_{i=1}^{m} D_i\left(1 - e^{\left(\frac{-(t-\Delta t)}{\tau_i}\right)}\right)\right] \tag{5}$$

$$\Delta\varepsilon^{t} = \Delta\sigma^{t} * \left[g_0 * D_0 + g * \sum_{i=1}^{m} D_i * \left(1 - e^{-\frac{t}{\tau_i}}\right)\right] + \sigma^{t-\Delta t} * \left[g * \sum_{i=1}^{m} D_i * \left(e^{-\frac{t-\Delta t}{\tau_i}} - e^{-\frac{t}{\tau_i}}\right)\right] \tag{6}$$

This relation allows the evaluation of the strain increment $\Delta\varepsilon^{t}$ as a function of the stress increment $\Delta\sigma^{t}$, the model parameters (D_i,τ_i, g_0, g), and the stress history at the previous time step $\sigma^{t-\Delta t}$. The first term represents the instantaneous contribution due to the stress increment, weighted by the creep functions, while the second term accounts for the variation of the viscoelastic memory functions between $t - \Delta t$ and t. This formulation is well-suited for implicit numerical implementations, particularly in UMAT-type subroutines for simulating time-dependent viscoelastic behavior.

The residual R^t is defined as the difference between the ABAQUS strain increment $\Delta\boldsymbol{\varepsilon}^{abaqus}$ and the estimated strain increment $\Delta\boldsymbol{\varepsilon}^{estimated}$ calculated from the estimated stress increment $\Delta\boldsymbol{\sigma}^{estimated}$.

$$R^{t} = \Delta\varepsilon^{estimeted} - \Delta\varepsilon^{abaqus} \tag{7}$$

- ✓ $\Delta\varepsilon^{estimeted}$ **:** is calculated from the incremental strain equation of the Schapery model.
- ✓ $\Delta\varepsilon^{abaqus}$:is the strain increment provided by Abaqus.

$$\Delta\varepsilon^{estimeted} = \Delta\sigma^{estimeted} * \left[g_0 * D_0 + g * \sum_{i=1}^{m} D_i * \left(1 - e^{-\frac{t}{\tau_i}}\right)\right] + \sigma^{t-\Delta t} * \left[g * \sum_{i=1}^{m} D_i * \left(e^{-\frac{t-\Delta t}{\tau_i}} - e^{-\frac{t}{\tau_i}}\right)\right] \tag{8}$$

$$R^{t} = \Delta\sigma^{estimeted} * \left[g_0 * D_0 + g * \sum_{i=1}^{m} D_i * \left(1 - e^{-\frac{t}{\tau_i}}\right)\right] + \sigma^{t-\Delta t} * \left[g * \sum_{i=1}^{m} D_i * \left(e^{-\frac{t-\Delta t}{\tau_i}} - e^{-\frac{t}{\tau_i}}\right)\right] - \Delta\varepsilon^{abaqus} \tag{9}$$

The residual R^t is used to check the convergence of the estimated stress increment $\Delta\sigma^{estimé}$. The objective is to determine $\Delta\sigma^{estimé}$ such that $R^t = 0$, thereby ensuring that the estimated strain increment exactly matches the strain increment provided by Abaqus.

In the UMAT routine, the residual R^t is also used to iteratively update the estimated stress increment $\Delta\sigma^{estimé}$ until convergence is achieved, while following the steps below:

1. Calculate $\Delta\varepsilon^{\text{estimé}}$:

Use the incremental strain equation to calculate $\Delta\varepsilon^{\text{estimé}}$ based on $\Delta\sigma^{\text{estimé}}$.

2. Calculate the residual R^t :

$R^t = \Delta\varepsilon^{\text{estimeted}} - \Delta\varepsilon^{\text{Abaqus}}$

3. Check convergence: if $|R^t| \leq$ tol, where tol is a defined tolerance, then convergence is achieved.
4. Update $\Delta\sigma^{\text{estimé}}$: if $|Rt|>$tol, update $\Delta\varepsilon^{\text{estimeted}}$ using an iterative method, such as the Newton-Raphson method:

$$\Delta\sigma^{\text{estimeted}} = \Delta\sigma^{\text{estimeted}} - \frac{R^t}{\frac{dR^t}{d\Delta\sigma^{\text{estimeted}}}} \qquad (10)$$

