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Numerical investigation of the consolidation behaviour of soft organic and inorganic clays stabilised with alkali-activated material deep mixed columns

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Abstract. Deep soil mixing (DSM) columns are widely used to enhance the mechanical performance of very soft clays that exhibit high compressibility, low shear strength, and significant long-term settlement. However, limited studies have investigated the consolidation behaviour of soft clays improved with alkali-activated material (AAM) based DSM columns, even though AAM is a novel and sustainable alternative to conventional binders. This study presents a numerical investigation of the consolidation behaviour of organic and inorganic soft clays stabilised with AAM-DSM columns, considering a unit cell. Finite element analyses were carried out to evaluate the dissipation of excess pore pressure, time-dependent settlement, and secondary consolidation (creep) characteristics of the improved ground. The soft clay was modelled as an elastic-viscoplastic material to capture time-dependent deformation, while the DSM column was modelled as a linear elastic perfectly plastic material, using material properties obtained from laboratory investigation. A parametric study was conducted to investigate the effect of area replacement ratio, column stiffness, and column length on the consolidation responses. Additionally, the field performance of the improved ground was evaluated through numerical analysis of an embankment supported on AAM-DSM column-stabilised ground. Overall, the study highlights the potential of AAM-DSM columns as an effective method to improve the consolidation behaviour of soft organic and inorganic clays.

1 Introduction

Soft clays are characterised by low shear strength, high compressibility, high sensitivity and high natural water content, and the presence of organic matter further deteriorates their engineering properties. Post construction settlements associated with high secondary consolidation of soft clays pose significant threats to any infrastructure development in such soft clays [1, 2]. Among the various ground improvement techniques, deep soil mixing (DSM) has been widely employed for soft clay stabilisation through the in-situ mixing of soil with binders to form stabilised columns, resulting in enhanced strength and reduced compressibility [3–5]. In recent years, concerns regarding the CO_2 emissions associated with ordinary Portland cement (OPC) have motivated the investigation of supplementary cementitious materials as partial or complete alternatives for soil stabilisation [6, 7]. Alkali-activated materials (AAMs) have emerged as a sustainable alternative to OPC for soil stabilisation, providing enhanced mechanical performance along with reduced environmental impact [8–10].

Several experimental and numerical investigations have examined the enhancement of bearing capacity and reduction in consolidation settlement of soft clays stabilised with OPC-DSM columns [3, 11–15]. Horpibulsuk et al. [15] investigated the consolidation mechanism of composite soft clay ground stabilised with OPC-DSM columns through laboratory-scale model testing and finite element (FE) modelling using PLAXIS 2D. The settlement behaviour of an embankment constructed on marine clay improved with OPC-DSM columns was carried out by Fulambarkar et al. [11], and reported that the consolidation settlement decreases with increasing area replacement ratio, column length and column stiffness. Using both two-dimensional and three-dimensional FE analyses, Chai et al. [16] demonstrated that floating OPC-DSM columns not only reduce total settlement but also accelerate the consolidation process. Recently, Gupta et al. [17] investigated the performance of embankments supported by AAM-DSM columns under static and cyclic loading conditions, reporting a significant reduction in consolidation settlement, along with improved stability.

Although the performance of DSM columns in reducing consolidation settlement has been extensively investigated, the existing literature predominantly focuses on OPC-based systems. The consolidation behaviour of soft clays stabilised with AAM-DSM columns, particularly through numerical modelling, remains relatively unexplored, and the influence of these columns on secondary consolidation behaviour has not been adequately addressed. Therefore, the present study

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investigates the consolidation response of organic and inorganic soft clays stabilised with AAM-based DSM columns using a two-dimensional unit-cell finite element model. The influence of parameters such as column length, column stiffness and area replacement ratio on the consolidation response is investigated, with secondary consolidation captured using the Soft Soil Creep model. In addition, the role of soil organic content on the consolidation response is assessed through a comparative evaluation of the two clay types, and the field performance of the improved ground is further analysed by numerically modelling an embankment supported on AAM-DSM column-stabilised ground.

2 Methodology

2.1 Validation study

For model validation, the unit cell experiment and numerical study reported by Horpibulsuk et al. [15] on soft clay stabilised with OPC-DSM columns were considered. The numerical model was developed using PLAXIS 2D. The material model, properties and geometry of the column improved composite ground described in the study were used, and the resulting time-settlement response is presented in Fig. 1. A close agreement is observed between the validation study and those reported by Horpibulsuk et al. [15], particularly in terms of the consolidation trend and ultimate settlement. This confirms the reliability of the developed numerical model and validates its use for further parametric studies on the consolidation behaviour of soft clay improved with DSM columns.

