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Simplified stiffness models: numerical optimization of story-dependent calibration for RC moment frames

Caetano José Freitas[1], *Yoyong* Arfiadi[2*]

[1]Master student, Department of Civil Engineering, Universitas Atma Jaya Yogyakarta, Indonesia.
[2]Professor, Department of Civil Engineering, Universitas Atma Jaya Yogyakarta, Indonesia.

Abstract. Classical analytical formulations for lateral story stiffness in reinforced concrete (RC) moment-resisting frames (MRFs) are widely used in simplified seismic analysis but are known to overestimate elastic stiffness, particularly in multi-span configurations. This study proposes a numerical calibration framework to improve analytical story stiffness estimation while preserving the closed-form structure of classical formulations. Forty-nine RC MRF portal frame configurations with span numbers ranging from one to seven were analyzed using linear elastic numerical models to obtain reference story stiffness values. The analytical formulation is enhanced through a beam–column interaction factor and a span-dependent correction factor calibrated using nonlinear least-squares optimization with story-dependent parameters for base, middle, and top stories. The calibrated model shows excellent agreement with numerical results, achieving a coefficient of determination of $R^2 = 0.9998$ with negligible systematic bias. The proposed approach provides a practical and computationally efficient tool for estimating lateral story stiffness in RC MRF structures for preliminary design and simplified seismic analysis.

1. Introduction

Accurate estimation of lateral story stiffness is essential for seismic analysis and design of reinforced concrete (RC) moment-resisting frame (MRF) structures, as it directly governs inter-story drift demand and lateral force distribution in simplified seismic analysis [1], [2]. Classical analytical stiffness formulations, typically derived from column flexural stiffness under linear elastic assumptions, are therefore widely used due to their analytical simplicity and computational efficiency [3]. However, when applied to multi-story RC MRFs, these formulations are known to systematically overestimate actual elastic stiffness obtained from numerical analysis, primarily due to beam–column interaction effects and frame configuration characteristics that are not fully captured in classical models [4], [5].

To address this limitation, this study proposes a rational calibration framework for analytical lateral story stiffness in RC MRF portal frames. The proposed approach correlates classical analytical stiffness expressions with numerically obtained elastic stiffness values and introduces optimized correction parameters while preserving the closed-form analytical

*Corresponding author: yoyong.ar@uajy.ac.id

structure. A key contribution of this work is the implementation of story-dependent calibration with separate parameter sets for base, middle, and top stories, resulting in significantly improved stiffness prediction accuracy while maintaining practical applicability for preliminary design and simplified seismic analysis.

2. Materials and Methods

2.1. Structural Models and Numerical Analysis

This study investigates the lateral story stiffness of reinforced concrete-moment-resisting frame (RC MRF) portal structures through numerical elastic analysis. A total of forty-nine (49) planar RC portal frame configurations were considered, representing variations in structural height and span arrangement. The configurations range from single-story single-span frames to multi-story multi-span frames, enabling a systematic investigation of the influence of structural geometry on story stiffness behaviour, as shown in Figure 1. Other structural properties are shown in Table 1.

All structural models were idealized as two-dimensional portal frames with rigid beam–column joints and fixed column bases [6]. Beams and columns were modelled as linear elastic members, assuming small deformation behaviour and neglecting material nonlinearity. The elastic modulus and sectional properties were assigned consistently across all models to isolate the effects of geometric configuration on stiffness response [1]. In the structural models, the own weight of the structural member is neglected. Numerical analyses were performed using linear static loading. A lateral force was applied at each story level, and the corresponding lateral displacements were obtained from the analysis results. All structural models were analysed using MIDAS software [7]. The applied loading pattern was selected to ensure elastic structural response and to allow direct evaluation of inter-story stiffness characteristics [2], [3].

