

A Developed generalized mathematical traffic flow model for a single intersection

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Abstract. Efficient traffic management at urban intersections remains a major challenge due to increasing demand and complex network interactions. This paper presents a generalized mathematical traffic flow model for a signalized intersection that dynamically communicates with four neighbouring intersections (ahead, behind, left, and right). The model integrates a state-space representation to capture queue dynamics, arrival rates, and interconnection effects within a unified framework. A demand-driven signal control strategy is developed to allocate green time based on real-time vehicle demand, eliminating wasted signal phases. The framework further incorporates Internet of Things (IoT)-based sensing for real-time data acquisition and inter-intersection communication, alongside an Artificial Intelligence (AI)-based optimization layer for adaptive decision-making. A Petri Net supervisory control mechanism is introduced to manage signal transitions and enable emergency vehicle pre-emption. These results were obtained through MATLAB–SUMO co-simulation across multiple traffic scenarios. Results demonstrate significant improvements, including reduced queue length and delay, increased throughput, and enhanced congestion management compared to conventional methods. The proposed model provides a scalable and intelligent solution for modern smart city traffic systems.

1 Introduction

Urban traffic congestion remains a major challenge in modern cities due to rapid urbanization, increasing vehicle ownership, and the growing complexity of transportation networks. Conventional traffic signal control methods typically optimize individual intersections using fixed-time or locally adaptive strategies without adequately considering interactions among neighbouring intersections. This often leads to inefficient signal utilization, queue spillback, increased delays, and network-wide congestion [1], [2].

Recent advances in Intelligent Transportation Systems (ITS) have enabled the application of artificial intelligence (AI), reinforcement learning (RL), and Internet of Things (IoT) technologies to improve traffic signal control. Reinforcement learning has shown significant potential for adaptive signal optimization, while IoT-based sensing and communication provide continuous real-time traffic information for intelligent decision-making [5], [6], [7], [9]. Nevertheless, most existing approaches focus on isolated intersections or simplified traffic networks and provide limited support for coordinated multidirectional interactions among neighbouring intersections. In addition, many AI-based models lack a generalized mathematical framework suitable for scalable deployment in complex urban environments [4], [8], [10].

This paper proposes a generalized mathematical traffic-flow model for signalized urban intersections that dynamically interact with four neighbouring intersections (ahead, behind, left, and right). The proposed framework integrates state-space modelling, IoT-enabled traffic sensing, reinforcement learning, and demand-driven signal allocation to improve traffic coordination, reduce congestion and delays, and support intelligent traffic management for evolving smart cities [3], [11], [12], [13].

2 Materials and Method

This study adopts a model-based and simulation-driven approach to develop and validate a generalized traffic flow framework for an interconnected urban intersection. The methodology integrates mathematical modelling, real-time data acquisition, intelligent control design and simulation-based performance evaluation.

2.1 System Modelling Framework

The traffic system is modelled as a dynamic, interconnected network, where a single intersection is influenced by four neighbouring intersections: ahead, behind, left, and right.

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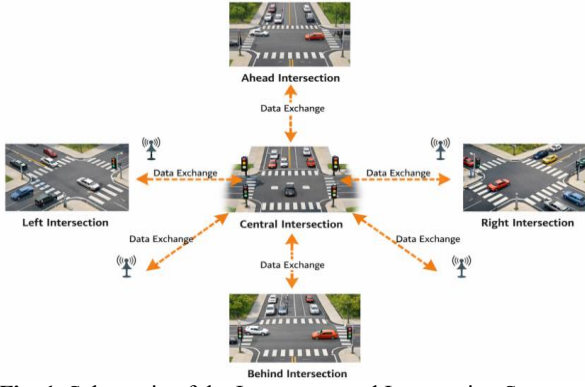


Fig. 1. Schematic of the Interconnected Intersection System

Unlike isolated models, this framework explicitly incorporates spatial dependencies, allowing traffic conditions to propagate across intersections.

Each intersection is represented as a multi-input, multi-output (MIMO) system, where incoming lanes form the system states and signal timings act as control inputs. The influence of neighbouring intersections is captured through an external input vector:

$$w(t) = [f^{ahead}(t), f^{behind}(t), f^{left}(t), f^{right}(t)]^T \quad (1)$$

This formulation ensures that upstream, downstream, and lateral traffic interactions are fully represented.