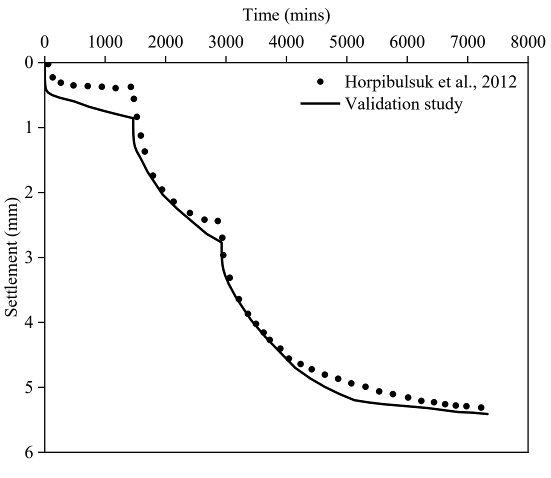

Fig. 1. Comparison of time–settlement response from the validation study and Horpibulsuk et al. [15].

2.2 Unit cell model

To study the influence of AAM-DSM columns on the consolidation response of very soft organic and inorganic clays, numerical analyses were carried out using PLAXIS 2D. The AAM stabilisation considered in this study used ground granulated blast furnace slag (GGBFS) as the precursor and NaOH as the activator. An axisymmetric unit cell model with dimensions identical to those reported by Horpibulsuk et al. [15] was adopted, as shown in Fig. 2. Owing to symmetry, only half of the unit cell was modelled. The vertical boundaries were restrained against horizontal displacement while allowing vertical displacement. At the bottom horizontal boundary, both horizontal and vertical displacements were restricted. Drainage was permitted at the top and bottom horizontal boundaries. Fifteen-nodded triangular elements with fine meshing were used. Incremental loading was applied and maintained until excess pore pressure dissipated, with additional time allowed to capture secondary consolidation. The adopted loading sequence consisted of 10, 20, 40, 60 and 80 kPa. Settlement was monitored at the midpoint on the top surface of the soft clay layer, while excess pore pressure was measured at the mid-depth of the soft clay layer.

The parameters considered in the analysis include soil organic content, column stiffness, area replacement ratio (Ar) and length-to-diameter ratio (L/d) as listed in Table 1. Column stiffness was varied by changing the activator molarity from 8M to 10M. The influence of these parameters on the consolidation settlement response of soft clay stabilised with an AAM-DSM column was investigated. Initially, analyses were carried out on unstabilised soft clays, and the results were compared with those for AAM-DSM column-stabilised clays. The Soft Soil Creep (SSC) model was used for the soft clay, while the DSM column and sand layer were modelled using the Mohr-Coulomb (MC) model. The material properties of soft clays and AAM-DSM columns were obtained from laboratory experiments, whereas the properties of the sand layer were adopted from Horpibulsuk et al. [15]. The material properties of sand, organic clay, and inorganic clay are given in Table 2, and the properties of the AAM-DSM columns are provided in Table 3.

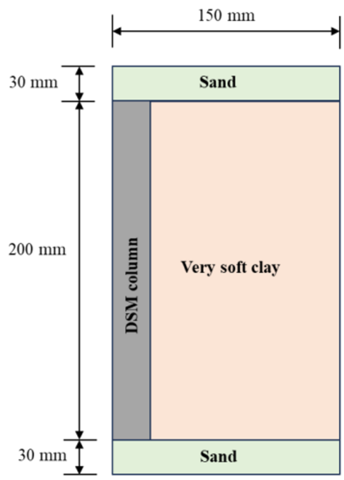

Fig. 2. Geometry of the unit cell model.

