

Towards physics-consistent machine learning models: a geomechanics-based artificial neural network for high-cyclic soil response

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Abstract. Estimating long-term soil deformation is crucial for the safe design of offshore foundations, railway substructures, pavement layers, and other cyclically loaded geotechnical systems. Traditional constitutive formulations, such as the High-Cycle Accumulation (HCA) model, provide reliable simulations but rely on empirically calibrated parameters that require extensive experimental campaigns for their determination. To overcome this shortcoming, it is proposed to design a Geomechanics-based Artificial Neural Network (GANN) that bypasses the need for calibration parameters and instead uses common soil descriptors. Two steps are required for this purpose: (i) develop a foundational GANN capable of accurately predicting accumulated strain evolution for a given soil; and (ii) generalize this GANN so that it can operate for a wide variety of soils using simple input variables such as the ones related to grain size distribution and state boundary surface. Accordingly, this manuscript aims to address this first step by considering two different soils as examples, namely Karlsruhe fine sand and Australian superfine silica sand. For each soil, a synthetic database is generated from the HCA model to train a GANN architecture, enabling it to learn the fundamental relationships governing strain accumulation. The methodology integrates data-driven learning with a physics-informed loss function, ensuring that the predicted strain evolution remains consistent with soil mechanics principles. This GANN comprises multiple fully connected layers using several activation functions, with outputs structured to capture strain accumulation over a high number of cycles. Validation against synthetic and experimental data confirms excellent performance, supporting this first step toward a generalized GANN framework.

1 Introduction

Soils exposed to high-cycle mechanical loading undergo intricate responses controlled by nonlinear and history-dependent deformation processes [1]. These loading regimes are frequently encountered in different engineering applications, such as offshore foundation systems, railway trackbeds, pavement structures, and facilities subjected to repetitive filling and discharge, including silos and reservoirs [1–3]. In this regard, under high-cyclic loading, soils can exhibit progressive deformation even when no monotonic loading is applied. In undrained conditions, cyclic loading may additionally induce a gradual accumulation of excess pore water pressure [4–6].

Among the modelling strategies proposed to represent the high-cycle behaviour of granular soils, the High-Cycle Accumulation (HCA) formulation introduced by Niemunis et al. [7] is commonly recognized as a robust and reliable approach. Within this framework, accumulated strain is described through empirically derived evolution functions that depend on stress amplitude and cycle count. Rather than resolving incremental plasticity at each cycle, the model condenses the long-term response into a scalar accumulation rule. This feature renders the HCA model

particularly suitable for simulations involving very large numbers of loading cycles. Fig. 1 describes the working principle of this approach.

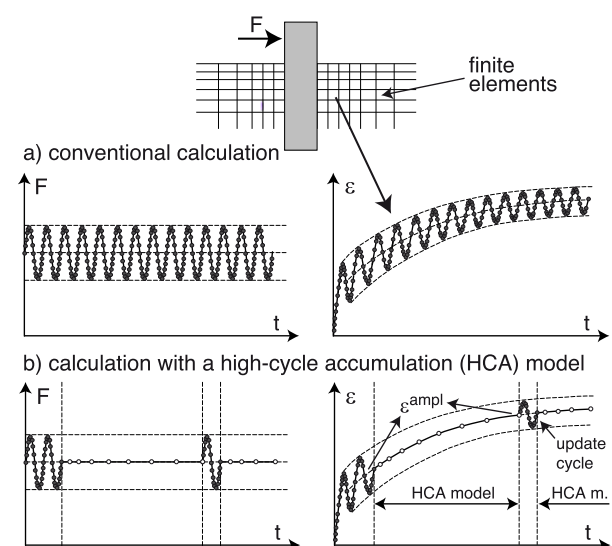


Fig. 1. Working principle of the HCA model.

The HCA model is well-suited for simulating long-term cyclic loading due to its low computational cost, even when millions of cycles are involved.