Where: $\frac{dR^t}{d\Delta\sigma^{estimeted}}$

denotes the derivative of the residual with respect to the estimated stress increment. The derivative of the residual is given by:

$$\frac{dR^t}{d\Delta\sigma^{\text{estimeted}}} = g_0 * D_0 + g * \sum_{i=1}^{m} D_i * \left(1 - e^{-\frac{t}{\tau_i}}\right) \qquad (11)$$

And the transformation Jacobian is: $\frac{d\sigma}{d\varepsilon} = \left[\frac{dR^t}{d\Delta\sigma^{\text{estimeted}}}\right]^{-1}$

The flowchart for developing the creep UMAT (***Fig. 1***) starts with reading the input data, including material parameters (D_i,τ_i), state variables, stress, and strain increments. The first step is to calculate the nonlinear parameters (g_0 et g), which depend on the applied stress level. When the stress is less than or equal to 4 MPa, the simplified values g=1and g_0=1 are used. Otherwise, these parameters are evaluated using experimentally identified polynomial relations.

Once these coefficients are determined, an estimated strain increment $\Delta\varepsilon^{\text{estimeted}}$ is calculated. A convergence criterion is introduced based on the residual defined in Eq. (7)

If the norm of the residual is below a set tolerance ($|R^t| \leq \mathbf{10^{-8}}$), the state variables are updated: the total creep strain and the constitutive derivatives (tangent matrix DDSDDE). Otherwise, a correction iteration is performed using the Newton-Raphson method. This correction relies on the derivative of the residual with respect to the strain increment, allowing the estimated value of $\Delta\varepsilon^{\text{estimeted}}$ to be updated.

When convergence is achieved, the internal variables are updated and returned to Abaqus, ensuring the continuity of the simulation. This incremental scheme allows an accurate representation of the time-dependent evolution of nonlinear viscoelastic creep strain according to the Schapery model.

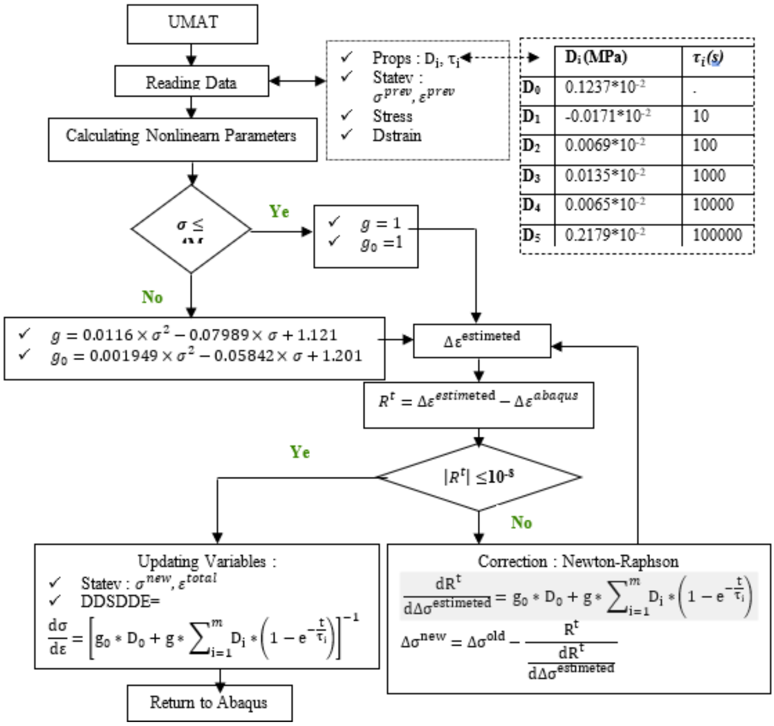

Fig. 1. Flowchart of the UMAT for creep

3 Results and discussion

Fig. 2 shows the variation of the experimental and simulated strain of the mirror epoxy resin under different applied stress levels, namely 4 MPa, 7 MPa, 9 MPa, 11 MPa, and 13 MPa. In all cases, a progressive increase in strain over time is observed, which is characteristic of viscoelastic creep behavior. The simulated curves obtained from the Schapery model (implemented in the UMAT) generally follow the same trend as the experimental results, demonstrating the model's ability to reproduce the time-dependent evolution of strain under different loading conditions. This direct comparison provides a solid basis for quantitatively evaluating the model's accuracy, which will be discussed using statistical indicators such as the mean error, the coefficient of determination (R^2), and the standard deviation [14] [15].