2.3 Embankment model

To assess the field applicability of AAM-DSM columns for improving the consolidation response of soft clays, a two-dimensional FE analysis was carried out in PLAXIS 2D by modelling an embankment constructed

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over AAM-DSM column-stabilised soft clay ground. The model geometry was adopted from Suganya and Sivapullaiah [18] and is shown in Fig. 3. The model consists of a 10 m thick very soft clay layer underlain by a 5 m thick sand layer. A 3 m high embankment was constructed in two stages, with each lift having a height of 1.5 m. AAM-DSM columns with a diameter of 1 m and centre-to-centre spacing of 1.5 m were provided beneath the embankment, as illustrated in Fig. 3. The vertical boundaries were restrained against horizontal displacement while allowing vertical displacement, whereas the bottom boundary was fixed in both horizontal and vertical directions. The embankment fill, sand layer and DSM columns were modelled using the MC failure criterion, while the soft clay layer was modelled using the SSC model to capture time-dependent consolidation behaviour. The material properties of soft clays were adopted from Table 2 for both organic and inorganic clays, and DSM columns with higher stiffness were considered, with properties listed in Table 3. The material properties of the sand layer and embankment were adopted from Suganya and Sivapullaiah [18]. The analysis was performed for both unstabilised and AAM-DSM column-stabilised conditions for organic and inorganic clays, and the corresponding time-settlement responses were obtained.

Table 1. Parameters of AAM-stabilised columns used in the unit cell numerical model analysis.

Soil type	Case	q_u (kPa)	d (mm)	L (mm)	Ar (%)	L/d
Organic clay	1	1070	50	200	17	4
	2	846	50	200	17	4
	3	1070	75	200	25	4
	4	1070	100	200	33	4
	5	1070	50	150	17	3
Inorganic clay	6	1585	50	200	17	4
	7	1570	50	200	17	4
	8	1585	75	200	25	4
	9	1585	100	200	33	4
	10	1585	50	150	17	3

q_u – UCS of column, d – column diameter, L – column length

Table. 2. Material properties of sand, organic clay and inorganic clay.

Material	Sand	Organic clay	Inorganic clay
Model	MC	SSC	SSC
γ_{unsat} (kN/m^3)	17	6.17	14
γ_{sat} (kN/m^3)	20	6.17	14
K_v (m/day)	0.49	5.16E-5	1.95E-5
K_v (m/day)	0.49	5.16E-5	1.95E-5
E (kPa)	13000	-	-
C (kPa)	1	7	7
Φ (degrees)	37	1	1
ν	0.3	0.3	0.3
λ^*	-	0.11	0.07
κ^*	-	0.011	0.007
μ^*	-	0.04	0.02

Table 3. Material Properties of AAM-DSM columns.

Material	AAM-DSM Columns			
Soil	Organic	Organic	Inorganic	Inorganic
UCS (kPa)	1070	846	1585	1570
Model	MC	MC	MC	MC
γ_{unsat} (kN/m^3)	14	14	16	16
γ_{sat} (kN/m^3)	14	14	16	16
K_v (m/day)	2.35E-6	2.78E-5	2.73E-4	1.04E-5
K_v (m/day)	2.35E-6	2.78E-5	2.73E-4	1.04E-5
E (kPa)	160,500	126,900	237,750	235,500
C (kPa)	535	423	793	785
Φ (degrees)	1	1	1	1
ν	0.3	0.3	0.3	0.3

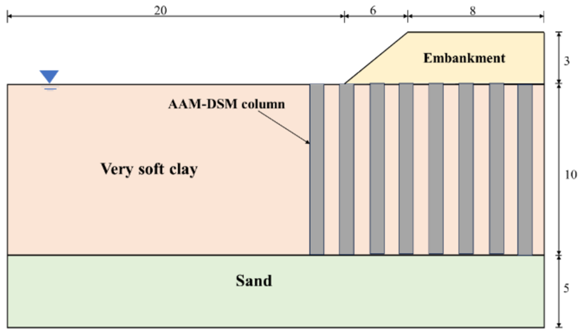

Fig. 3. Geometry of the AAM-DSM column supported embankment on soft clay (all dimensions are in meters).

