Quantifying parameter uncertainty in empirical correlations for soft clay characterisation

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Abstract. Preliminary estimation of the undrained shear strength of soft clays commonly relies on empirical or semi-empirical correlations incorporating soil index properties and stress history. In practical design situations where laboratory or field data are limited, such correlations are widely applied, although their reliability depends strongly on parameter uncertainty and inherent data variability. This study applies Global Sensitivity Analysis (GSA) to quantify how uncertainty in key parameters propagates through a semi-empirical undrained strength relation representative of Swedish practice. A combined Sobol variance-based analysis and Monte Carlo Filtering (MCF) approach is used to assess both parameter importance and model performance. Experimental datasets digitised from published Swedish investigations provide data-informed bounds for the input parameters. Results from both GSA methods consistently indicate that soil plasticity, represented by the liquid limit, is the dominant contributor to output variability and predictive performance. In contrast, the overconsolidation ratio and exponent parameter show limited influence. MCF further reveals that behavioural model responses are mainly associated with medium- to high-plasticity clays, corresponding to liquid limits of approximately 45-75%. The proposed framework offers a transparent basis for uncertainty assessment of empirical geotechnical correlations and provides practical insight for engineering design.

1 Introduction

Empirical relations for estimating the undrained shear strength of natural soft clays are deeply embedded in geotechnical practice, particularly in situations where laboratory or field data are limited due to constraints related to equipment availability, project scale, or economic considerations. Such relations commonly estimate undrained shear strength by linking it to index properties and stress history parameters, providing a practical basis for preliminary design and assessment in soft clay deposits.

A wide range of empirical correlations for undrained shear strength has been proposed in the literature, reflecting differences in geological conditions, soil fabric, and depositional history. Several studies have linked undrained shear strength to index properties such as liquid limit, often calibrated for specific regions or soil types, e.g. [1, 2]. In Swedish practice, semi-empirical relations incorporating both plasticity and stress history parameters are commonly used for soft clays and form an important basis for routine design [3]. In parallel, Global Sensitivity Analysis (GSA) techniques have increasingly been applied in geotechnical engineering to investigate parameter influence and uncertainty propagation in numerical and data-driven models, e.g. [4, 5].

Despite the widespread use of empirical undrained shear strength correlations in practice, there is a lack of systematic assessment of how uncertainty in their input parameters influences both model variability and predictive performance. In particular, it remains unclear which parameters primarily govern the response of these relations and under what conditions they provide consistent or inconsistent predictions when applied to natural soft clays.

The objective of this study is to address this gap by applying two complementary GSA methods—Sobol variance-based analysis and Monte Carlo Filtering—to a semi-empirical undrained shear strength relation representative of Swedish practice. Experimental datasets compiled from published investigations of Swedish soft clays are used to derive data-informed parameter bounds and to support the sensitivity analyses. By combining variance-based and behaviour-based perspectives, the study aims to quantify parameter importance, identify dominant controls on model performance, and delineate the conditions under which the semi-empirical relation produces reliable predictions.

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2 Theory

2.1 Semi-empirical relation for undrained shear strength

Preliminary estimation of the undrained shear strength of soft clays often relies on empirical or semi-empirical correlations incorporating soil index parameters and stress history. In Swedish practice [3], the undrained shear strength in direct shear is commonly expressed as:

$$c_u = a\,\sigma_c'\,\mathrm{OCR}^{(b-1)} \tag{1}$$

where σ_c' is the preconsolidation pressure and OCR is the overconsolidation ratio. The parameters a and b are material-dependent coefficients. For clays in direct shear, parameter a is empirically related to the liquid limit w_L as:

$$a = 0.125 + \frac{0.205\,w_L}{1.17} \tag{2}$$

Parameter b is typically assumed to lie between 0.7 and 0.9, with a common representative value of 0.8. EQ. 1 captures the influence of stress history through σ_c' and OCR, while soil plasticity effects are incorporated via parameter a.

2.2 Variance-based Global Sensitivity Analysis (GSA)

To systematically quantify how uncertainty in the model parameters influences the predicted undrained shear strength, a variance-based Global Sensitivity Analysis (GSA) is performed using the Sobol method [6, 7]. Considering a deterministic analytical function f with k number of parameters:

$$Y = \mathrm{f}(X_1, \dots, X_k) \tag{3}$$

Sobol approach decomposes the total variance of the model response into contribution associated with individual parameters and their interactions.