Nevertheless, a main shortcoming remains. Its formulation is governed by heuristic evolution equations with empirically calibrated parameters. Unfortunately, the required parameter calibration typically relies on extensive laboratory datasets, and extending it to general multiaxial stress states or arbitrary strain paths is challenging. The preceding limitations can be surmounted by means of Machine Learning (ML)-based constitutive frameworks constrained by physical and/or phenomenological principles, which provide a systematic remedy to these issues by reconciling empirical adaptability with theoretical soundness [8].

When Artificial Neural Networks (ANNs) are embedded within a physics-based framework through the introduction of energy potentials, internal variables, or structured dissipation mechanisms, the resulting models exhibit improved interpretability and consistency with classical soil mechanics [9]. Such hybrid strategies enable physics-consistent surrogate models capable of reproducing key features of soils, including path dependence, anisotropy, and cyclic degradation, while preserving numerical efficiency.

In this regard, the present investigation aims to develop a Geomechanics-based Artificial Neural Network (GANN) model to predict the strain accumulation evolution of any given granular soil. For this purpose, two different soils are considered as case studies, namely Karlsruhe Fine Sand (KFS) and Australian Superfine Silica Sand (ASSS). For each soil, a synthetic database is generated from the HCA model and subsequently used to train the designed GANN architecture. Consequently, this framework combines the efficiency and mechanistic insight of the HCA formulation with the representational capacity and extrapolation capability of ML. The primary contribution of the proposed approach is the explicit linkage between data-driven learning techniques, constitutive descriptions tailored for high-cyclic soil behaviour, and physics principles.

2 Background

Below, the HCA model and associated expressions are described. For this purpose, a classic tensorial notation is employed, in which the tensors are denoted with bold letters. For instance, the unit second-order tensor is represented by $\mathbf{1}_{ij} = \mathbf{1}$.

Equation (1) shows the mechanical formulation of the HCA model as proposed by Niemunis et al. [7]. According to this mathematical expression, the HCA model separates the description of accumulation from traditional plasticity frameworks by employing a strain variable that develops progressively, depending on both the applied stress trajectory and the number of loading cycles. In this framework, $\dot{\boldsymbol{\sigma}}$ is the objective stress rate and $\boldsymbol{\sigma}$ is the effective Cauchy stress. As can be seen in Equation (2), the strain is thus divided into an accumulation rate ($\dot{\boldsymbol{\varepsilon}}^{acc}$) and a plastic strain rate ($\dot{\boldsymbol{\varepsilon}}^{pl}$), following a stress-strain relation that obeys the pressure-dependent elastic tangent stiffness tensor ($\mathbf{E}$).

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{acc} - \dot{\boldsymbol{\varepsilon}}^{pl}) \quad (1)$$

$$\dot{\boldsymbol{\varepsilon}}^{acc} = \dot{\boldsymbol{\varepsilon}}^{acc} \mathbf{m} = f_{amp} \dot{f}_N f_e f_p f_Y f_\pi \mathbf{m} \quad (2)$$

As can be noted in Equation (2), the HCA model incorporates several factors that control the intensity at which the strain accumulates. These factors are the strain amplitude function (f_{amp}), the cyclic preloading function ($\dot{f}_N$), the current void ratio function (f_e), the average mean effective stress function (f_p), the normalized average stress ratio function (f_Y), and finally, f_π controls the polarization changes, which is set as equal to 1 for guaranteeing a constant direction of cyclic loading. The preceding functions are described in Equations (3)-(7). It is also important to highlight that the unit tensor $\mathbf{m}$ is employed to describe the direction of strain accumulation given by the ratio between volumetric and deviatoric strain accumulation rates.