3 Results and discussions

3.1 Unit cell model

The consolidation settlement of unstabilised organic and inorganic clays is provided in Fig. 4 (a). The organic clay exhibits a total settlement of 47 mm, whereas the inorganic clay undergoes a settlement of 24 mm. The higher consolidation settlement observed in the organic clay can be attributed to its higher compressibility, reflected by its higher compression index. The corresponding excess pore pressure variation of unstabilised clays is shown in Fig. 4 (b), which indicates that the time required for excess pore pressure dissipation is significantly higher for unstabilised clays, which can be associated with their lower permeability values. For AAM-DSM column-stabilised composite ground, the total settlement reduces to 14 mm for organic clay and 7.5 mm for inorganic clay (Fig. 5 (a)), demonstrating the beneficial influence of AAM-DSM columns on the consolidation behaviour of these clays. The time required for excess pore pressure dissipation is also significantly reduced in stabilised composite ground (Fig. 5 (b)), indicating an accelerated consolidation response. Excess pore pressure contour plots of AAM-DSM column stabilised inorganic clay at different time intervals are provided in Fig. 5(c). From the time-settlement behaviour of the AAM-DSM column stabilised composite ground, it is evident that the columns effectively reduce the total settlement and accelerate the rate of consolidation. The reduction in consolidation settlement of the AAM-DSM stabilised composite ground can be attributed to the higher stiffness of the column, which enables transfer of a major portion of the applied load to the column, thereby reducing the settlement of the surrounding clay [15, 17]. For both stabilised and unstabilised cases, settlement continues to increase even after complete dissipation of excess pore pressure, indicating the contribution of secondary consolidation. The time-settlement plot of organic clay (Fig. 4 (a)) exhibits a steeper slope compared to that of inorganic clay, which is consistent with the higher secondary compression index of organic clay. Although the AAM-DSM column significantly reduces primary consolidation settlement, creep deformation of the surrounding clay continues to contribute to long-term settlement (Fig. 5 (a)).

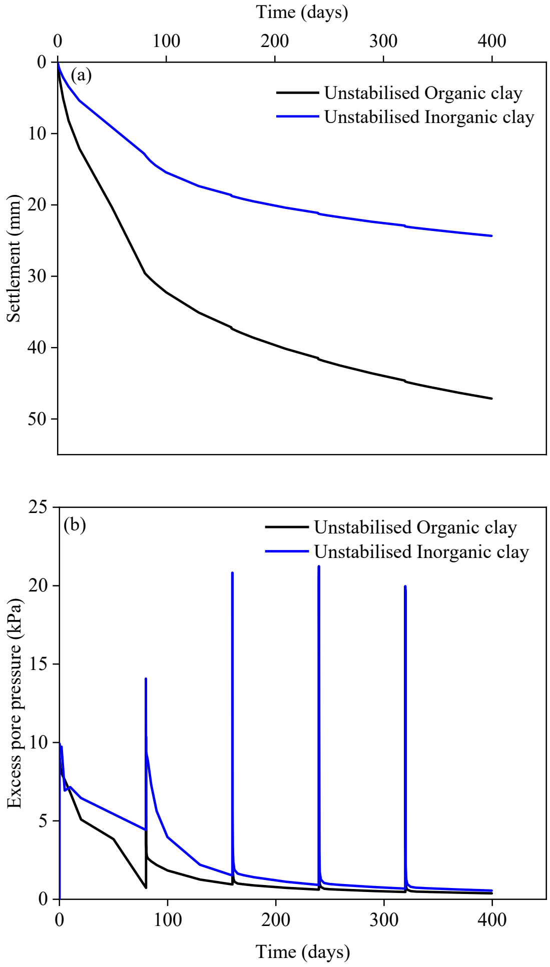

Fig. 4. (a) Time-settlement response and (b) excess pore pressure dissipation of unstabilised organic and inorganic clays.

The influence of column stiffness on the consolidation response of AAM-DSM column-stabilised composite ground was analysed, considering columns with different UCS values. The obtained time-settlement response of these cases is shown in Fig. 6. From Fig. 6 (a), it can be observed that column stiffness has no significant influence on the consolidation settlement of AAM-DSM column-stabilised organic clays. A similar trend is observed for inorganic clay also, where column stiffness does not noticeably affect the magnitude of total settlement of the stabilised composite ground (Fig. 6 (b)). This behaviour suggests that, within the range of stiffness values considered in the present study, the overall consolidation response is governed primarily by the compressibility of the surrounding soft clay and the load-sharing mechanism between the column and soil, rather than the stiffness variation of the column. Once the column stiffness is sufficiently higher than that of the surrounding clay, further increase in stiffness does not significantly alter stress distribution or settlement behaviour. Similar observations were reported by Horpibulsuk et al. [15] for cement-DSM column-stabilised composite ground.