For a model response Y depending on k uncertain parameters, the total variance is expressed as:

$$V(\mathrm{f}) = \sum_{1}^{k} V_i + \sum_{1\le i\le j\le k} V_{ij} + \cdots + V_{1,\dots,k} \tag{4}$$

Where V_i represents the first-order contribution of i^{th} parameter, V_{ij} denotes the interaction effect between parameters i and j, and higher order terms account for interactions among multiple parameters.

The first-order Sobol index is defined as:

$$S_i = \frac{V_i}{V} \tag{5}$$

Which measures the direct contribution of parameter i to the output variance. The total-effect index is given by:

$$S_{Ti} = S_i + S_{ij} + S_{ij\dots} \tag{6}$$

and represents the overall contribution of parameter i, including all interaction effects. The difference between the S_{Ti} and S_i therefore indicated the extent to which parameter interactions influence the model response.

2.3 Monte Carlo Filtering (MCF)

Monte Carlo Filtering (MCF) is a distribution-based GSA technique that identifies which uncertain input parameters most strongly influence whether a model output satisfies a predefined performance criterion [7]. Unlike variance-based methods, MCF focuses on distinguishing between behavioural and non-behavioural model responses.

In the MCF framework, a large number $N$of input parameter sets are sampled from their prescribed probability distributions and propagated through the model to generate corresponding output realisations. The model responses are then classified into two subsets according to a chosen performance threshold: a behavioural set B, containing outputs that satisfy the criterion, and a non-behavioural set $\overline{\mathrm{B}}$, containing those that do not.

For each input parameter X_i, the conditional empirical distributions $\mathrm{f_n}(X_i|\mathrm{B})$ and $\mathrm{f_n}(X_i|\overline{\mathrm{B}})$ are constructed from the two subsets [7]. Sensitivity is evaluated by comparing these conditional distributions. If the two distributions differ significantly, it indicates that the parameter plays an important role in determining whether the model output falls within the behavioural region. Conversely, if the conditional distributions are similar, the parameter has little influence on the model's ability to meet the performance criterion.

The degree of separation between the two distributions is commonly quantified using non-parametric statistical measures such as the Kolmogorov–Smirnov (KS) statistic. A larger KS value implies greater divergence between behavioural and non-behavioural samples, and thus higher parameter importance.

In contrast to variance-based methods such as Sobol analysis, which attribute contributions to output variance, MCF evaluates parameter importance in relation to a specific target behaviour. This makes the method particularly suitable for problems where the objective is to understand which parameters control whether a model prediction satisfies engineering performance requirements.

3 Data basis and uncertainty characterisation

3.1 Experimental data

The experimental data used in this study were digitised from published investigations of Swedish soft clay deposits. Part of the dataset was obtained from [8] and relates to the West Link railway project in Gothenburg, specifically the E02 Central Station area. The compiled variables include natural unit weight (γ_n), liquid limit (w_L), overconsolidation ratio (OCR), preconsolidation pressure (σ'_c), and undrained shear strength (c_u) obtained from laboratory testing.

The selected measurements, summarised in Table 1, cover depths between 8 m and 32 m. Within this interval, the liquid limit varies approximately between 58% and 78%, while OCR values are generally close to unity to slightly above 1.3. The measured undrained shear strength ranges from approximately 22 kPa to 61 kPa.

Table 1. Experimental data compiled from [8].

Depth (m)	γ_n (kN/m³)	w_L (%)	OCR	C_u (kPa)	σ_c' (kPa)
8	16	65	1.45	21.90	69.5
12	15.6	73	1.35	29.00	90.4
14	16.5	58	1.23	25.91	112.2
15	16	59	1.15	29.93	103.7
17	15.9	73	1.26	29.01	126.2
20	16.2	68	1.08	40.15	134.5
22	16.3	66	1.14	45.26	157.7
25	16.2	71	1.05	40.15	163.5
30	15.7	69.2	1.06	55.29	181.8
32	16.2	78.13	1.04	61.30	206.6

Additional experimental data were digitised from boreholes 14T21 and 14T27 reported by [9] located approximately 3 km north of central Uppsala, Sweden. The extracted datasets for boreholes 14T21 and 14T27 are summarised in Table 2 and Table 3, respectively. The same set of geotechnical variables was extracted for these boreholes, comprising index properties, stress history parameters, and undrained shear strength determined from Direct Simple Shear (DSS) tests. The measurements cover elevations between 6 m and 16 m in the Swedish national reference system RH2000. Within this range, the liquid limit varies approximately between 37% and 77%, while OCR ranges from about 1.1 to slightly above 2.0. The measured undrained shear strength values range between approximately 12 kPa and 35 kPa.

