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Application of Bayesian settlement prediction with creep on a high embankment on stabilised soft soil

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Abstract. The Norwegian Public Roads Administration requires that future settlements of a road be less than 40 cm after its opening. This in turn requires a proper settlement prediction analysis. For road embankments on sensitive clay, material properties may be improved either by lime-cement columns, pre-fabricated vertical drains or both. The soil improvement alters the coefficient of consolidation, and as such, physics-based deterministic modelling of the settlement development is difficult. Continuous assessment of the settlements under an embankment on improved sensitive clays showed that the physics-based models underestimated the final total settlements. In his paper on an observational auto-regressive method, Asaoka included two extensions to the proposed method: a) creep effects and b) a Bayesian approach to the regression, which will output a probabilistic distribution instead of a point estimate. However, he did not include a combination of creep and Bayesian approach. The main benefit of a Bayesian model is that it allows for a probabilistic assessment of whether the final settlements will exceed a given threshold. This work extends the original work by Asaoka by including secondary settlements (creep) to the Bayesian auto-regressive method for improving settlement predictions to be used in practice where soil consolidation properties are altered or otherwise unknown.

1 Introduction

Settlement predictions remain one of the most challenging aspects of geotechnical engineering practice. While there exist several theoretical models for estimating the rate of consolidation and final settlements, the assumptions that form the foundation of such models are often simplified due to the complexity of the settlement process.

Advanced material models in finite element (FE) analysis require fitting several parameters, but amongst others, Ayeldeen et al. [1] found that the FE analysis underestimated the measured settlements. See for instance Filippo et al. [2] for a thorough back-calculation of settlements that require many parameters to be set, estimated or tuned. They conclude that the predictions are noticeably influenced even by small variations in c_{h} and in the soil layering.

The above analyses assumed that the soil parameters are intact, but with ground stabilization methods such as lime-cement columns or pre-fabricated vertical drains (PVD), the stiffness and consolidation parameters are significantly altered. There are models developed for PVD to account for both the combination of radial and vertical drainage and potential installation effects [3]. Those models require soil material properties typically derived from laboratory tests (i.e. oedometer test) such as the compression modulus M and the coefficients of consolidation c_{v} and c_{h}. It adds to the complexity that the soil parameters are expected to change with time and degree of consolidation, which in turn requires additional estimates, like the diameter of the disturbed smear zone around the installed PVD, drainage path and soil layering.

For areas subject to vertical drains, monitoring is recommended [1,2]. When monitoring settlement data is available, statistical regression methods such as the approaches described in Asaoka [4] can be applied to the problem to predict the total settlement at the end of the consolidation period. In addition, a Bayesian approach provides uncertainty quantification. The Asaoka method relies on assumptions that may be too simplistic, and the method is highly susceptible to the choice of time interval Δt selected for the interpolation step and how far in the consolidation process the soil has gotten at the stage of prediction. Nevertheless, the author is of the opinion that despite the shortcomings, the method does hold value in practice and specifically for cases with altered soil parameters.

This paper applies Bayesian linear second-order autoregression to settlement measurement data spanning more than 1.5 years. The source of the data is a road infrastructure project located north of Trondheim, Norway where a 14-meter-high embankment with an additional 2-meter preloading embankment has been established on 40 m thick sensitive clay with PVD. This work does not attempt to solve the difficult task of understanding primary and secondary consolidation but aims to demonstrate that relatively simple regression methods perform reasonably well from a practical standpoint. Furthermore, the purpose of this method is to aid the practitioner in estimating the final settlements,

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by quantifying the uncertainty in such an estimate, to decide whether the future settlements are acceptable according to current guidelines for road design [5].

2 Asaoka autoregressive method

The approach for the Bayesian final settlement estimates, is to draw distributions for all curve parameters, a, b, and in the case of second order regression, c. The draws are made from a normal distribution where the mean is the corresponding point estimates of the ordinary Asaoka method. In this work, creep is seen as a parallel, but independent, process to primary consolidation, in line with the conclusions put forth by Degago [6].