To account for the inherent variation of stiffness along the height of the structure, story responses were categorized into three groups: base story, middle story, and top story. This classification allows a focused evaluation of stiffness behaviour at different elevation levels and supports subsequent scaling and calibration procedures [8]. The results of MIDAS computer modelling provide story shear forces and lateral displacements, which are subsequently used to evaluate the actual elastic story stiffness and to assess the accuracy of classical analytical stiffness formulations, as described in the following sections.

Table 1. Summary of analyzed frame configurations

Parameter	Range/Value
Number of stories	1 to 7
Number of spans	1 to 7
Base Story height (H)	5 m
Upper Story height (H)	4 m
Span length (L)	5 m
Column dimensions	0.4 x 0.4 m
Beam dimensions	0.3 x 0.5 m
Concrete elastic modulus	25742.96 MPa
Total configurations	49

Note: All frames analyzed as 2D portal systems with rigid joints and fixed base conditions.

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2.2. Actual Elastic Story Stiffness

The actual elastic story stiffness of the portal frame structures was evaluated based on the results obtained from the MIDAS analysis. For each story, the elastic story stiffness was defined as the ratio between the story shear force and the corresponding lateral displacement at that story level [1]. The actual elastic story stiffness, denoted as K_{actual}, is expressed as (see Figure 1):

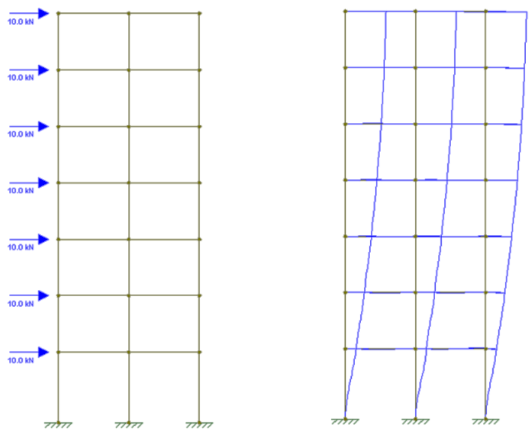

Fig. 1. Representative portal frame model from 49 analyzed configurations

Schematic representation of story shear force (V) and interstory drift (δ) for calculation of actual elastic story stiffness. Left: undeformed structure under lateral loading; Right: deformed configuration showing interstory displacements (Δ_i at story top, Δ_{i-1} at story bottom, and $\delta = \Delta_i - \Delta_{i-1}$), where Δ_i = ith floor displacement and Δ_{i-1} = (i-1) floor displacement

$$K\ actual\ = V\ /\ \delta \tag{1}$$

where V is the story shear force obtained from the computer analytical model, and δ is the lateral interstory drift at the corresponding story level. This definition represents the effective elastic stiffness of each story under lateral loading and reflects the combined contribution of flexural stiffness, structural geometry, and load distribution [2]. The stiffness values obtained using this approach are regarded as reference values for evaluating the accuracy of approximate stiffness formulations.

The calculated values of K_{actual} were extracted for all stories and subsequently grouped into base, middle, and top story categories. This grouping allows a consistent comparison between actual stiffness results and estimated stiffness models [8] as well as the assessment of stiffness variation along the height of the structure [8]. The actual elastic story stiffness values serve as the benchmark against which analytical stiffness predictions and calibrated stiffness formulations are evaluated in the subsequent analysis stages.

2.3. Classical Analytical Story Stiffness Formulation

The analytical story stiffness of reinforced concrete moment-resisting frames was evaluated using a classical column-based stiffness formulation commonly adopted in structural analysis and seismic design practice. In this approach, the lateral stiffness of a story is primarily governed by the flexural stiffness of the columns within that story, assuming rigid beam-column joints and fixed-end column conditions [1].