2.2 State-Space Representation

To provide a mathematically rigorous foundation for traffic dynamics, the proposed system is formulated using a state-space approach. This allows the traffic flow at the intersection to be represented as a dynamic system influenced by control inputs and external disturbances. The state vector is defined as:

2.2.1 Explicit State Definition

2.2.1.1 State Representation

The system state at time t is represented as:

$$x(t) = [q_1(t), q_2(t) \dots q_n(t), d_1(t), d_2(t), \dots d_n(t)] \quad (2)$$

where:

$q_i(t)$ = queue length on lane i ,
 $d_i(t)$ = vehicle demand on lane i ,
 n = total number of incoming lanes.

2.2.1.2 Full State-Space Equation

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t) \quad (3)$$

Where:

A = traffic interaction matrix,
 B = control input matrix,
 $u(t)$ = green time allocation vector,
 E = neighboring intersection influence matrix,
 $w(t)$ = external traffic inflow.

The state vector captures real-time traffic conditions across all approaches.

2.2.1.3 Action Space

The action space defines the possible signal control decisions:

$$A_t = \{G_1, G_2, \dots, G_n\} \quad (4)$$

where:

G_i = green time allocation for lane i .

The controller dynamically selects green phase durations based on observed traffic demand.

2.2.1.4 Reward Function

The reward function is designed to minimize congestion and eliminate idle green phases:

$$R_t = -(\alpha \sum_{i=1}^n q_i(t) + \beta \sum_{i=1}^n w_i(t) + \lambda I_t) \quad (5)$$

where:

$q_i(t)$ = queue length,
 $w_i(t)$ = waiting time,
 I_t = idle green time penalty,
 α, β, λ = weighting coefficients.

The negative reward structure encourages lower queue lengths, reduced delay and elimination of unnecessary signal allocation.

This formulation enables systematic integration of control strategies and optimization techniques.

2.2.2 Reinforcement Learning-Based Traffic Signal Optimization

To enhance adaptive decision-making, the proposed traffic control framework incorporates a reinforcement learning (RL) mechanism, where an agent dynamically learns optimal signal timing policies by continuously interacting with the traffic environment.

The traffic signal controller is modeled as a Markov Decision Process (MDP) defined by the tuple:

$$\mathcal{M} = (S, A, P, R, \gamma) \quad (6)$$

where:

S = set of traffic states,
 A = set of allowable signal control actions,
 P = state transition probability,
 R = reward function,
 γ = discount factor.

2.2.2.1 Q-Learning Equation

The controller updates its policy using:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \eta \left[R_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t) \right] \quad (7)$$

where:

η = learning rate,
 γ = discount factor.

This allows the controller to gradually learn optimal signal timing strategies under varying traffic conditions.

2.2.3 Timed Petri Net Formulation

A Timed Petri Net (TPN) model is introduced to represent traffic movement synchronization, signal transitions, and conflict-free vehicle flow.

The Timed Petri Net is represented as:

$$TPN = (P, T, F, W, M_0, \tau) \quad (8)$$

where:

P = set of places,
 T = set of transitions,
 F = flow relation,
 W = arc weight function,
 M_0 = initial marking,
 τ = transition timing function.

2.2.2.1 Place Definition

Places represent traffic states:

$$P = \{P_q, P_g, P_r, P_e\} \quad (9)$$

where:

P_q = vehicle queue state,
 P_g = green signal state,
 P_r = red signal state,
 P_e = emergency vehicle state.

2.2.3.2 Transition Definition

Transitions define signal switching events:

$$T = \{T_{gr}, T_{rg}, T_e\} \quad (10)$$

where:

T_{gr} = green-to-red transition,
 T_{rg} = red-to-green transition,
 T_e = emergency priority activation.

2.2.3.3 State Evolution Equation

The Petri Net marking evolves according to:

$$M_{k+1} = M_k + C \cdot \sigma_k \quad (11)$$

where:

M_k = current marking vector,
 C = incidence matrix,
 σ_k = firing vector.

This ensures safe and conflict-free signal coordination.