These datasets provide a representative experimental basis for evaluating the semi-empirical relation under Swedish geological conditions and for deriving data-informed parameter ranges employed in the subsequent sensitivity analyses.

Table 2. Experimental data (Borehole 14T21) compiled from [9].

Elevation	γ_n (kN/m³)	w_L (%)	OCR	C_u (kPa)	σ_c' (kPa)
16	15.8	77.4	2.05	17.66	79.23
14	17.5	58.5	1.49	19.25	86.80
12	17	54.7	1.20	22.07	106.5
10	17.4	49.6	1.11	24.73	132.84
8	17.8	40.8	1.11	31.06	166.85
6	18.5	36.7	1.13	28.03	206.80

Table 3. Experimental data (Borehole14T27) compiled from [9].

Elevation	γ_n (kN/m³)	w_L (%)	OCR	C_u (kPa)	σ_c' (kPa)
16	15.9	73.7	2.17	22.23	80.67
14	17.2	49.7	1.52	12.29	85.02
10	17.4	43.9	1.12	15.74	129.57
6	17.8	42.8	1.12	34.73	202.76

3.2 Experimental prediction error

The measured undrained shear strength values digitised from literature are compared with the predications of the semi-empirical relation in Fig. 1. Both measured and predicted values are normalised by the corresponding preconsolidation pressure σ_c', allowing comparison across different depths and stress levels. Although the overall trend follows the 1:1 reference line, a noticeable spread is observed. This dispersion reflects the combined influence of parameter variability, measurement uncertainty, and potential sample disturbance on the predictive performance of the semi-empirical model.

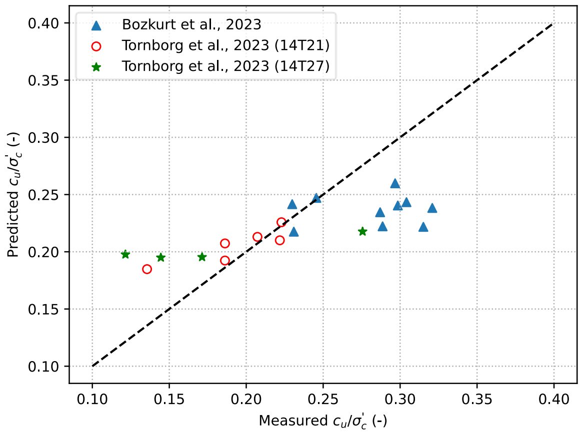

Fig. 1. Measured versus predicted normalised undrained shear strength c_u/σ_c'. The dashed line indicates the 1:1 reference line.

3.3 Definition of parameter uncertainty bounds

Parameter uncertainty bounds were derived from the pooled experimental database described in Section 3.1. The datasets were combined to obtain representative statistics for Swedish soft clays, from which the mean (μ) and standard deviation (σ) were calculated for the liquid limit (w_L) and overconsolidation ratio (OCR). The resulting statistics were μ = 60.4% and σ = 13.0% for w_L, and μ = 1.29 with σ = 0.32 for OCR.

To capture a realistic range of parameter variability while avoiding excessive extrapolation beyond the observed data, the uncertainty bounds were defined using the interval μ ± 2σ. This range approximately encompasses the majority of expected field conditions and is therefore considered representative for sensitivity analysis. The resulting bounds are $w_L \in [34.4\%, 86.4\%]$ and OCR ∈ [0.66, 1.92]. The lower statistical bound for OCR resulted in values below unity, which are not physically consistent with the normally to slightly overconsolidated clay conditions represented in the database. Therefore, the OCR range was adjusted to [1.0, 2.0] to ensure geotechnical realism while remaining representative of Swedish soft clay deposits.

The exponent parameter b was assigned a uniform range between 0.7 and 0.9 based on commonly reported values in Swedish practice [3].

These data-informed bounds were adopted for the Sobol analysis and subsequently used to generate Monte Carlo samples for the behaviour-based sensitivity assessment.

3.4 Sensitivity analysis implementation

Global sensitivity analyses were conducted using both variance-based (Sobol) and behaviour-based (Monte Carlo Filtering) approaches. The implementation aimed to ensure consistent parameter treatment across both methods while maintaining physically realistic sampling domains.