For a single column with flexural rigidity EI and story height H, the lateral stiffness contribution is expressed as:

$$K_c = \frac{12EI}{H^3} \tag{2}$$

For a story consisting of multiple columns, the analytical story stiffness is obtained by summing the stiffness contributions of all columns within the story:

$$K_{\text{analytical}} = \sum \frac{12EI}{H^3} \quad (3)$$

This formulation is widely used in simplified analytical models and is often attributed to classical stiffness-based approaches employed in frame analysis literature [3]. Equation (3) has been widely used in the literature, e.g., in [1]. Although the expression itself originates from fundamental beam theory and classical structural mechanics[6]. While this column-based stiffness formulation provides a convenient analytical estimate of story stiffness, it neglects the influence of beam flexural stiffness, span configuration, and frame continuity effects. As a result, it tends to overestimate the actual elastic story stiffness when compared with numerically obtained stiffness values, particularly for multi-span and multi-story frame configurations [4], [5]. Therefore, this analytical stiffness formulation is adopted in this study as a baseline model, which is subsequently modified through correction factors to improve its agreement with the actual elastic story stiffness obtained from computer model analysis.

2.4. Frame Interaction Modification Factor (α)

The classical analytical story stiffness formulation presented in Section Classical Analytical Story Stiffness Formulation considers only the flexural stiffness of columns and assumes a fully rigid beam (floor). However, in reinforced concrete moment-resisting frames, the relative stiffness between beams and columns significantly influences the overall lateral response of the frame. Neglecting this interaction may lead to an overestimation of the story stiffness.

To account for the effect of beam-to-column stiffness interaction, a beam–column stiffness modification factor, denoted as $\boldsymbol{\alpha}$ is introduced. This factor represents the relative contribution of beam flexural stiffness to the lateral deformation compatibility of the frame at a given story.

The modification factor $\boldsymbol{\alpha}$ is defined as:

$$\boldsymbol{\alpha} = \frac{\Sigma\left(\frac{\boldsymbol{EI_b}}{\boldsymbol{L_b}}\right)}{\Sigma\left(\frac{\boldsymbol{EI_{ct}}}{\boldsymbol{H_{ct}}}\right)+\left(\frac{\boldsymbol{EI_{cb}}}{\boldsymbol{H_{cb}}}\right)} \quad (4)$$

where:

- EI_b and L_b denote the beam flexural rigidity and span length,
- EI_{ct} and H_{ct} are the flexural rigidity and height of columns above,
- EI_{cb} and H_{cb} those of columns below the considered story.

This formulation reflects the relative stiffness balance between beams and columns that governs the rotational compatibility at beam-column joints [1], [6]. A higher value of α indicates a greater influence of beam stiffness relative to column stiffness, resulting in reduced lateral stiffness contribution from the columns.

It is important to note that the factor α is derived purely from structural geometry and sectional properties, without any reference to numerical or elastic stiffness obtained from analysis results. Therefore, α remains an independent analytical parameter and does not rely on the actual elastic story stiffness [1], [4].

The modified analytical story stiffness incorporating the beam-to-column stiffness interaction is then expressed as:

$$K_\alpha = \alpha \cdot K_{analytical} \quad (5)$$

where $K_{\text{analytical}}$ is the classical column-based analytical story stiffness defined in the classical analytical story stiffness formulation [2], [5], [9]

2.5. Empirical Span-Dependent Correction Factor (β) with Story-Dependent Calibration

Although the analytical stiffness is modified by the beam–column interaction factor, α provides an improved representation of frame behavior; systematic discrepancies with numerically evaluated story stiffness remain. These discrepancies indicate the presence of global structural effects that are not explicitly captured by classical analytical formulations [4], [5].

Parametric investigation of multiple reinforced concrete moment-resisting frame configurations reveals that the number of spans (N_b) plays a significant role in influencing lateral story stiffness. As the number of spans increases, changes in global deformation patterns, load redistribution, and redundancy effects become more pronounced, particularly at the middle story, where boundary influences are minimized [2], [9], [10].

To account for this behaviour, an empirical correction factor β is introduced as a nonlinear function of the number of spans:

$$\beta_s = c_s + a_s {N_b}^{b_s} \tag{6}$$

where a_s, b_s, and c_s are empirical parameters associated with a specific story category s. In this study, three group categories are considered: base, middle, and top stories [8].