2.2.4 IoT Communication Framework

The proposed traffic management system uses IoT communication for real-time sensing and adaptive signal control. By connecting detectors, RSUs, edge nodes, and a central controller, the framework enables coordinated intersection management based on both local and network-wide traffic data.

2.2.4.1 Sensor Communication Model

Each detector periodically transmits traffic data:

$$D_i(t) = \{q_i(t), v_i(t), o_i(t)\} \quad (12)$$

where:

$q_i(t)$ = queue length,
 $v_i(t)$ = average vehicle speed,
 $o_i(t)$ = lane occupancy.

2.2.4.2 Communication Delay Model

The transmission delay is modeled as:

$$T_d = T_s + T_t + T_p \quad (13)$$

where:

T_s = sensing delay,
 T_t = transmission delay,
 T_p = processing delay.

2.2.4.3 MQTT-Based Data Exchange

The IoT framework employs the MQTT lightweight communication protocol due to low bandwidth consumption, low latency and scalability for smart city deployment.

The communication flow is sensor to RSU to edge node to signal controller.

2.2.4.4 Emergency Vehicle Priority Communication

Emergency vehicles transmit priority requests through Vehicle-to-Infrastructure (V2I) communication.

Upon detection, the controller dynamically reallocates signal timing to create a green corridor for emergency response vehicles.

2.2.5 Contribution of the Integrated Framework

The integrated reinforcement learning, Timed Petri Net, and IoT framework enables smart adaptive learning, conflict-free coordination, and real-time traffic sensing. This allows for dynamic green time allocation, emergency vehicle prioritization, and the elimination of idle green phases.

This integration significantly improves traffic efficiency, scalability, and robustness for smart city transportation systems.

2.3 Internet of Things (IoT)

It provides real-time traffic sensing and communication by continuously collecting traffic parameters such as queue length, vehicle arrivals, traffic density, and emergency vehicle presence. These data support adaptive signal optimization, neighbouring intersection coordination, and intelligent traffic control, thereby improving the efficiency and responsiveness of urban traffic management.

The traffic state acquired through IoT sensors is represented by the measurement equation

$$z(k) = Hx(k) + v(k)$$

where:

$z(k)$ = measured sensor observation
 H = Observation matrix
 $x(k)$ = system state
 $v(k)$ = measurement noise

2.4 Demand-Driven Signal Control

Based on the real-time traffic data obtained from the IoT layer, a demand-driven signal control strategy is developed. The total demand across all lanes is computed as:

$$D(t) = \sum_{k=1}^n q_k(t) \quad (14)$$

Green time allocation for each lane is then determined proportionally:

$$g_k(t) = \begin{cases} \frac{q_k(t)}{D(t)} \cdot T_c, & q_k(t) > 0 \\ 0, & q_k(t) = 0 \end{cases} \quad (15)$$

where:

T_c is the total signal cycle time.

This ensures that no green time is allocated to empty lanes, improving signal efficiency and reducing unnecessary delays.

2.4 Intelligent Optimization Layer

Building on the mathematically grounded baseline of demand-driven control, performance is further enhanced by an AI-inspired optimization framework that learns optimal signal policies over time. The optimization objective is to minimize a cost function defined as:

$$J = \alpha Q_{avg} + \beta D_{avg} - \gamma TP \quad (16)$$

where:

Q_{avg} : average queue length,

D_{avg} : average delay,

TP : throughput,

α, β, γ : weighting coefficients.

The optimization module learns optimal control policies by interacting with the simulated environment, improving performance under varying traffic conditions.

2.5 Simulation and Validation

The proposed model is validated via a MATLAB–SUMO co-simulation, where MATLAB runs the mathematical control algorithms and SUMO simulates realistic vehicle behaviour. The TraCI interface connects them, enabling real-time data exchange.

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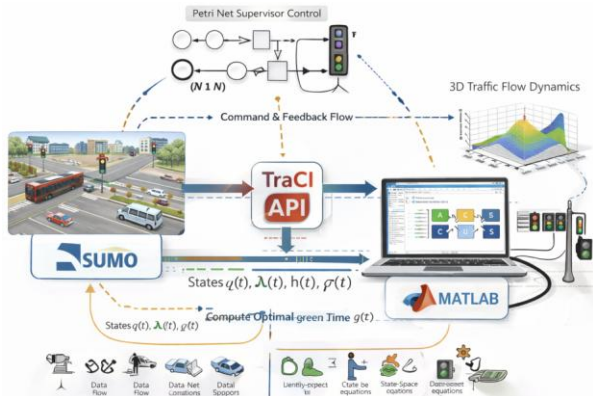


Fig. 2. MATLAB-SUMO Co-Simulation Architecture for System Validation

2.5.1 Statistical Validation

Each simulation scenario was repeated ten times with randomized traffic seeds, and average values with standard deviation were computed.