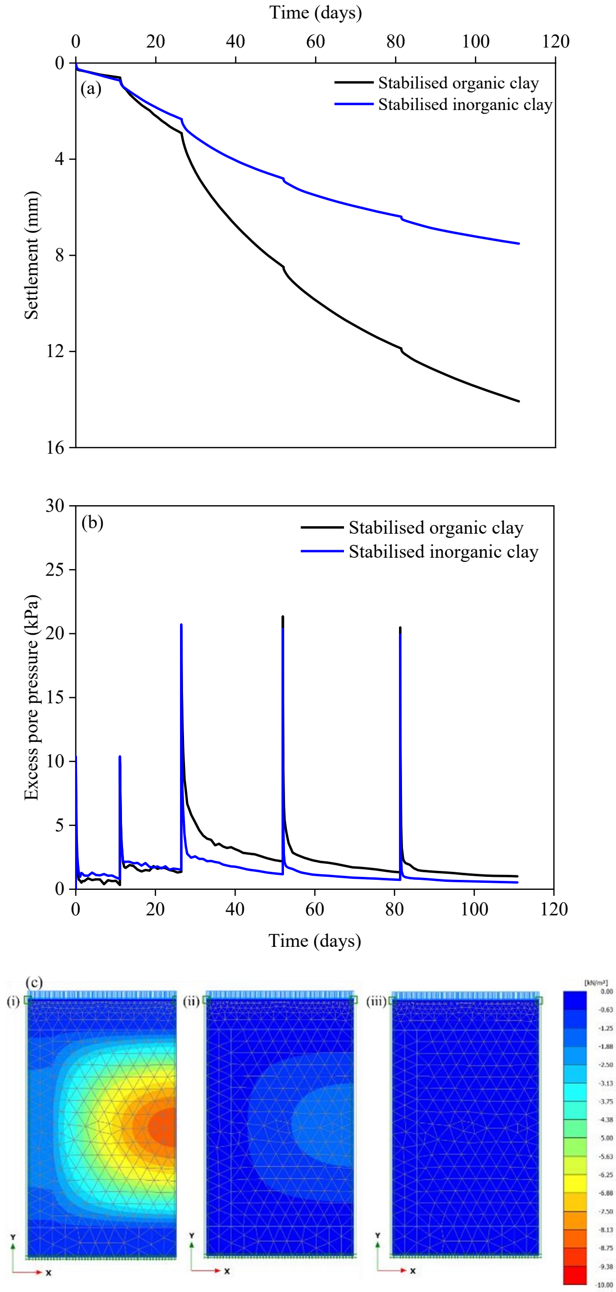

Fig. 5. (a) Time-settlement response, and (b) excess pore pressure dissipation of AAM-DSM column stabilised organic and inorganic clays, (c) Excess pore pressure contour plot of stabilised inorganic clay at (i) start of consolidation; (ii) after 2 days, and (iii) after 30 days.

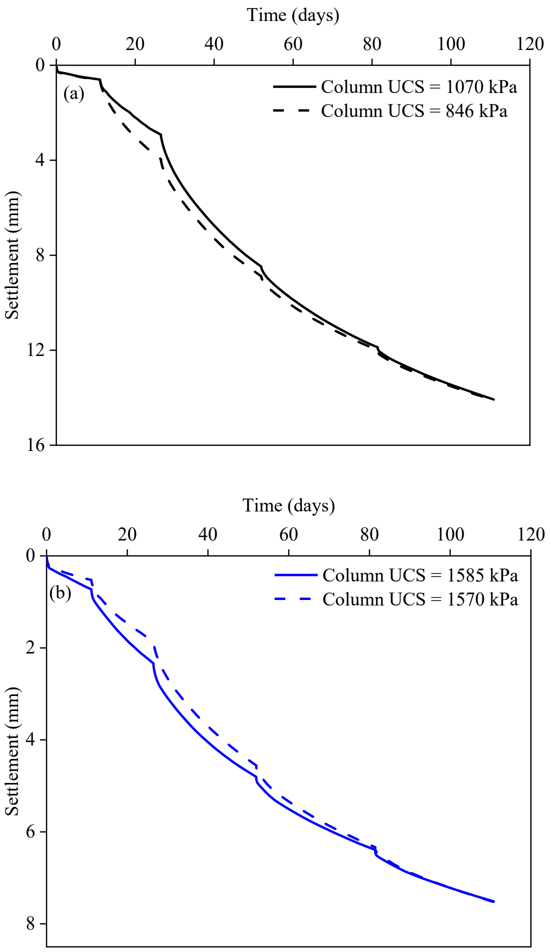

Fig. 6. Influence of varying column stiffness on the time-settlement response of AAM-DSM column stabilised (a) organic clay and (b) inorganic clay.