Rather than introducing additional scaling factors, the same functional form of $\beta(N_b)$ is retained for all story levels, while the empirical parameters are calibrated independently for each story category. This story-dependent calibration strategy allows the analytical formulation to capture systematic stiffness variation along the building height while preserving a unified modelling framework [3].

The calibration of the empirical parameters is performed through numerical optimization by minimizing the discrepancy between the analytically modified stiffness and the numerically evaluated elastic story stiffness for each story category. This approach results in a configuration-based and story-sensitive correction factor without altering the analytical structure of the stiffness formulation. The final calibrated analytical story stiffness is expressed as:

$$K_{proposed} = \alpha \cdot \beta_s \cdot K_{analytical} \tag{7}$$

This formulation provides a consistent and physically interpretable analytical model for estimating lateral story stiffness across different structural configurations and story levels [1], [6].

2.6. Optimization Procedure

The empirical parameters ($\boldsymbol{a_s}$, $\boldsymbol{b_s}$, $\boldsymbol{c_s}$) for each story category were determined through numerical optimization by minimizing the discrepancy between the calibrated analytical stiffness and the numerically evaluated elastic story stiffness. The optimization was performed independently for base, middle, and top story groups to allow story-dependent calibration while maintaining a consistent functional form [8].

The objective function for each story category $s$is defined as the sum of squared errors:

$$J_s = \sum_{i=1}^{N_s} \left(K_{proposed\,i} - K_{actual,i}\right)^2 \tag{8}$$

where N_s is the number of story samples in the category s, $K_{\text{proposed},i}$ is the calibrated analytical stiffness, and $K_{\text{actual},i}$ is the corresponding numerical elastic stiffness from finite element analysis [6].

Substituting the expression for K_{proposed} from Equation (7) yields:

$$J_s(a_s, b_s, c_s) = \sum_{i=1}^{N_s} \left[\alpha_i \cdot \left(c_s + a_s N_{b,i}^{b_s}\right) \cdot K_{analytical,i} - K_{actual,i}\right]^2 \quad (9)$$

The optimization problem is formulated as:

$$\min_{a_s, b_s, c_s} J_s(a_s, b_s, c_s) \quad (10)$$

The calibration dataset consisted of story samples extracted from the 49 frame configurations analyzed, with the number of spans ranging from single-bay to multi-bay arrangements [3], [4]. The optimized parameters for each story category are presented in the Results section.

The nonlinear least squares optimization was implemented using the lsqnonlin solver in GNU Octave. Physically meaningful parameter bounds were imposed to ensure numerical stability. Convergence was achieved when the relative change in the objective function between successive iterations fell below a tolerance of 10^{-6}.

3. Results and Discussion

3.1. Performance of Classical Analytical Story Stiffness

The classical column-based analytical formulation was first evaluated by comparing the analytically estimated story stiffness, $K_{\text{analytical}}$, with the actual elastic story stiffness obtained from computer model analysis, K_{actual}. The comparison indicates that the classical formulation consistently overestimates lateral story stiffness for all investigated frame configurations [3], [4].

This overestimation becomes more pronounced as the number of spans and stories increases, highlighting the limitations of simplified column-based stiffness expressions when applied to multi-span RC MRF frames [5], [9]. Figure 2 shows the plot of K_{actual} versus $K_{\text{analytical}}$ for all frame configurations. Blue, red, and green markers indicate top, middle, and base story stiffness, respectively. The classical formulation systematically overestimates actual stiffness, with the linear regression ($y = 0.3085x$, $R^2 = 0.9962$) showing strong correlation but consistent deviation above the perfect agreement line.

The observed discrepancy confirms that classical formulations are unable to adequately capture system-level deformation characteristics, particularly those arising from frame continuity and global load redistribution effects.

By comparing the results, the classical formulation overestimates the elastic story stiffness by an average of 38.6%, with deviations ranging from 21.4% to 57.9% depending on frame configuration. The coefficient of determination between $K_{\text{analytical}}$ and K_{actual} $R^2 = 0.9962$, indicating a very strong linear correlation; however, the data points consistently lie above the perfect agreement line, confirming systematic overestimation bias rather than random scatter [2].