2.1 First order settlements

Asaoka [4] proposed a graphical, statistical approach to settlement predictions based on measurement data on layered soil or soil with PVD where the coefficient of consolidation was not readily obtainable. The graphical method is a first order autoregressive method with 2 steps: (i) Interpolating all data points to a given constant time interval Δt; (ii) plotting the interpolated data points in a S_j by S_{j-1} plot and do linear regression on them. Then, the fitted curve along with the final settlement value is given by

$$S_j = a + bS_{j-1} + \varepsilon \tag{1}$$

where ε is the Bayesian error term having the standard normal distribution. The total settlement is found from the asymptote estimation using the assumption that $S_j = S_{j-1}$ at steady state, which gives

$$S_\infty = \frac{a}{1-b}. \tag{2}$$

where a is the fitted intercept and b is the fitted slope. These parameters can either be point estimates if omitting the error term or posterior distributions if including it and having both be distributions themselves.

2.2 Second order settlements

Asaoka [4] also proposed a solution for second order settlements, i.e. creep. However, the combination of Bayesian modelling and creep was not presented. In the following section we will outline the extension of the Bayesian approach to second-order settlement prediction.

Modelling the second order settlements as a Voigt model as well, we have to solve the following differential equation:

$$S_{II} + c\dot{S}_{II} = P \tag{3}$$

Linearly combining the solution of Equation (1) with the solution of the similar differential equation for the primary settlement, we obtain:

$$\begin{gathered} S = S_I + S_{II} = \\ (S_{I,\infty} + S_{II,\infty}) - (S_{I,\infty} - S_{I,0})\exp\left(-\frac{t}{c_I}\right) \\ -(S_{II,\infty} + S_{II,0})\exp\left(-\frac{t}{c_{II}}\right) \end{gathered} \tag{4}$$

This in turn is rewritten into a second order autoregressive equation:

$$S_j = a + bS_{j-1} + cS_{j-2} \tag{5}$$

The above form is the same form as any Bayesian linear regression formulation, so we convert Equation (3) by adding a Bayesian error term $\varepsilon \sim N(\mu = 0, \sigma)$ as was done for the primary settlement in Equation (1).

Assuming, as for the first order settlements, that

$$S_j = S_{j-1} = S_{j-2} \tag{6}$$

is true at steady state when $t \to \infty$. Similarly, we can then derive the total settlement by combining Equations (5) and (6), effectively finding the asymptote:

$$S_\infty = \frac{a}{1-b-c} \tag{7}$$

2.3 Limitations of the method

The method is derived from the Terzaghi one-dimensional consolidation equation. The underlying assumption is therefore that the coefficients of consolidation are time independent. This is a well-known simplification of the actual physical process involved, as when the in-situ conditions change, so do the consolidation parameters.

Furthermore, the method is highly dependent on choice of the interpolation time interval Δt and which historical measurements are used in the regression analysis. Asaoka recommends that a certain degree of consolidation should already have been taken place at time of analysis, with some authors claiming that a rough consolidation rate of 60% is good [7]. While a sensitivity analysis on the selection of Δt would be of great interest, it is not the scope of this paper. For a minor sensitivity analysis on the effect of historical measurement selections, the reader is referred to [8].

3 Application on a case study

A large road infrastructure project north of Trondheim is constructed on multiple large road embankments on sensitive clay. To improve the strength properties of the sensitive clay, increase the rate of consolidation, and reduce future settlements of the finished road, a combination of preloading and PVD were applied. The estimated time for primary consolidation without any ground improvement, was several decades. An alternative soil improvement technique would have been lime-cement columns, but that is linked with high CO_2 emissions. Consequently, lime-cement columns were limited to strengthening the foot of the embankment. Deformation sensors and piezometers were also

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installed. For a more detailed description of the design and installation process, consult Amdal et al. [9].