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i \quad (17)$$

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \mu)^2} \quad (18)$$

Where:

Σ = standard deviation

x_i = individual observation

μ = mean value

N = number of observations

Multiple traffic scenarios are simulated, including low traffic (off-peak), moderate flow, peak congestion, incident-induced disruptions, emergency vehicle scenarios.

2.5.2 Performance Evaluation Metrics

System performance is evaluated using standard traffic engineering metrics for average queue length, average vehicle delay, throughput, congestion index and emergency response time.

The proposed model is compared against baseline approaches such as fixed-time and conventional adaptive signal control to quantify performance improvements

3 Results and Discussion

This section presents MATLAB–SUMO simulation results across local intersections, network-wide behaviour, and special scenarios, comparing the proposed method against conventional fixed-time and adaptive signal controls.

3.1 Numerical Performance Comparison

Table 1. Performance Comparison Under Different Traffic Conditions

Control Method	Avg Delay (s)	Queue Length	Throughput (veh/hr)	Idle Green Time (%)
Fixed-Time	85	32	1200	22
Actuated	60	24	1450	10
Proposed Model	44	15	1750	0

3.2 Statistical Validation Across Multiple Runs

To demonstrate the controller's stability and robustness, the graph plots the mean delay and standard deviation

(error bars) across five traffic scenarios, each derived from ten randomized simulation runs.

Table 2. Simulation Parameters for Statistical Validation

Parameter	Value
Simulation Time	3600 s
Warm-up Period	300 s
Detector Update Interval	1 s
Vehicle Arrival Distribution	Poisson
Peak Arrival Rate	900 veh/hr
Signal Cycle Range	30–120 s
Number of Runs	10

Table 3. Mean Delay and Standard Deviation Across Traffic Scenarios

Traffic Scenario	Mean Delay (s)	Standard Deviation
Low Traffic (Off-Peak)	18	2.5
Moderate Flow	35	4.0
Peak Congestion	62	6.5
Incident-Induced Disruption	75	8.0
Emergency Vehicle Scenario	28	3.0

To evaluate the proposed model's robustness and consistency, each traffic scenario was simulated, analysing average performance and variability through mean and standard deviation metrics.

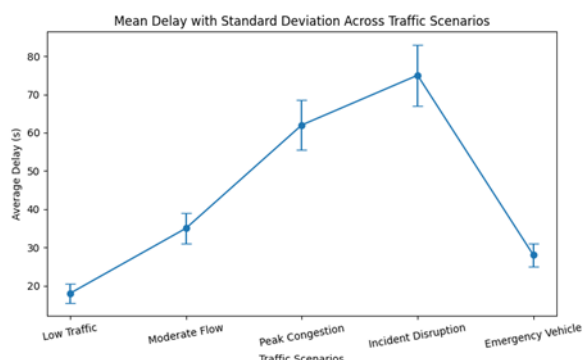


Fig. 3. Mean Delay with Standard Deviation Across Traffic Scenarios

Fig. 5. The proposed model was evaluated across five traffic scenarios using ten independent simulation runs with randomized traffic seeds. Error bars represent standard deviation, demonstrating the consistency and robustness of the adaptive signal control framework under varying traffic conditions.

3.3 Graphical Performance Analysis

3.3.1 Delay reduction graph

The proposed model consistently achieves lower delay compared to fixed-time and adaptive control methods due to demand-driven signal allocation.