$$f_{amp} = \min \left[\left(\frac{\varepsilon^{amp}}{10^{-4}} \right)^{C_{amp}} ; 10^{C_{amp}} \right] \quad (3)$$

$$\dot{f}_N = C_{N1} C_{N2} \exp \left(\frac{-g^A}{C_{N1} f_{amp}} \right) + C_{N1} C_{N3} \quad (4)$$

$$f_e = \frac{(C_e - e)^2}{(1 + e)} \frac{1 + e_{max}}{(C_e - e_{max})^2} \quad (5)$$

$$f_p = \exp \left[-C_p \left(\frac{p^{av}}{100 \text{ kPa}} - 1 \right) \right] \quad (6)$$

$$f_Y = \exp [C_Y \bar{Y}^{av}] \quad (7)$$

According to Equations (3)-(7), seven material constants are incorporated within the HCA model, namely C_{amp} , C_{N1} , C_{N2} , C_{N3} , C_p , C_Y , and C_e . As can be seen in Table 1, these material constants have already been calibrated for KFS and ASSS in previous studies [10,11].

Table 1. Material constants of the HCA model.

Material Constants	Unit	Soil	
		KFS	ASSS
C_{amp}	[-]	1.33	1.93
C_{N1}	[10 ⁻⁴]	2.95	4.85
C_{N2}	[-]	0.41	0.093
C_{N3}	[10 ⁻⁵]	1.9	7.5
C_p	[-]	0.23	0.11
C_Y	[-]	1.68	1.24
C_e	[-]	0.6	0.45

3 Methods

The proposed GANN model aims to capture the constitutive behaviour of soils subjected to high-cycle loading while remaining consistent with physical constraints. The approach establishes a functional relationship between selected mechanical descriptors and either stress increments or the evolution of internal state variables, depending on the adopted formulation. The input is defined by variables that govern cyclic soil

response, including the number of cycles (N), initial void ratio (e_0), mean effective pressure p^{av} , stress ratio (η^{av}), and strain amplitude (ε^{ampl}).

The proposed GANN model is built upon supervised learning on a synthetic database generated from the HCA model. Basically, the calibrated HCA model was implemented to simulate a comprehensive spectrum of cyclic loading scenarios for two soils (i.e., KFS and ASSS as reported in Table 1). For each addressed soil, 100,000 samples were synthetically produced to evaluate the inelastic strain-accumulation behaviour by randomly altering the magnitude of the input variables within the following ranges: $N \in [10^4, 10^6]$, $e_0 \in [e_{min}, e_{max}]$, $p^{av} \in [50, 800]$ kPa, $\eta^{av} \in [0.6, 1.0]$, and $\varepsilon^{ampl} \in [10^{-5}, 7 \cdot 10^{-4}]$.

The selected ranges of the input variables reflect representative operational conditions commonly encountered by geomaterials forming the foundation of civil engineering systems exposed to high-cyclic actions. Accordingly, these intervals capture realistic in-situ loading conditions that characterize the mechanical environment of soils underlying functioning infrastructure.

The supervised learning workflow was carried out separately for each studied soil. To this end, each complete database was randomly partitioned into three mutually exclusive subsets: 80% for model training, 10% for testing, and 10% for validation. Model optimization was conducted using mini-batch gradient descent with the Adamax optimizer. In order to mitigate overfitting and improve the model's ability to generalize, the early stopping technique and validation-driven learning rate adjustment were implemented.

It is important to highlight that physics-consistent predictions of the GANN model are enforced through a tailored loss function, see Equations (8)-(11). This loss function integrates a purely data-driven component (i.e., Equation (9)) with a physics-informed component (i.e., Equation (10)).

$$\mathcal{L} = \mathcal{L}_{data} + (\mathcal{L}_{\varepsilon^{acc}} \cdot \lambda_{dyn}) \quad (8)$$

$$\mathcal{L}_{data} = \frac{1}{N} \sum_{i=1}^N (pred \ \varepsilon_i^{acc} - true \ \varepsilon_i^{acc})^2 \quad (9)$$

$$\mathcal{L}_{\varepsilon^{acc}} = \sum_i 2 \cdot \mathbf{1}_{\{pred \ \varepsilon_i^{acc} < 0\}} \quad (10)$$

$$\lambda_{dyn} = 10^{\left(\frac{\ln(\mathcal{L}_{data} + c_{stab})}{\ln(10)} - \frac{\ln(\mathcal{L}_{\varepsilon^{acc}} + c_{stab})}{\ln(10)} \right)} \quad (11)$$