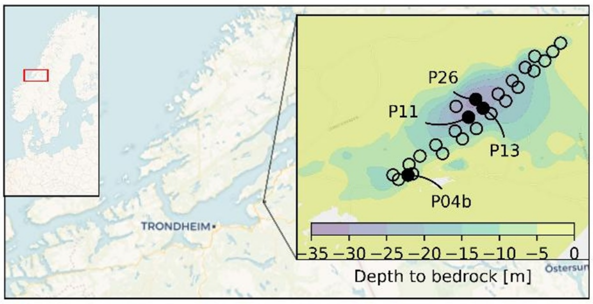

Fig. 1. Map over the location of the case study with selected settlement sensors P04b, P11, P13, and P26, highlighted.

3.1 Dataset

Settlement monitoring plates are installed at several locations along the road infrastructure project. For this analysis, the data from the area Langsteindalen is used, see location in Fig. 1. Here, a 14-meter-high embankment with an additional 2 meters of pre-loading, on top of a roughly 35-meter-thick layer of sensitive clay, is constructed. The data consists of a timeline with the corresponding deformations created the embankment. There are 25 deformation sensors installed in the area. The depth to bedrock vary considerably over the area, from visible bedrock to around 35 m depth. For the present study, 4 sensors are selected for analysis. Sensors P11 and P13 are considered since they show the largest settlement development, being placed in an area with the largest soil thickness, as shown in Fig. 1. Sensor P04b is included as a representative for the sensors with the smallest settlement development, and sensor P26 is included because it is the sensor having the largest depth to bedrock. The depths to bedrock for the relevant sensors are summarised in Table 1.

Table 1 Approximate depths to bedrock for the sensors used analysed in this paper.

Sensor	Depth to bedrock*
P04b	8 m
P11	25 m
P13	28 m
P26	30 m

*Approximate value based on the bedrock ground model.

For a finished road, The Norwegian Public Roads Administration (NPRA) requires that the road does not exceed 40 cm of total settlements after 40 years [5]. The geotechnical design chosen aimed at allowing primary consolidation to be finished during construction period, leaving only secondary consolidation settlements for the operation phase. In the design, the estimate was to allow 6 months to up to a year for the consolidation.

3.2 Data processing

For application of the Asaoka method, each deformation sensor is fitted independently. Then, by means of the fitted curve parameters, a, b, and c, whether they are point predicted or a distribution in the Bayesian case, the total settlement or asymptote can be computed from Equation (7). The priors for curve parameters a, b, and c, are normal distributions with a mean of the corresponding fitted parameters for the non-Bayesian regression. Following recommendations from [4,7], the initial settlement measurements are not included in the regression.

For this analysis, the 30 most recent measurements and a Δt of 15 days for the interpolation step was chosen. In addition to comparing the performances of the first and second order regressions, the differences in the added value of the second order term are demonstrated by comparing measurements from areas with great depth to bedrock with measurements from areas with small depth to bedrock, and sensors directly under the planned road center-line and in the outskirt.

Figs. 2-5 show the historic measurement data of sensors P04b, P11, P13, and P26, respectively, and whether they have been used in the subsequent regression analyses. Sensor P04b shows that steady state is more or less met at the time of the last measurement, at a final settlement of approximately 3.5 cm. As this sensor is on the shallowest end of the road embankment, this is as expected. This is contrasted to the remaining three sensors where the settlement process is very much ungoing at the time of the last recorded measurement.

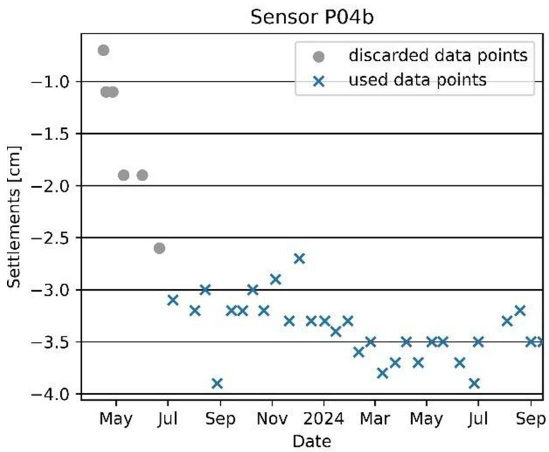

Fig. 2. Settlement measurements for sensor P04b. See location of this sensor in Fig. 1.