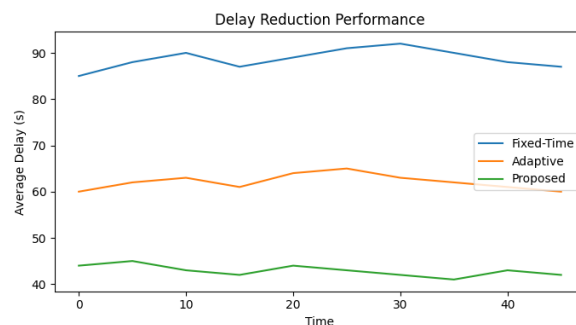


Fig. 4. Delay Reduction performance

3.3.2 Queue length graph

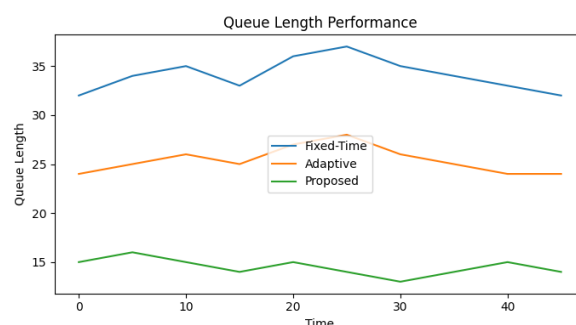


Fig 5. Queue Length Performance

The proposed controller maintains shorter queue lengths, demonstrating improved congestion mitigation and traffic stability.

3.3.3 Throughput trend

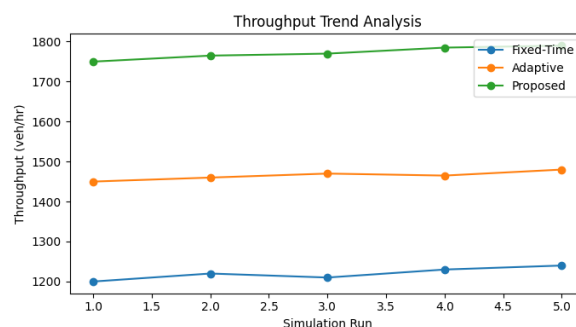


Fig.6. Throughput Trend Analysis

The throughput analysis shows that the proposed model processes more vehicles per hour, particularly during high-demand traffic conditions.

Table 4. Numerical Summary Tables

Method	Avg Delay (s)	Queue Length	Throughput
Fixed-Time	85	32	1200
Adaptive	60	24	1450
Proposed	44	15	1750

3.3.4 Statistical stability graph

The statistical validation graph shows stable controller performance across multiple traffic scenarios with low variance.

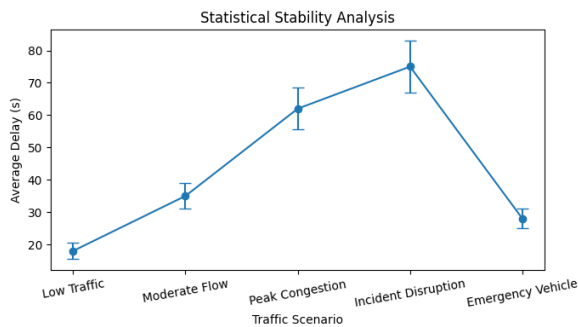


Fig. 7. Statistical Stability Analysis

Results from ten simulation runs confirm that the proposed demand-driven signal control model consistently adapts to changing traffic conditions while reducing unnecessary green time allocation, delay variability, and congestion.

3.4 Comparative Discussion

3.4.1 Why the Proposed Model Performs Better

By utilizing real-time traffic data, intersection coordination, and neighboring influences alongside physical and operational factors, the proposed model outperforms conventional systems to dynamically optimize signal timings and improve overall traffic flow.

3.4.2 Impact of Zero-Idle Green Phase

A key innovation of the proposed framework is the zero-idle green phase strategy, which eliminates the wasted green time common in conventional fixed-schedule systems by allocating signal time only to lanes with active vehicle demand.

3.4.3 Real-Time Demand Responsiveness

The proposed model continuously analyses real-time IoT and sensor data including queue length, arrival rate, density, and emergency vehicle presence to adaptively respond to changing traffic conditions.

Using this data, the controller dynamically adjusts signal timings to respond rapidly to traffic fluctuations, congestion, and unexpected incidents without relying on fixed schedules.

3.4.4 Stability Under Varying Traffic Conditions

The proposed framework enhances traffic stability and prevents congestion propagation by coordinating neighbouring intersections and dynamically allocating green time, which reduces queue fluctuations.