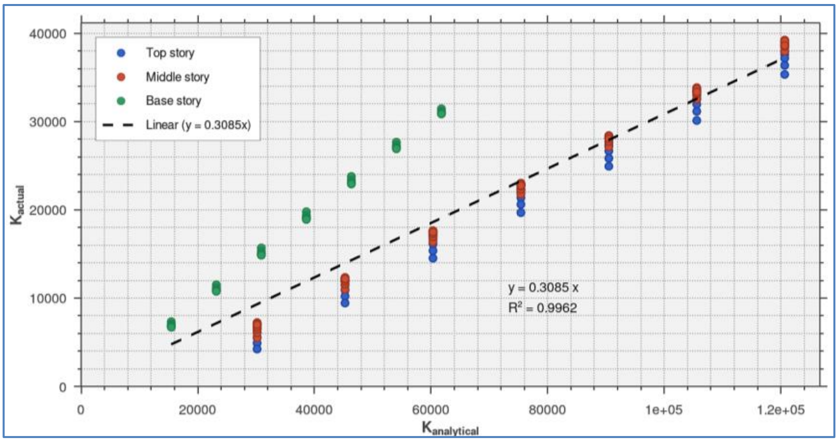

Fig.2. K_{actual} vs $K_{\text{analytical}}$

3.2. Effect of Beam–Column Interaction Modification (α)

Upon introduction of the beam-column interaction modification factor α, a noticeable reduction in analytical stiffness is observed. The modified stiffness $\alpha \cdot K_{\text{analytical}}$ shows improved agreement with numerical stiffness values obtained from the computer model, as reflected by a coefficient of determination of $R^2 = 0.9636$ (see Figure 3). This indicates that the relative stiffness contribution of beams plays an important role in governing lateral response. However, despite this improvement, systematic deviations between the modified analytical stiffness and K_{actual} remain, especially for frames with larger numbers of spans. This result suggests that local beam–column interaction alone is insufficient to fully capture global stiffness effects and that additional configuration-dependent correction factors must be considered.

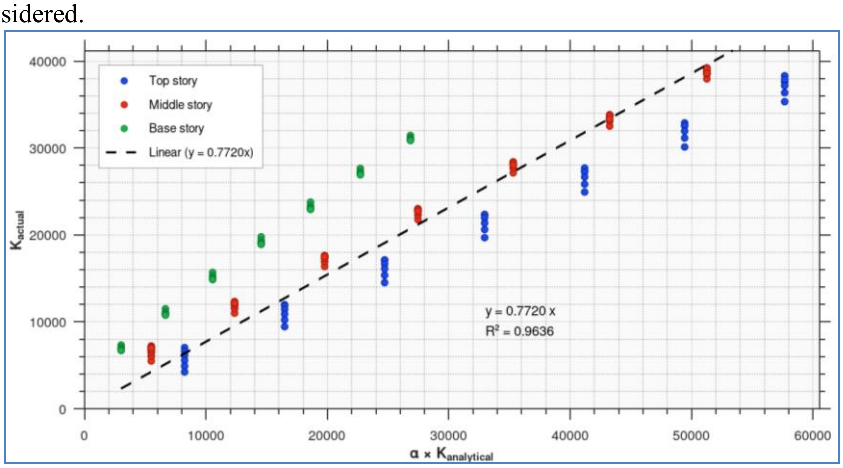

Fig. 3. K_{actual} vs α. $K_{\text{analytical}}$

3.3. Influence of Structural Configuration on Story Stiffness

Further examination of numerical results reveals that geometric ratios, such as the story height-to-span ratio (H/L), do not produce a consistent correlation with the observed stiffness discrepancy. Consequently, such ratios were not pursued further in the calibration process [2], [3]. In contrast, the number of spans (N_b) exhibits a clear and stable influence on lateral story stiffness. As N_b increases, the calibrated analytical story stiffness decreases relative to the classical analytical prediction. From a physical perspective, this trend reflects increased structural redundancy and deformation compatibility effects in multi-span frames. With a larger number of bays, lateral deformation is distributed among more columns, reducing the effective contribution of individual columns to story stiffness compared with classical column-based assumptions.