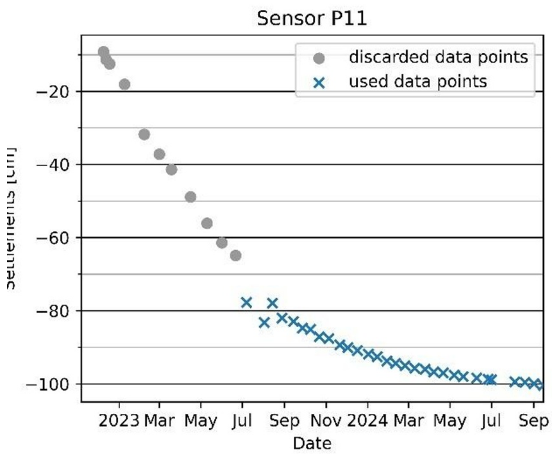

Fig. 3. Settlement measurements for sensor P11. See location of this sensor in Fig. 1.

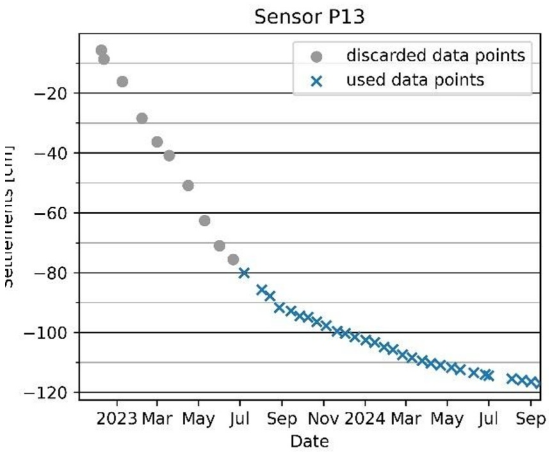

Fig. 4. Settlement measurements for sensor P13. See location of this sensor in Fig. 1.

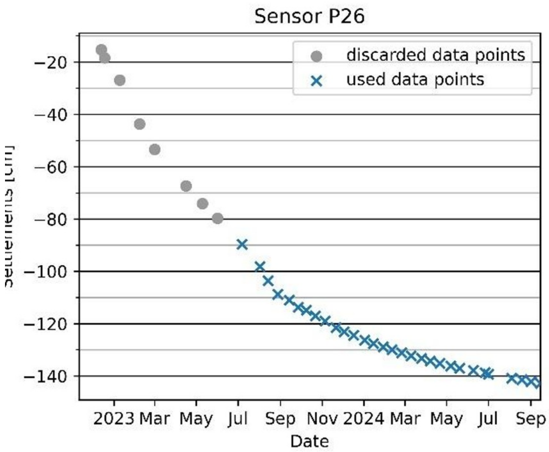

Fig. 5. Settlement measurements for sensor P26. See location of this sensor in Fig. 1.

4 Results and discussion

4.1 Primary versus secondary order Bayesian regression

The means of the posterior distributions of the predicted final settlement follow the same tendency: the resulting mean from the second order regression is greater than or equal to the resulting mean from the first order regression, see Figs. 6-9 for sensors P04b, P11, P13, and P26, respectively. For sensors P04b and P26 the final settlements are roughly the same regardless of regression order. For sensor P04b this is not surprising given the relatively small depth to bedrock where steady state may have been reached by the time of the final settlement measurement, as seen in Fig. 2. Despite having the greatest depth to bedrock, sensor P26 also gives the same mean for both first and second order regressions, despite no visible steady state in the measurements (Fig. 4).