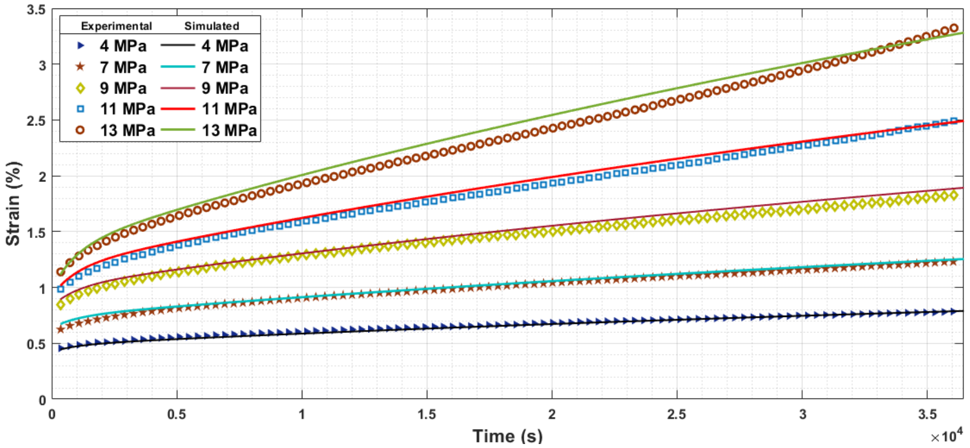

Fig. 2. Variation of experimental and simulated strain of mirror epoxy resin for different stress levels (4MPa, 7MPa, 9MPa, 11MPa and 13MPa).

The analysis of relative errors as a function of stress level (**Table 1**) highlights the model's ability to faithfully reproduce the experimentally observed mechanical behavior. At low stresses (4 and 7 MPa), the average errors are below 2 %, with an R^2 close to unity, indicating an almost perfect fit and confirming that the model effectively captures the initial creep kinetics. The moderate increase in error at 9 MPa (2,93 %) may reflect the material's increased sensitivity to nonlinear phenomena or secondary micro-deformation mechanisms, although the low standard deviation and high R^2 indicate a strong correlation.

At 11 MPa, the decrease in average error (2,42 %) and the optimal R^2 (0,9869) suggest that the model describes the material's response very accurately within this stress range. Finally, at 13 MPa, the highest average error (3,49 %) and increased standard deviation indicate a slight loss of accuracy, likely related to the complexity of high-stress creep mechanisms (localized damage, microstructural rearrangements). Despite this, the R^2 remains above 0,97, confirming that the overall trend is well reproduced.

These results demonstrate that the model exhibits high robustness across the entire stress range, with small and controlled deviations, making it reliable for predictive simulations of long-term behavior.

Table 1. Error analysis at different loading levels

Stress (MPa)	**Mean Error (%)**	**Standard Deviation (%)**	**R^2**	**Analysis**
4 MPa	1.5938	1.1343	0.9806	Excellent accuracy with low error and R^2 close to 1, indicating a good fit.
7 MPa	1.9895	1.3225	0.9811	Performance similar to 4 MPa, slight increase in error but R^2 remains high.
9 MPa	2.9313	1.0507	0.9637	Noticeable increase in mean error, but standard deviation remains controlled. R^2 slightly lower.

11 MPa	2.4173	0.9403	0.9869	Reduction in mean error compared to 9 MPa, with excellent R^2.
13 MPa	3.4879	1.3924	0.9775	Highest mean error, but R^2 remains high, suggesting the trend is well captured despite deviations.

Conclusion

The comparative analysis between experimental and simulated creep curves highlights the ability of the Schapery model, implemented through a UMAT subroutine, to accurately reproduce the viscoelastic behavior of the studied material. The relative errors observed in creep remain low and generally stable across the investigated stress levels, with a high correlation coefficient ($R^2 > 0.9$), indicating good agreement between numerical predictions and experimental trends.