This behaviour is most evident at the middle story level, where boundary effects from the base and roof are minimized, and global deformation mechanisms dominate [4], [10]. These observations justify the selection of N_b as the primary configuration parameter for empirical stiffness correction.

3.4. Calibration Results of the Empirical β Function

The empirical correction factor β, formulated as a nonlinear function of the number of spans, was calibrated independently for base, middle, and top stories while retaining a consistent functional form. The calibrated parameters demonstrate distinct characteristics for each story category, reflecting the influence of boundary conditions and load transfer mechanisms at different elevations [1]. The middle story exhibits the most stable and monotonic relationship between β and N_b, making it a suitable reference for initial calibration [8]. Base and top stories show systematic deviations from the middle-story trend, which are effectively captured through story-dependent parameter sets [2], [10]. The consistency of the functional form across all story levels confirms the robustness of the proposed calibration framework [6].

Table 2. Calibrated empirical parameters for the β correction factor

Story Category	a	b	c
Base	1.37659	-0.90001	0.92272
Middle	0.58986	-0.76186	0.62194
Top	0.06167	-1.14735	0.63674

Parameters calibrated through nonlinear least squares optimization using 49 frame configurations with N_b ranging from 1 to 7 spans.

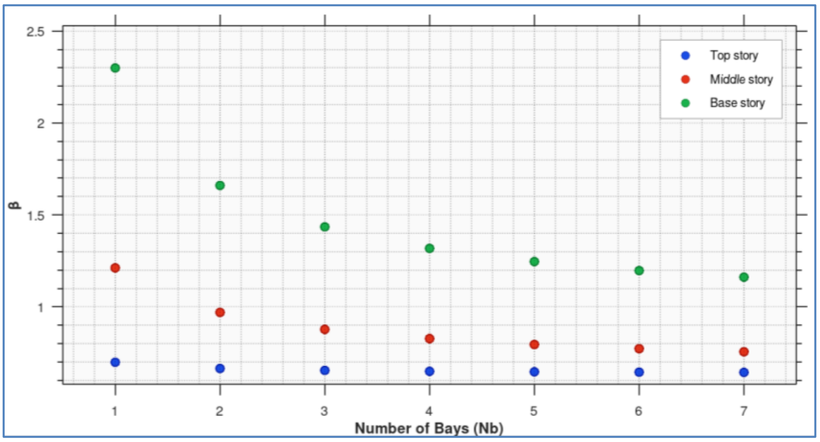

Fig. 4. β vs Nb

Variation of the calibrated β correction factor with the number of spans N_b for base, middle, and top story categories. Figure 4 illustrates the systematic variation of the calibrated β factor with the number of spans, highlighting distinct trends for different story categories [5], [9].

3.5. Performance of the Calibrated Stiffness Model

The final calibrated analytical stiffness, expressed as:

$$\boldsymbol{K}_{\boldsymbol{Proposed}} = \boldsymbol{\alpha} \cdot \boldsymbol{\beta}_{\boldsymbol{s}} \cdot \boldsymbol{K}_{\boldsymbol{analytical}}$$

was compared with the numerical stiffness K_{actual} for all frame configurations and story levels, where α is obtained from equation (4) and β_s is computed using equation (6). The parameters a_s, b_s, and c_s have been computed as in Table 2. The results show a substantial improvement, with the calibrated model closely estimating the actual stiffness, which can be seen from the trend across the stiffness of base, middle, and top stories [3], [4].