The second order regression gives higher standard deviations. Consequently, the second order model will estimate larger settlements than the first order model. Reporting the likelihood within 2 standard deviations when estimating the final settlements to meet the NPRA requirement, will therefore be a conservative estimate.

The second order posterior distributions do in general display more right-skewness than the first order distributions.

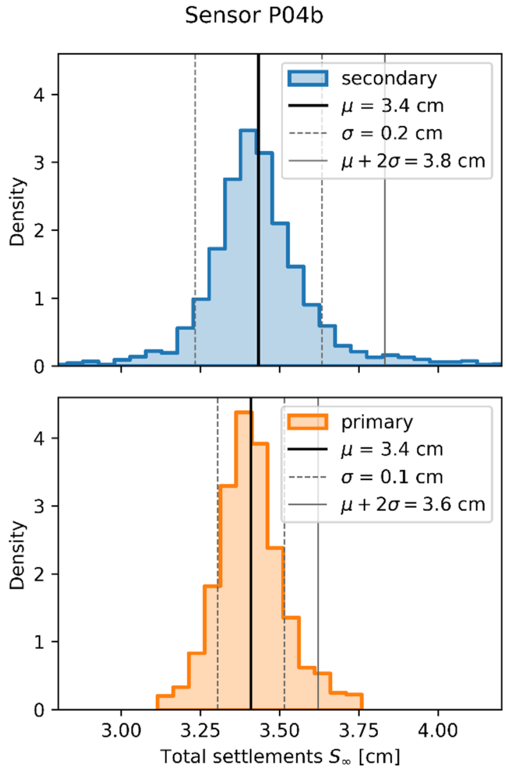

Fig. 6. Histogram of the posterior distribution of the predicted final settlement for sensor P04b for both primary and secondary settlements (first and second order regression, respectively).

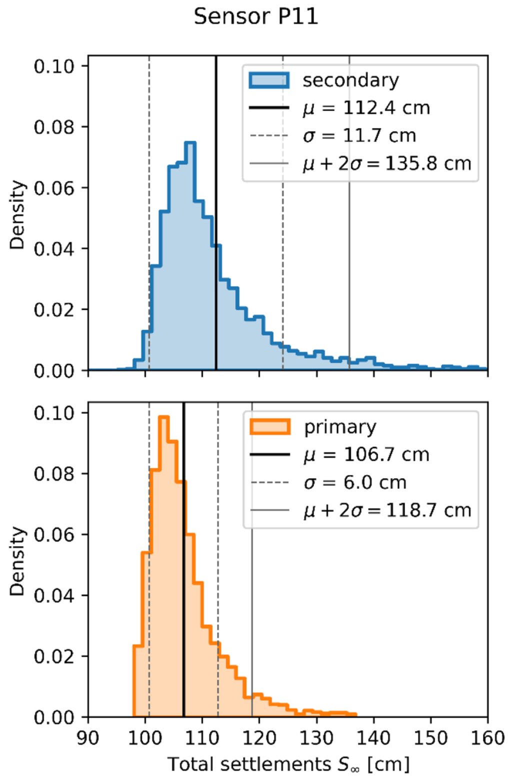

Fig. 7. Histogram of the posterior distribution of the predicted final settlement for sensor P11 for both primary and secondary settlements (first and second order regression, respectively).