The influence of area replacement ratio (Ar) on the settlement of AAM-DSM column-stabilised composite ground was investigated by considering Ar values of 17%, 25% and 33% and the corresponding time-settlement responses are provided in Fig. 7. The column diameters corresponding to these Ar values are provided in Table 1. For stabilised organic clay (Fig. 7 (a)), the total settlement reduced from 14 mm for Ar = 17%, to 8 mm for Ar = 25% and 3.5 mm for Ar = 33%. Similarly, for inorganic clay (Fig. 7 (b)), the total settlement decreased from 7.5 mm for Ar = 17%, to 4.4 mm for Ar = 25%, and 2.3 mm for Ar = 33%. For both organic and inorganic clays, settlement decreases significantly with increasing area replacement ratio. Higher area replacement ratio values also accelerate excess pore pressure dissipation, as shown in Fig. 8. As the Ar increases, the proportion of stiffer column material increases, leading to higher stress concentration in the columns and a corresponding reduction in the stress acting on the surrounding soft clay, resulting in lower total settlement of the composite ground [15, 17, 19]. Although the permeability of the soft clay and the AAM-DSM column is similar, accelerated excess pore pressure dissipation is observed at higher Ar values. Similar observations were reported by Horpibulsuk et al. [15] for cement-DSM column-stabilised composite ground.

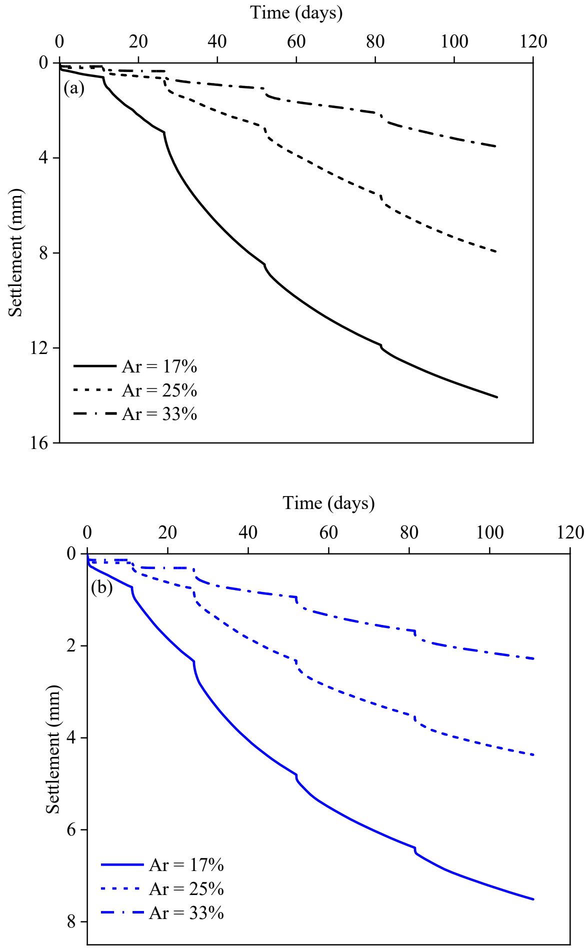

Fig. 7. Influence of area replacement ratio on time-settlement response of AAM-DSM column stabilised (a) organic clay and (b) inorganic clay.

Further, the consolidation behaviour of end-bearing and floating AAM-DSM columns was evaluated by considering the length-to-diameter (L/d) ratios of 4 and 3, respectively. The time-settlement response of column-stabilised organic and inorganic clays with different L/d ratios is shown in Fig. 9 (a) and Fig. 9 (b), respectively. For organic clay (Fig. 9 (a)), the total settlement of the end bearing column improved ground was 14 mm (L/d = 4), whereas the floating column improved ground exhibited a higher settlement of 25 mm (L/d = 3). A similar trend was observed for inorganic clay also (Fig. 9 (b)), where the end bearing column improved ground resulted in a settlement of 7.5 mm (L/d = 4), compared to 14 mm for the floating column improved ground (L/d = 3). The reduced settlement observed in the end-bearing column cases can be attributed to higher stress concentration in the columns due to direct load transfer to the underlying stiff stratum, which enables the columns to carry a larger portion of the applied load and thereby reduces deformation in the surrounding soft clay [17].