Model inputs were sampled using quasi-random Sobol sequences to achieve efficient and uniform coverage of the parameter space. The liquid limit and overconsolidation ratio were bounded using data-informed ranges derived from the pooled experimental database (Section 3.3), while the exponent parameter *b* was assigned a uniform distribution between 0.7 and 0.9 based on commonly reported values in Swedish practice.

A base sample size of N = 4,096 was adopted, resulting in 32,768 model evaluations after applying the Sobol sampling scheme. Second-order effects were included to allow assessment of potential parameter interactions. The Sobol analysis was performed on the normalized undrained shear strength to quantify parameter influence on the intrinsic variability of the semi-empirical model.

For Monte Carlo Filtering, simulations were classified according to the distribution of relative prediction errors. Rather than imposing an arbitrary tolerance threshold, a quantile-based approach was adopted to reflect the observed variability of semi-empirical strength correlations. Simulations falling within the 20th–80th percentile range were defined as behavioural, representing typical model performance, while simulations in the outer tails were categorized as non-behavioural. An ensemble of 5,000 parameter realisations was generated within the defined uncertainty bounds to ensure stable estimation of behavioural and non-behavioural distributions.

Model performance was evaluated by comparing predicted and measured normalized undrained shear strength. The reference value was defined as the median of the pooled measured c_u/σ_c' dataset, providing a robust representation of typical experimental behaviour. The relative error function was defined as:

$$\varepsilon = \left|\frac{(c_u/\sigma_c')_{pred} - (c_u/\sigma_c')_{meas}}{(c_u/\sigma_c')_{meas}}\right| \qquad (7)$$

4 Results and discussion

4.1 Sobol analysis

Results from variance-based method of Sobol are summarised in Table 4. The results show that the liquid limit w_L dominates the variance of the predicted undrained shear strength, contributing approximately 96% of the total variance, indicating a predominantly direct influence on the model response. The overconsolidation ratio (OCR) exhibits a secondary but clearly smaller contribution, while the exponent parameter *b* has negligible effect within the investigated range. The close agreement between first-order index S_1 and total-order index S_T indicates limited interaction effects among the parameters.

Table 4. Sobol sensitivity results.

	S_1	S_T
w_L	0.963	0.966
OCR	0.025	0.028
b	0.008	0.010

4.2 Monte Carlo Filtering

Figs. 2-4 presents the Empirical Cumulative Distribution Functions (ECDF) of the liquid limit w_L, OCR and parameter b obtained from the Monte Carlo Filtering analysis. In these figures, behavioural simulations (Q20-Q80) are indicated by solid lines, while non-behavioural simulations (tail quantiles) are shown by dotted line. The degree of separation between behavioural and non-behavioural distributions is quantified using the Kolmogorov–Smirnov (KS) statistic.

Fig. 2 presents the ECDFs of liquid limit for behavioural and non-behavioural simulations. A clear separation is observed, with behavioural outcomes primarily concentrated within an intermediate liquid limit range of approximately 45–75%, corresponding broadly to medium- to high-plastic clays. In contrast, non-behavioural outcomes occur predominantly at lower liquid limits and are also present at higher values. This strong separation is reflected by a high KS-statistic (KS ≈ 0.43), indicating a substantial difference between the two distributions. This observation confirms that liquid limit exerts a dominant influence on model performance, indicating that the semi-empirical correlation produces the most consistent predictions within an intermediate plasticity range.

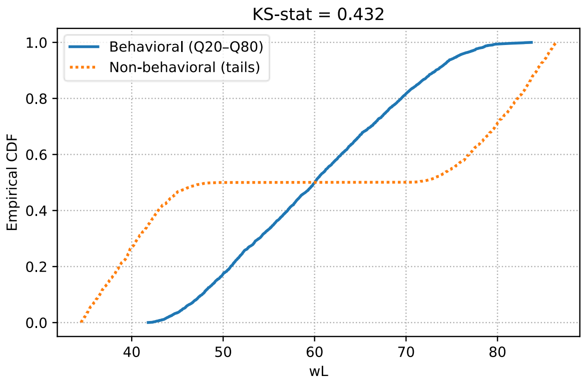

Fig. 2. Empirical cumulative distribution functions (ECDFs) of liquid limit for behavioural and non-behavioural simulations identified using Monte Carlo Filtering.