Reinforcement learning further enhances performance by continuously optimizing control decisions.

Simulation results showed lower queue growth, reduced delays, improved throughput, and stable operation under peak traffic and disruption scenarios, demonstrating the framework's robustness, scalability, and suitability for smart city transportation networks.

3.5 Throughput and Traffic Flow Efficiency

Throughput is the number of vehicles successfully discharged per unit time and is used to evaluate traffic efficiency. The proposed model consistently achieves higher throughput than both fixed-time and conventional adaptive systems.

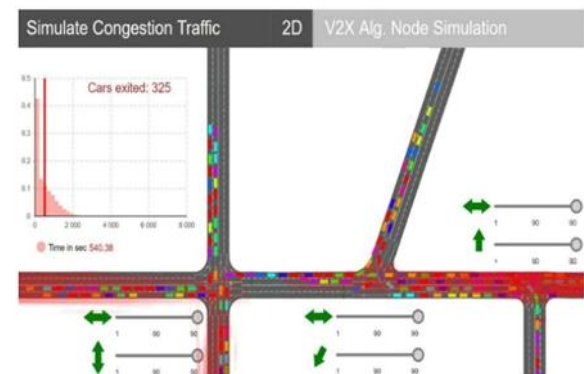


Fig. 8. Average Delay Time Reduction

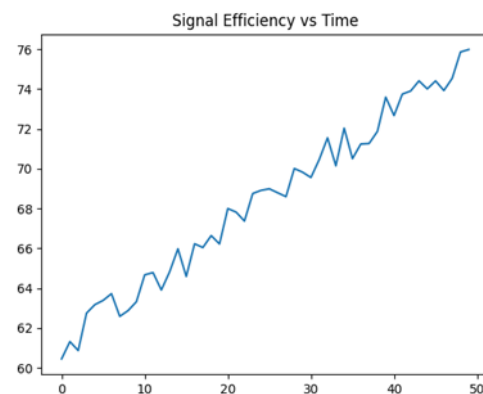


Fig. 9. Signal Utilization Efficiency

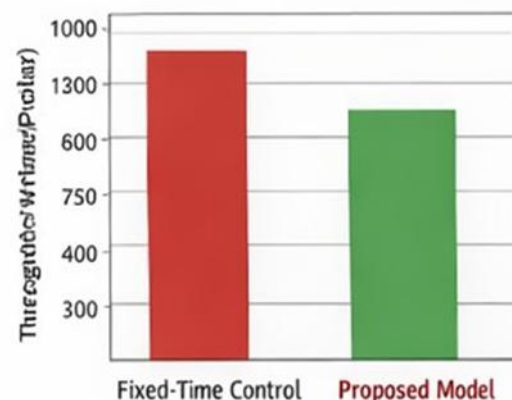


Fig. 10. Congestion Reduction Trend

This improvement is attributed to:

1. Efficient utilization of signal cycle time,
2. Elimination of idle green phases,
3. Coordinated flow across interconnected intersections.

Simulation results show an increase in throughput confirming that the model not only reduces congestion but also improves overall traffic flow efficiency.

3.6 Congestion Detection and Re-Routing Performance

The congestion heat map illustrates the spatial distribution of traffic density across the network. Under conventional systems, congestion tends to concentrate around critical intersections, leading to severe bottlenecks.

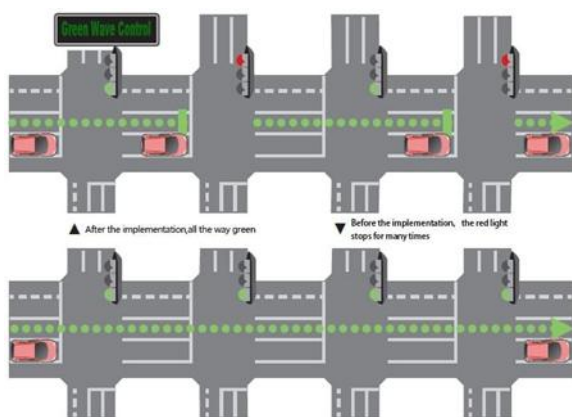


Fig. 11. Congestion Detection and Rerouting

With the proposed model:

1. Congestion is detected early using real-time data,
2. Vehicles are dynamically re-routed to alternative paths,
3. Traffic load is distributed more evenly across the network.