Although minor local fluctuations may appear, particularly at low strain levels, the results confirm the robustness and accuracy of the model in predicting long-term creep behavior. These findings demonstrate that the developed UMAT constitutes a reliable and effective numerical tool for modeling the nonlinear viscoelastic creep response of the studied resin.

References.

[1] J. J. Fekiač *et al.*, “Comprehensive Review: Optimization of Epoxy Composites, Mechanical Properties, & Technological Trends,” *Polymers (Basel).*, vol. 17, no. 3, 2025, doi: 10.3390/polym17030271.

[2] D. Meng, S. Yang, H. Yang, A. M. P. De Jesus, J. Correia, and S.-P. Zhu, “Intelligent-inspired framework for fatigue reliability evaluation of offshore wind turbine support structures under hybrid uncertainty,” *Ocean Eng.*, vol. 307, p. 118213, Sep. 2024, doi: 10.1016/j.oceaneng.2024.118213.

[3] K. Abdessemed, O. Allaoui, B. Guerira, and L. Ghelani, “Characterization of the thermal, water absorption, and viscoelastic behavior of short date palm fiber reinforced epoxy,” *Mech. Time-Dependent Mater.*, 2023, doi: 10.1007/s11043-023-09656-2.

[4] H. Leaderman, “Elastic and creep properties of filamentous materials,” 1941,

[5] L. Bernstein, B.; Kearsley, E. A.; Zapas, “A Study of Stress Relaxation with Finite Strain,” *Rubber Chem. Technol.*, pp. 38(1), 76-89., 1963, doi: doi:10.5254/1.3535640.

[6] A. E. Green and R. S. Rivlin, “The mechanics of non-linear materials with memory,” *Arch. Ration. Mech. Anal.*, vol. 4, no. 1, pp. 387–404, 1959, doi: 10.1007/BF00281398.

[7] M. Jafaripour and F. Taheri-Behrooz, “Creep behavior modeling of polymeric composites using Schapery model based on micro-macromechanical approaches,” *Eur. J. Mech. A/Solids*, vol. 81, no. July 2019, p. 103963, 2020, doi: 10.1016/j.euromechsol.2020.103963.

[8] M. Henriksen, “Nonlinear viscoelastic stress analysis - a finite element approach,” *Comput. Struct*, vol. 18, pp. 133–139, 1984.

[9] A. Lai, J., Bakker, “3-D Schapery representation for non-linear viscoelasticity and finite element implementation,” *Comput. Mech*, vol. 18, pp. 182–191, 1996.

[10] R. M. Haj-Ali and A. H. Muliana, “Numerical finite element formulation of the

Schapery non-linear viscoelastic material model," *Int. J. Numer. Methods Eng.*, vol. 59, no. 1, pp. 25–45, 2004, doi: 10.1002/nme.861.

[11] M. Dardouri, A. Fellah, F. Gmir, and A. Aloui, "Long-term viscoelastic behavior and evolution of the Schapery model for mirror epoxy," *J. Mech. Behav. Mater. 33 20240012*, 2024.

[12] D. H. Asif Abdul Azeez , Kyong Yop Rhee, Soo Jin Park, "Epoxy clay nanocomposites – processing, properties and applications: A review," *Compos. Part B Eng.*, pp. 308–320, 2013.

[13] A. Horoschenkoff, "Characterization of the Creep 22 and J Compliances J Orthotropic Composites with PEEK and Epoxy Matrices Using the Nonlinear Viscoelastic Response of the Neat Resins," *J. Coposite Mater.*, vol. 24, pp. 879–891, 1989.

[14] R. M. Guedes and J. L. Morais, "Comparison of the Performance of Nonlinear Time-Dependent Constitutive Models Calibrated with Minimal Test Data Applied to an Epoxy Resin," *Materials (Basel).*, vol. 18, no. 2, pp. 1–20, 2025, doi: 10.3390/ma18020404.

[15] D. Chicco, M. J. Warrens, and G. Jurman, "The coefficient of determination R-squared is more informative than SMAPE, MAE, MAPE, MSE and RMSE in regression analysis evaluation," *PeerJ Comput. Sci.*, vol. 7, pp. 1–24, 2021, doi: 10.7717/PEERJ-CS.623.