The ECDFs of OCR in Fig. 3 exhibit only small separation between behavioural and non-behavioural simulations. The corresponding KS statistic value is comparatively low (KS ≈ 0.07), indicating limited discriminatory power of OCR relative to the liquid limit. Behavioral outcomes are more frequently associated with slightly higher OCR values, reflecting a modest distributional shift rather than a clear separation. Overall, within the investigated domain, OCR primarily acts as a secondary scaling parameter rather than a dominant control on model performance.

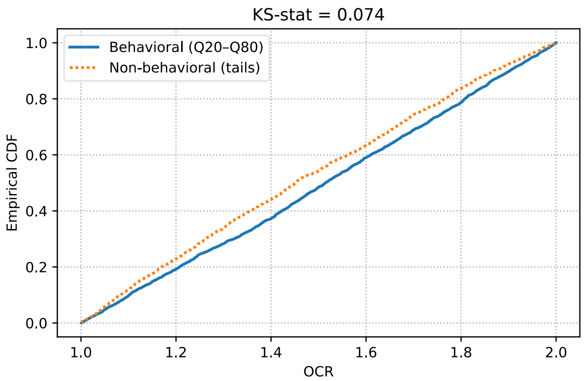

Fig. 3. Empirical cumulative distribution functions (ECDFs) of overconsolidation ratio (OCR) for behavioural and non-behavioural simulations identified using Monte Carlo Filtering.

Fig. 4 presents the ECDFs of the exponent parameter b for behavioural and non-behavioural simulations. The near-complete overlap of the distributions is quantified by a very small KS statistic (KS ≈ 0.03), indicating that variations in b within the investigated range have negligible influence on model performance. This observation is consistent with the Sobol analysis, which identified parameter b as having a minor contribution to the overall variance of the predicted undrained shear strength.

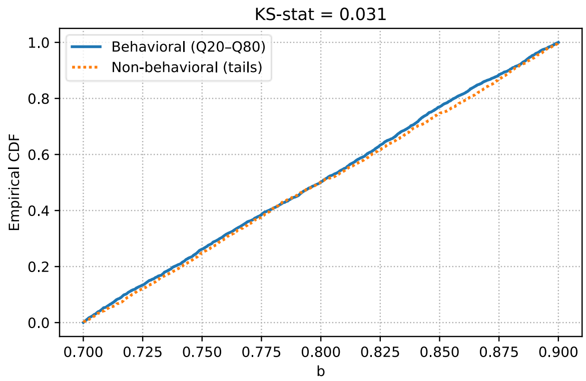

Fig. 4. Empirical cumulative distribution functions (ECDFs) of parameter b for behavioural and non-behavioural simulations identified using Monte Carlo Filtering.

5 Conclusions

The objective of this study was to apply two complementary Global Sensitivity Analysis (GSA) methods—Sobol variance-based analysis and Monte Carlo Filtering (MCF)—to quantify parameter uncertainty in a semi-empirical relation used for estimating the undrained shear strength of Swedish soft natural clays in direct simple shear. To support this analysis, experimental datasets digitised from published Swedish investigations were assembled, providing a representative basis for evaluating the semi-empirical relation under typical geological and stress conditions and for deriving data-informed parameter bounds.

The Sobol analysis demonstrates that the liquid limit w_L overwhelmingly dominates the variance of the predicted undrained shear strength, while the overconsolidation ratio (OCR) and the exponent parameter b contribute only marginally. The close agreement between first-order and total-order indices further indicates that interaction effects among the input parameters are limited.

Monte Carlo Filtering (MCF) reveals a pronounced separation between behavioural and non-behavioural model responses with respect to liquid limit, whereas only weak separation is observed for OCR and virtually none for parameter b. Behavioural simulations were defined as those falling within the central 60% of the prediction-error distribution, while the remaining simulations were classified as non-behavioural.

The convergence of the Sobol and MCF results confirms that soil plasticity, represented by the liquid limit, is the dominant control on both the variability and performance of the semi-empirical relation. In contrast, OCR primarily acts as a secondary scaling parameter, and uncertainty in parameter b has negligible practical influence within the investigated range. Behavioural model responses are mainly observed within a liquid limit range of approximately 45–75%. When interpreted using established Swedish plasticity classifications [10], the behavioural domain corresponds mainly to medium- and high-plasticity clays, while non-behavioural outcomes are more frequently associated with low-plasticity and very high-plasticity conditions. This suggests that the semi-empirical correlation produces the most consistent predictions within a limited plasticity range typical of Swedish soft clays.

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