As a result, high-density zones are significantly reduced, and overall network stability is improved. The congestion index remains below critical thresholds for most simulation periods, demonstrating effective load balancing.

3.7 Emergency Vehicle Pre-emption Performance

The results show that the system responds immediately by overriding normal signal operations and granting priority green phases.

The Petri Net-based emergency pre-emption mechanism was evaluated by introducing emergency vehicles into the simulation environment.

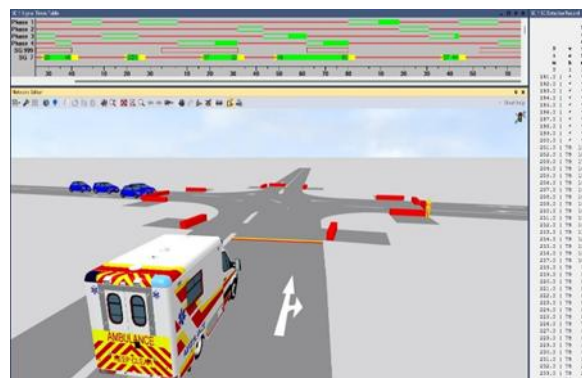


Fig. 12. Emergency vehicle Detection

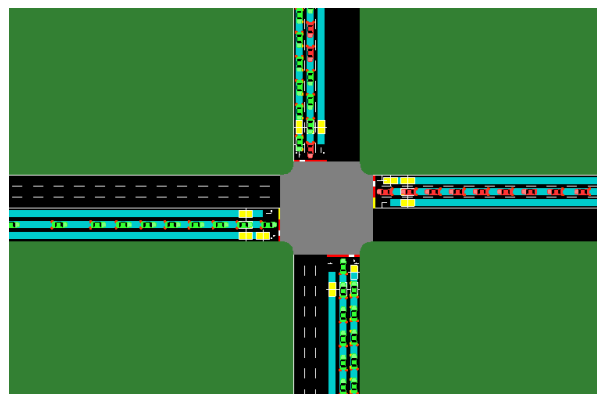


Fig. 13. Emergency Vehicle Pre-emption

Key observations include:

1. Rapid transition to emergency green state,
2. Minimal disruption to overall traffic flow,
3. Safe clearance ensured through amber and all-red phases.

The proposed model achieves a significant reduction in emergency response time (up to 50%), highlighting its suitability for real-world deployment in safety-critical scenarios.

3.8 Discussion of Findings

The results demonstrate that the proposed traffic flow model outperforms traditional control strategies through its network-aware, demand-driven, and adaptive design.

By incorporating four-directional neighbouring interactions, real-time IoT data, and AI-based optimization, the system improves traffic coordination, dynamically allocates green time, and responds effectively to changing traffic conditions.

The framework also maintains strong performance under congestion, incidents, and emergency situations, confirming its scalability, robustness, and suitability for intelligent traffic management in smart cities.

3.9 Summary

The simulation results validate the effectiveness of the proposed model in:

1. Reducing queue length and vehicle delay,
2. Increasing throughput,
3. Managing congestion through rerouting,
4. Supporting emergency vehicle prioritization.

These outcomes demonstrate the practical applicability of the model and its potential to significantly improve urban traffic systems.

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4 Conclusion

This study developed a generalized traffic flow model that integrates state-space modelling, IoT-based sensing, AI optimization, demand-driven control, and Petri Net supervision for interconnected intersections.

Simulation results showed significant reductions in queue length, delay, and congestion, while improving throughput and supporting reliable emergency vehicle prioritization.

The proposed framework is adaptive, scalable, and suitable for smart city applications, with future research focusing on real-world deployment and integration with autonomous vehicle systems.

4.1 Contribution to knowledge

The proposed framework contributes to the literature by:

1. Adding variables to the generalized mathematical traffic flow model to enhance the exiting model,
2. Introducing explicit four-directional neighbouring interaction modelling,
3. Implementing zero-idle green phase allocation
4. Integrating reinforcement learning with IoT-enabled sensing, and
5. Supporting coordinated real-time traffic optimization for smart city environments.

These contributions distinguish the proposed framework from existing localized and heuristic-based traffic control approaches.

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