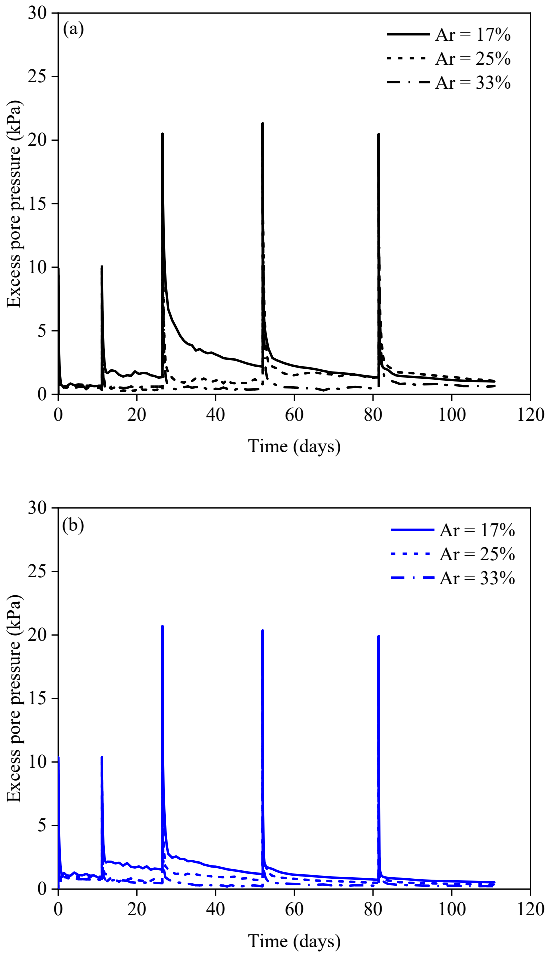

Fig. 8. Influence of area replacement ratio on excess pore pressure dissipation of AAM-DSM column stabilised (a) organic clay and (b) inorganic clay.

3.2 Embankment model

The behaviour of soft clay stabilised with AAM-DSM columns in the field was investigated by considering an embankment constructed over the stabilised ground. The settlement along the ground surface at the centre line of the embankment is given in Fig. 10 (a) and Fig. 10 (b) for unstabilised and AAM-DSM column-stabilised cases, respectively. From Fig. 10 (a), the settlement in organic clay was approximately 2.8 m, whereas inorganic clay exhibited a settlement of about 1.6 m under embankment loading in the unstabilised condition. The rate of settlement was high during the initial period due to primary consolidation, after which the settlement rate gradually decreased. However, both clays continued to deform with time at a slower rate, indicating secondary consolidation behaviour. The settlement in organic clay was approximately 1.8 times higher than that observed in inorganic clay, which can be attributed to the higher compressibility of organic clay. The sustained long-term deformation, associated with secondary consolidation, was also more pronounced in organic clay due to the presence of organic matter and its high void ratio [20, 21].

Upon stabilisation with AAM-DSM columns, the settlement of organic clay reduced to approximately 0.37 m, while that of inorganic clay reduced to 0.24 m (Fig. 10 (b)). A substantial reduction in the settlement of the foundation soil was observed for both organic and inorganic clays compared to the unstabilised condition. Stabilisation with AAM-DSM columns also accelerated the consolidation process, consistent with the observations from the unit cell analysis. The time required for excess pore pressure dissipation was reduced by half in the stabilised case. This improvement in consolidation behaviour can be attributed to the stiff AAM-DSM columns, which significantly increase the composite stiffness of the ground and cause stress concentration within the column. Consequently, a larger portion of the embankment load is transferred to the columns, reducing the stress carried by the surrounding soft clay and thereby reducing settlement [11]. These results indicate that stabilisation of soft clays with AAM-DSM columns significantly improves the consolidation behaviour and demonstrates their potential for field application.