The physics-informed term (Equation (10)) enforces the predicted values to comply with the second law of thermodynamics by penalizing negative accumulated strain-rate predictions. The preceding can be interpreted even more simply in terms of soil behaviour. Phenomenologically, this assumption reflects the irreversible nature of cyclic deformation in geomaterials subjected to repeated loading. Under high-cyclic stress paths, soils progressively rearrange their internal fabric through particle sliding, rotation, and

local contact rearrangement, which leads to a gradual accumulation of permanent deformation. Since these micromechanical processes are dissipative and associated with energy loss, the accumulated strain cannot decrease with the advancing number of cycles. Instead, it evolves incrementally as loading progresses, although the rate of accumulation may gradually diminish as the soil structure approaches a more stable configuration. Thus, enforcing a non-negative strain accumulation rate ensures that the model predictions remain consistent with the irreversible and dissipative behaviour that characterizes high-cyclic soil response.

In simple terms, Equation 10 evaluates whether the predicted increment of accumulated strain becomes negative through the indicator function $\mathbf{1}_{\{pred \ \varepsilon_i^{acc} < 0\}}$, which returns a value of 1 when the predicted strain evolution violates the admissible condition and 0 otherwise. The constant factor equal to 2 represents the magnitude of the penalty assigned to each previously identified violation. Therefore, whenever the model estimates a negative strain accumulation for any observation within the current training mini-batch, a fixed penalty of 2 is added to the physics-informed loss component. This weighting increases the optimization cost associated with predictions that contradict the irreversible nature of high-cyclic soil deformation and encourages the learning algorithm to avoid such responses during parameter updates. The summation over all observations ensures that each physically inconsistent prediction contributes to the overall penalty, thereby systematically steering the model toward physically consistent outputs.

The dynamic scaling factor (λ_{dyn}) is introduced to regulate the relative contribution of the physics-informed penalty with respect to the data-driven term during the optimization process. According to Equation (11), this factor assesses the relative magnitudes of the two loss components by comparing their logarithms. Specifically, the natural logarithms of the stabilized quantities (i.e., $\ln(\mathcal{L}_{data} + c_{stab})$ and $\ln(\mathcal{L}_{\varepsilon^{acc}} + c_{stab})$) are computed and then divided by $\ln(10)$. This normalization converts natural logarithm values to base-10 logarithms, which represent the order of magnitude of each loss component. The difference between these two normalized logarithmic values, therefore, quantifies the order-of-magnitude separation between the data-fitting error and the physics-based penalty. This difference is subsequently used as the exponent in a base-10 expression that defines λ_{dyn} . As a result, the scaling coefficient automatically adjusts the weight applied to the physics-informed term ($\mathcal{L}_{\varepsilon^{acc}}$) in the total loss function. When the data-driven loss ($\mathcal{L}_{data}$) becomes significantly larger than the physics penalty, the scaling factor elevates the effective contribution of the physics-informed term, ensuring that physical consistency remains influential during training. Conversely, if the penalty associated with negative accumulated strain rates grows disproportionately, the scaling factor reduces its relative influence and prevents it from dominating the optimization procedure. In this manner, the formulation maintains a balanced trade-off between accurate data fitting and the physically

admissible condition that accumulated strain must evolve increasingly with the number of loading cycles. Besides, including $c_{stab} = 10^{-8}$ prevents undefined logarithmic operations when any loss component approaches zero, thereby ensuring numerical robustness throughout the training process.

The central component of the developed GANN model is its deep learning architecture, illustrated in Fig. 2, which is specifically structured to achieve an effective balance between representational capacity and computational stability. The proposed structure corresponds to a fully connected deep neural network composed of six hidden layers arranged symmetrically, with neuron counts of b , $2b$, $4b$, $4b$, $2b$, and b , where b was set to 280 based on systematic trial-and-error assessment. The symmetric distribution of neurons ensures a gradual expansion of feature representation toward the central layers, enabling the extraction of high-level abstractions, followed by a controlled contraction that promotes effective dimensionality reduction prior to the output stage. Such a mirrored-layer design also improves training stability by maintaining a balanced information flow across network depth, thereby reducing the risk of vanishing gradient-related problems and enhancing convergence.