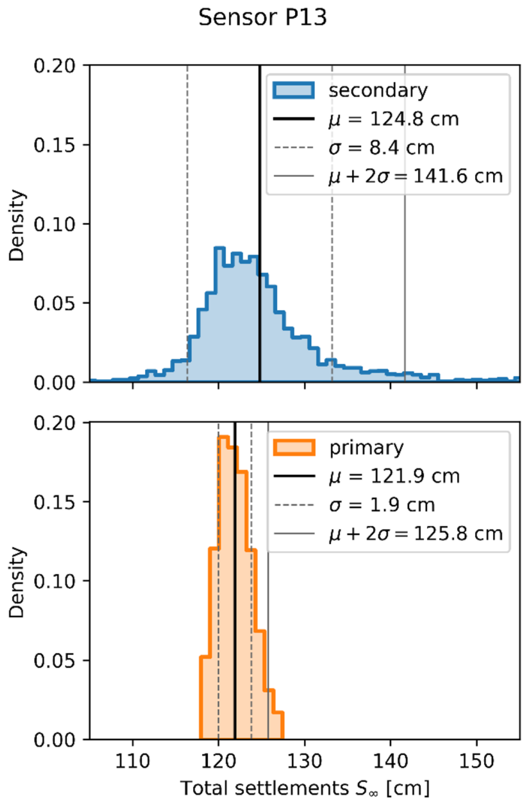

Fig. 8. Histogram of the posterior distribution of the predicted final settlement for sensor P13 for both primary and secondary settlements (first and second order regression, respectively).

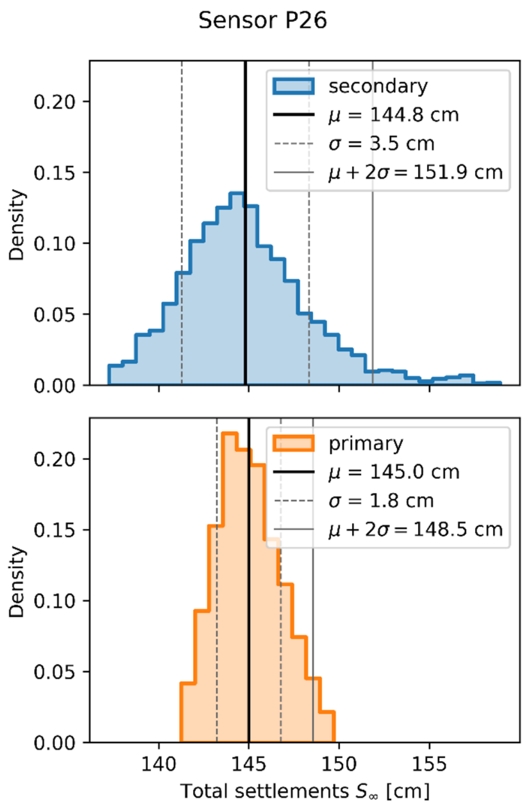

Fig. 9. Histogram of the posterior distribution of the predicted final settlement for sensor P26 for both primary and secondary settlements (first and second order regression, respectively).

4.2 Second order Bayesian vs non-Bayesian regression

What extra information will a Bayesian approach to the second order auto-regression provide compared to the standard approach described in [4]? For the selected sensors, the estimation of the total settlement at infinity was done with both approaches. The point estimate of S_∞ with creep is found from Equation (7). The posterior distribution of the Bayesian second order auto-regression together with its mean is plotted together with the point estimate in Figs. 10-13 for sensors P04b, P11, P13, and P26, respectively.

As with the comparison between first and second order approaches, the second order Bayesian method does overall produce estimates of the final settlement that are greater than or equal to the second order point estimate. In addition, the estimates are more or less the same for the sensors P04b and P26, where the former is likely to have reached steady state at the time of the last measurement (Fig. 2).

Compared to the regular first order auto-regression, the addition of the creep, or second order term, also shifts the results towards larger estimated final settlements.

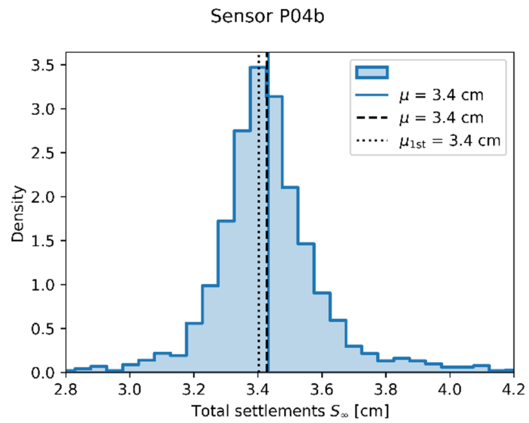

Fig. 10. Comparison of the expected final settlement obtained from either a Bayesian approach with creep (solid line), point estimate with creep (dashed line), or point estimate without creep (dotted line) for sensor P04b.