Compared to the classical formulation, the proposed model significantly reduces stiffness overestimation, yielding a near-linear correlation with the actual results. This demonstrates that the combined effect of beam–column interaction and span-dependent correction successfully captures the dominant mechanisms governing lateral story stiffness in RC MRF frames. The results show a substantial improvement in agreement, with the calibrated model achieving $R^2 = 0.9998$, representing a significant enhancement over both the classical ($R^2 = 0.9962$) and α-modified ($R^2 = 0.9957$) formulations. As shown in Fig. 5, the calibrated model closely follows the numerical stiffness trend across base, middle, and top stories, with data points clustering tightly around the perfect agreement line. The near-unity regression slope (1.0001x) indicates virtually no systematic bias, confirming that the combined effect of beam–column interaction and span-dependent correction successfully captures the dominant mechanisms governing lateral story stiffness in RC MRF frames [11], [12], [13].

Comparison between actual elastic story stiffness (K_{actual}) and fully calibrated analytical stiffness $\alpha . \beta . K_{\text{analytical}}$. The calibrated model demonstrates excellent agreement across all frame configurations and story levels, with data points clustering closely around the perfect agreement line [14].

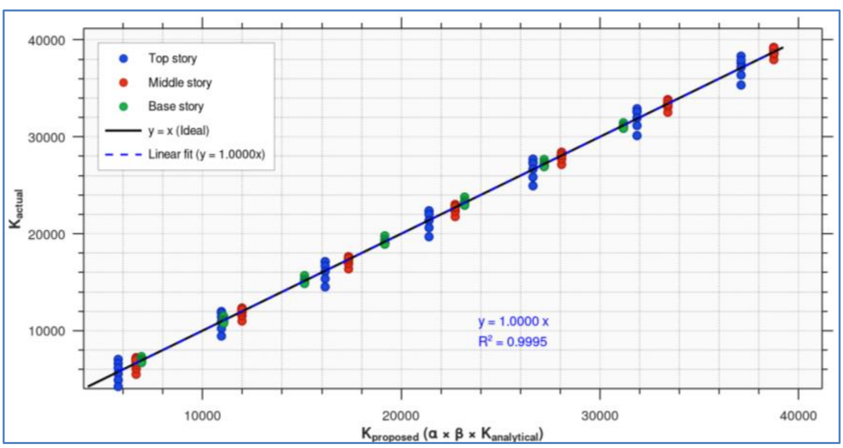

Fig. 5. K_{actual} vs $\alpha . \beta . K_{\text{analytical}}$

3.6. Engineering Interpretation and Implications

The results indicate that stiffness overestimation in classical analytical formulations arises primarily from neglecting global frame configuration effects rather than local member behaviour alone [3], [4], [5]. By incorporating a span-dependent correction through β and allowing story-dependent calibration, the proposed analytical model achieves improved accuracy without sacrificing simplicity [2], [10].

From an engineering perspective, the calibrated stiffness formulation provides a practical and computationally efficient tool for estimating lateral story stiffness in RC MRF structures [1]. It is particularly suitable for preliminary design, simplified seismic analysis, and parametric studies where full numerical modelling may be impractical [6], [15].

The proposed calibrated stiffness formulation may serve as a rational correction tool for simplified seismic analysis procedures that still employ classical stiffness assumptions, particularly in preliminary design stages or educational applications before detailed numerical modeling is conducted.

4. Conclusion

This study proposed a calibrated analytical framework for estimating lateral story stiffness in reinforced concrete moment-resisting frames. Classical column-based stiffness formulations were shown to systematically overestimate actual elastic stiffness, particularly for multi-span configurations.

By incorporating a beam–column interaction factor and a span-dependent correction function with story-dependent calibration, the proposed formulation significantly improves agreement with numerical stiffness results across all investigated configurations. The results demonstrate that global frame configuration effects, especially the number of spans, play a critical role in governing lateral story stiffness and should be explicitly considered in simplified analytical models.

The proposed approach provides a practical and computationally efficient tool for preliminary design and simplified seismic analysis of RC MRF structures. It should be noted that the present study is limited to linear elastic analysis; nonlinear structural behaviour may influence effective story stiffness and warrants further investigation.

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