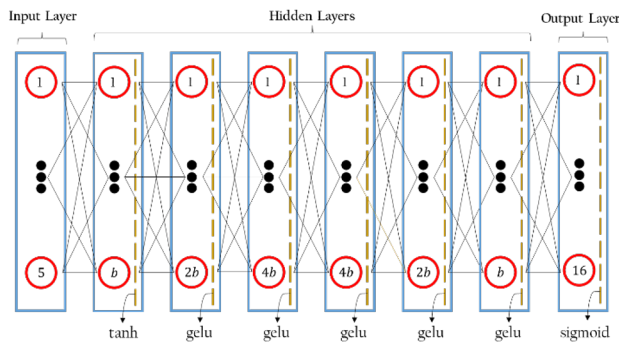


Fig. 2. Architecture of the GANN model.

As illustrated in Fig. 2, the input layer consists of 5 neurons, whereas the output layer comprises 16 neurons. Each neuron in the input layer corresponds to one of the variables defining the loading conditions (i.e., N , e_0 , p^{av} , η^{av} , and ε^{amp}). Meanwhile, the 16 neurons in the output layer represent discrete sampling points describing the evolution of strain accumulation in logarithmic space. Instead of predicting the accumulated strain rate for every loading cycle, the model records ε^{acc} values at selected representative locations along the deformation trajectory, thereby providing a compact yet informative description of ε^{acc} evolution.

It is also important to highlight that the proposed GANN integrates three distinct activation functions, namely the hyperbolic tangent (Tanh), the Gaussian Error Linear Unit (GELU), and the sigmoid function, each serving a specific role in the nonlinear transformation process. The preceding use of multiple activation mechanisms is intended to apply successive nonlinear mappings that progressively reshape the input feature space toward a physically consistent output domain. Initially, Tanh maps inputs to a symmetric range between -1 and 1, and its zero-centered response

supports stable optimization while preserving information from both positive and negative values [12]. Subsequently, GELU modulates each input according to its magnitude, softly suppressing small negative values without truncation and allowing dominant positive responses to propagate, which provides a smooth and context-aware alternative to piecewise linear functions [13]. At the output stage, a sigmoid activation is applied to restrict predictions to the open interval between 0 and 1, a requirement that directly reflects soil mechanics principles by ensuring that the estimated accumulated strain rates remain within feasible physical limits [14]. Through this ordered activation sequence, the network systematically conditions raw inputs by harmonizing scales, filtering salient features through nonlinear responses, and enforcing bounded outputs, thus producing predictions that are both numerically stable and consistent with the governing physical constraints.

4 Results and discussion

The training and testing procedure was carried out using an early stopping strategy, showing that the implementation for the KFS soil converged in approximately 60 epochs, whereas the ASSS soil required close to 90 epochs. The preceding represents a limited number of iterations and therefore indicates that the proposed GANN model exhibits rapid convergence, stable learning behaviour, and an efficient optimization process with no evidence of either overfitting or underfitting. For both soils, the associated loss values at the end of the training-and-testing process are remarkably low, reaching 10^{-8} for KFS and 10^{-4} for ASSS, implying minimal prediction error and confirming the high accuracy of the developed framework. Overall, this level of accuracy was described by coefficients of determination exceeding 0.96 across the training, testing, and validation subsets for both evaluated soils.