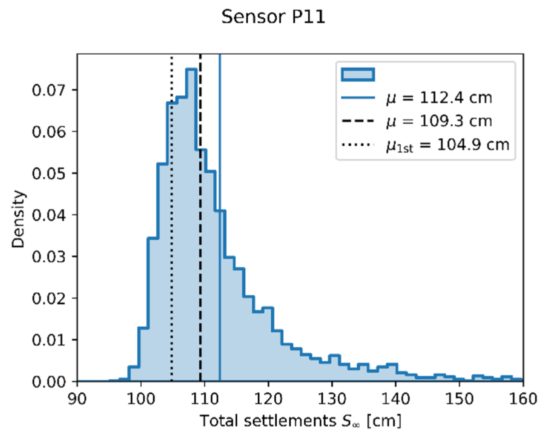

Fig. 11. Comparison of the expected final settlement obtained from either a Bayesian approach with creep (solid line), point estimate with creep (dashed line), or point estimate without creep (dotted line) for sensor P11.

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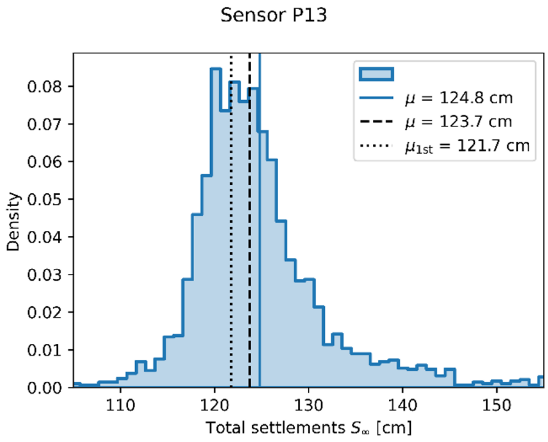

Fig. 12. Comparison of the expected final settlement obtained from either a Bayesian approach with creep (solid line), point estimate with creep (dashed line), or point estimate without creep (dotted line) for sensor P13.

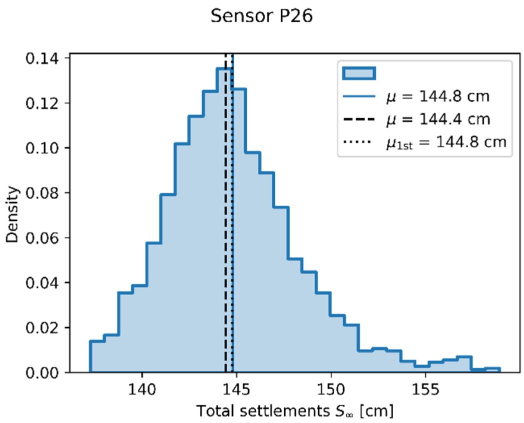

Fig. 13. Comparison of the expected final settlement obtained from either a Bayesian approach with creep (solid line), point estimate with creep (dashed line), or point estimate without creep (dotted line) for sensor P26.

5 Conclusion

In this paper, we have implemented and performed settlement predictions combining the proposed creep extension and the Bayesian linear regression approach as suggested by Asaoka [4]. The results here show that the addition of the second order displacement term in general give a larger than or equal estimation of the final settlement than the first order regression.

Despite the shortcomings of the method as is, the method holds value in practice, particularly the Bayesian approach, as competing models and FE analyses also fail to accurately predict total settlements. The Bayesian approach allows for adding an uncertainty estimate to the predicted final settlement, which is beneficial when determining whether the final settlement is likely to be within the required limit.

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