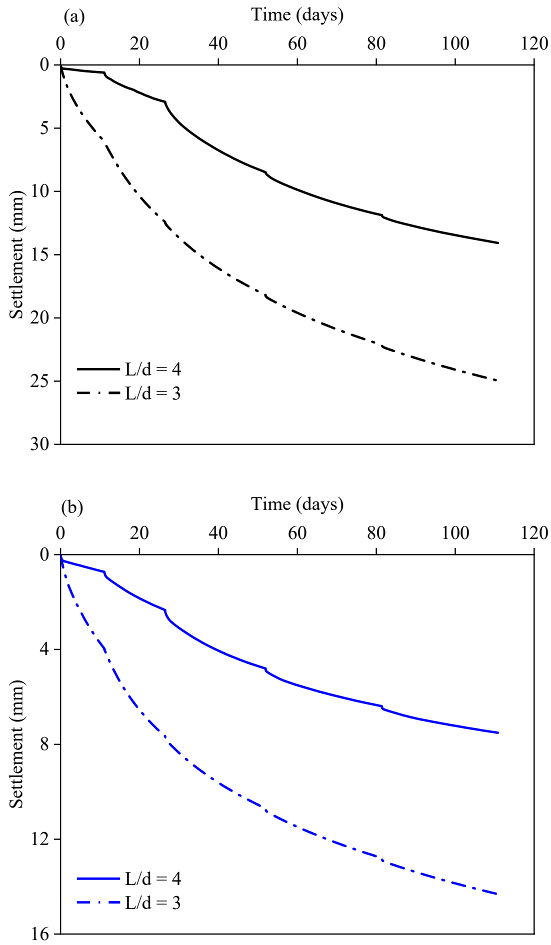

Fig. 9. Influence of length-to-diameter ratio on time-settlement response of AAM-DSM column stabilised (a) organic clay and (b) inorganic clay.

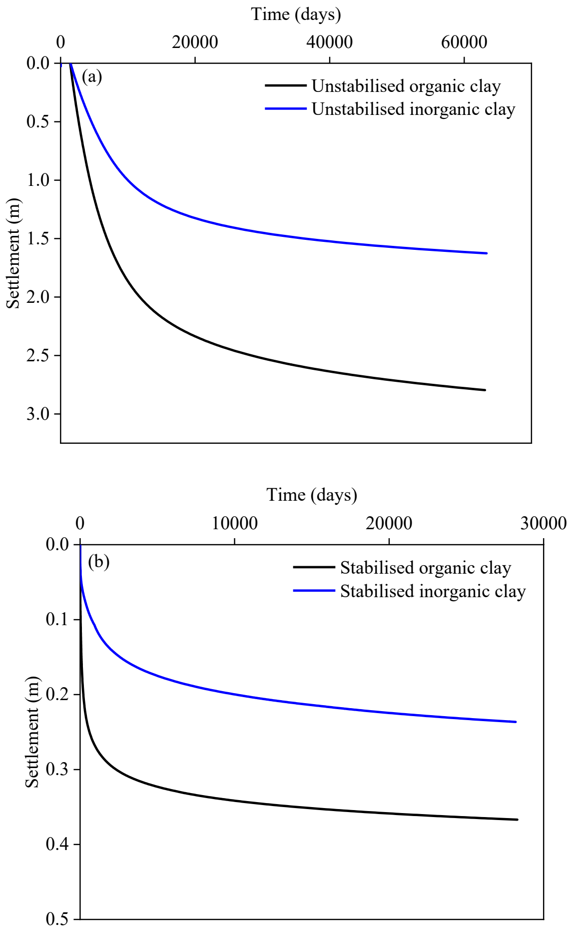

Fig. 10. Time - settlement response of embankment constructed on (a) unstabilised and (b) AAM-DSM column stabilised organic and inorganic clays.

4 Conclusions

The consolidation behaviour of very soft organic and inorganic clays stabilised using AAM-DSM columns was investigated through numerical analysis. Based on the results obtained in this study, the following conclusions were drawn:

- Unstabilised organic clay exhibited approximately twice the settlement of inorganic clay, with continued deformation after excess pore pressure dissipation, indicating more pronounced secondary consolidation.
- AAM-DSM column stabilisation significantly reduced settlement and accelerated consolidation in both organic and inorganic clays, although secondary consolidation remained evident in the stabilised composite ground.
- Column stiffness showed limited influence on consolidation response, whereas increasing the area replacement ratio significantly reduced settlement and accelerated excess pore pressure dissipation.
- End-bearing columns were more effective than floating columns in reducing settlement for both organic and inorganic clays due to improved load transfer to the underlying stiff stratum.
- Analysis of an embankment constructed over AAM-DSM column-stabilised ground showed that stabilisation significantly reduces consolidation settlement of the foundation soil and accelerates consolidation, highlighting its effectiveness for field applications.

Overall, the study demonstrates the potential of AAM-DSM columns as an effective and sustainable ground improvement method for enhancing the consolidation performance of very soft organic and inorganic clays.

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