Experimental data obtained from laboratory tests on KFS soil reported by *Wichtmann* [1] and on ASSS soil reported by *Canales et al.* [15] are used to assess the predictive capability of the proposed GANN model. Figs. 3-4 present comparisons between model predictions and experimental results for KFS soil under different p^{av} and η^{av} values, respectively. Meanwhile, Figs. 5-6 show the corresponding comparisons for ASSS soil. Across these graphs, p^{av} values range from 50 to 300 kPa and η^{av} values from 0.5 to 1.25, covering a broad spectrum of stress conditions representative of practical cyclic loading scenarios. Overall, both the conventional HCA model and the proposed GANN framework reproduce the experimental trends of accumulated strain with increasing number of cycles, and their predicted curves are generally close to each other, indicating that the underlying HCA framework already provides a solid representation of cyclic deformation. Nevertheless, a more detailed inspection reveals that the GANN model yields a systematically improved agreement with the experimental data, particularly in terms of curve shape and long-term evolution.

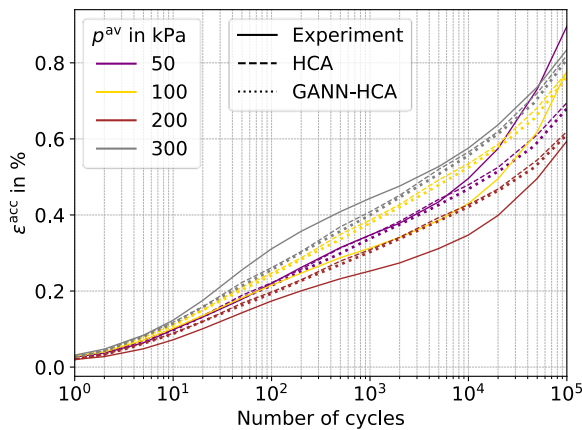


Fig. 3. Comparison between model predictions and experimental data for KFS soil under different p^{av} .

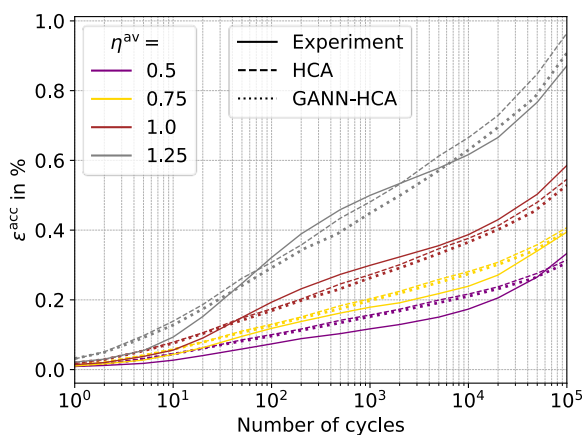


Fig. 4. Comparison between model predictions and experimental data for KFS soil under different η^{av} .

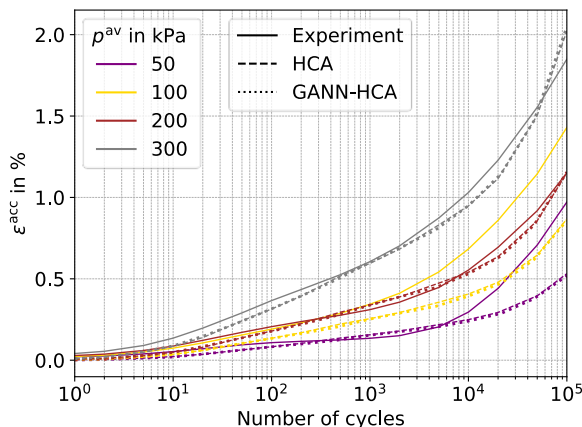


Fig. 5. Comparison between model predictions and experimental data for ASSS soil under different p^{av} .

While the HCA model tends to show small but persistent deviations in slope and curvature as the number of cycles increases, especially at higher p^{av} and η^{av} , the GANN predictions remain slightly closer to the experimental trajectories in the high-cycle regime. These differences, although moderate, are consistent across all figures and indicate a reduced cumulative error over extended loading histories. The preceding implies that the tailored loss function incorporated into the GANN formulation is effectively guiding the learning process toward a more accurate representation

of the long-term cyclic response, allowing the proposed model to better capture the progressive evolution of the experimental accumulated strain under high-cycle loading.

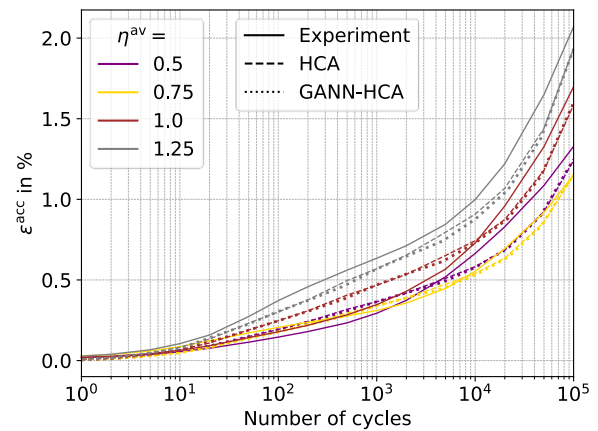


Fig. 6. Comparison between model predictions and experimental data for ASSS soil under different η^{av} .

Overall, the advantage of the GANN approach lies not in a drastic change of predicted behaviour but in a subtle yet systematic enhancement of robustness and consistency across different stress conditions and soils, leading to improved long-term predictive reliability while preserving the physical basis of the original HCA formulation. In this context, the results presented in this study should be interpreted as the first step toward the development of a generalized geomechanics-based ML framework. The present work demonstrates that a dedicated GANN architecture can successfully learn the strain-accumulation patterns for a specific soil. By reproducing the cyclic response of individual soils with high fidelity, the proposed approach establishes the feasibility of embedding soil mechanics principles within a neural network structure while maintaining a clear link to the underlying physical processes governing high-cycle deformation.

Building on this initial stage, the next step in the research is to extend the GANN formulation to operate across a broader spectrum of geomaterials. This will require incorporating soil descriptors that capture intrinsic material characteristics, such as parameters derived from grain-size distributions, state boundary surface concepts, and other fundamental indicators of soil behaviour. Through this extension, the model is expected to evolve from a soil-specific predictor into a generalized framework capable of representing strain accumulation across multiple soils under high-cyclic loading. Such a development would ultimately enable the replacement of empirically calibrated constitutive parameters with readily measurable soil properties, thereby significantly reducing the experimental effort traditionally required for high-cycle soil response simulations.

5 Summary and conclusions

This study developed a physics-consistent ML framework for modelling high-cyclic soil behaviour by

designing a geomechanics-based ANN trained on extensive synthetic datasets for two well-calibrated soils. The methodology relied on supervised learning using mechanically relevant input variables, including N , e_0 , p^{av} , η^{av} , and ε^{ampl} , which were systematically varied to span a wide range of high-cyclic loading conditions. A composite loss function was formulated by combining a data-driven error term with a physics-informed penalty enforcing thermodynamic admissibility, thereby preventing nonphysical strain accumulation rates during training. The proposed deep learning architecture was carefully designed to balance expressive power and numerical robustness through a symmetric deep layout and a structured sequence of activation functions, which ensured smooth nonlinear mappings, bounded outputs, and stable optimization. This integrated strategy enabled the network to efficiently learn the complex, history-dependent relationships governing strain accumulation while maintaining consistency with fundamental soil mechanics principles.

Thus, the main findings of this research demonstrate that the proposed GANN model accurately captures the long-term evolution of accumulated strain across very large numbers of loading cycles, rapidly achieving stable convergence and high coefficients of determination for both studied soils. A final verification against independent experimental data confirmed that the model faithfully reproduces observed trends of strain accumulation across different stress states, while preserving the characteristic response patterns of the original HCA formulation. Compared with the conventional HCA model, the GANN approach provides a slight improvement in the predicted slopes and curvatures. These results indicate that embedding physical constraints within a data-driven framework enhances robustness, generalization, and long-term reliability without sacrificing physical interpretability. Overall, the proposed model represents an efficient and physically grounded surrogate that advances the integration of constitutive geomechanics and ML for practical analysis and design of cyclically loaded geotechnical systems.

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