A new approach to the debate between hypotheses A and B based on the isotache concept

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Abstract. Consolidation settlement of an in-situ clay layer with a thickness of several meters to several tens of meters is generally predicted based on laboratory test results obtained from small specimens with a thickness of only 20 mm. Therefore, the similitude law with respect to layer thickness is of critical importance. This issue is directly related to the well-known debate between Hypothesis A and Hypothesis B. Against this background, the first author previously proposed a constitutive model describing the isotache law, in which the stress – strain relationship is uniquely determined as a function of strain rate in consolidation behaviour. The model is formulated using the consolidation yield stress as a key parameter and is essentially equivalent to the overstress power-law creep model. A simple finite-difference algorithm for practical uncoupled analysis was presented, in which compressive behaviour governed by the isotache law is solved using the temporal dissipation of excess pore water pressure. Furthermore, parametric analyses were conducted for various layer thicknesses, taking into account the initial effective stress distribution with depth due to the self-weight of the clay. The influence of layer thickness on consolidation settlement behaviour is discussed in relation to the debate between Hypothesis A and Hypothesis B.

1 Introduction

Prediction of consolidation settlement for an in-situ clay layer with a thickness of several meters to several tens of meters is generally performed based on laboratory test results obtained from small specimens with a thickness of only 20 mm. When making such predictions, similarity laws must be considered. The H^2-law, which states that required consolidation time is proportional to the square of the layer thickness (or, more precisely, the maximum drainage path length), is widely recognized as effective for predicting the dissipation behaviour of excess pore water pressure.

Meanwhile, the settlement behaviour expressed in terms of strain has long been discussed in the context of the debate between Hypothesis A and Hypothesis B, following Ladd et al. [1] and Jamiolkowski et al. [2]. This issue has remained one of the major topics in geotechnical engineering for more than half a century. A conceptual illustration of Hypothesis A and Hypothesis B is shown in Fig. 1.

Hypothesis A states that viscous effects (i.e., creep deformation) do not occur during primary consolidation; creep occurs only after primary consolidation has completed. Therefore, the strain at the end of primary consolidation should be independent of layer thickness. Mesri and Choi [3] strongly supported Hypothesis A based on their experimental results.

In contrast, Hypothesis B assumes that creep deformation occurs even during the primary consolidation. Because thicker layers require a longer time to complete primary consolidation, the amount of creep that develops during this stage increases with layer thickness, resulting in a larger strain at the end of primary consolidation.

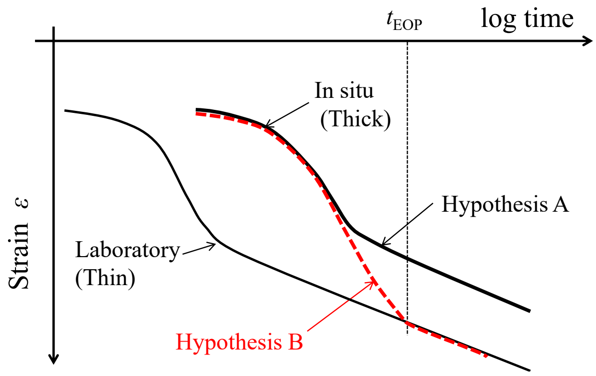

Fig. 1. Illustration of Hypothesis A and Hypothesis B in consolidation behaviour.

A well-known experimental dataset in this debate is a series of consolidation tests conducted by Aboshi [4], who used the same clay specimen with layer thicknesses ranging from a laboratory scale (0.02 m) to nearly field scale (1 m). However, these results were interpreted as exhibiting intermediate behaviour between Hypothesis A and Hypothesis B, and no definitive conclusion has yet been reached.

Degago et al. [5] stated that consolidation behaviour essentially supports Hypothesis B. However, they demonstrated that differences in settlement in strain

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during previous loading stages, smaller for thin specimens and larger for thick specimens, are carried over into consequent loading stages, making the overall behaviour appear to support Hypothesis A. This reasoning is essentially consistent with the argument presented by Imai et al. [6].

The isotache concept, originally proposed by Šuklje [7], describes a unique stress – strain – strain rate relationship observed in consolidation behaviour. Leroueil et al. [8] formalized the isotache concept by expressing it using normalized compression curve and consolidation yield stress. Watabe et al. [9] and Watabe and Leroueil [10] further modelled the isotache law using a rheological approach based on long-term consolidation tests and multi-stepwise CRS (constant rate of strain) consolidation tests, and proposed generalized expressions and parameter values applicable to a wide range of clays worldwide.

Furthermore, Watabe and Miyata [11] demonstrated that the viscoplastic strain description derived from the isotache law is mathematically equivalent to that of a power-law creep model in conjunction with an overstress formulation. They presented a time-marching numerical algorithm and clarified that the two formulations are essentially identical.

In the present study, the temporal variation of strain during consolidation is calculated based on the isotache law, taking into account the initial effective stress distribution (corresponding to depth in-situ) and the self-weight of the clay. The influence of layer thickness is examined in relation to Hypothesis A and Hypothesis B.

2 Model and calculation method

The isotache law applies to viscoplastic strain $\varepsilon^{\mathrm{vp}}$, obtained by subtracting the elastic strain ε^{e} from the total strain ε. In the conceptual model proposed by Leroueil et al. [8], the consolidation yield stress σ'^{p} (preconsolidation pressure) is expressed as a function of viscoplastic strain rate $\dot{\varepsilon}^{\mathrm{vp}}$, Watabe et al. [9] provided a specific formulation (Fig. 2) based on extensive experimental results. The plots in the figure show that the relationship between viscoplastic strain rate $\dot{\varepsilon}^{\mathrm{vp}}$ and the corresponding consolidation yield stress σ'^{p} obtained from long-term consolidation tests conducted at two different consolidation pressures.

The normalization parameter σ'^{p0} is defined as the consolidation yield stress corresponding to a viscoplastic strain rate of 1.0×10^{-7} s^{-1} at 24 hours in a standard incremental loading oedometer test. The strain-rate dependency of the consolidation yield stress σ'^{p} is expressed as:

$$\ln \frac{\sigma'^{\mathrm{p}} - \sigma'^{\mathrm{pL}}}{\sigma'^{\mathrm{pL}}} = c_1 + c_2 \ln \dot{\varepsilon}^{\mathrm{vp}} \qquad (1)$$

where σ'^{pL} is the lower limit of the consolidation yield stress corresponding to an infinitesimal viscoplastic strain rate, and c_1 and c_2 are material constants representing the reference shift and strain-rate sensitivity, respectively.

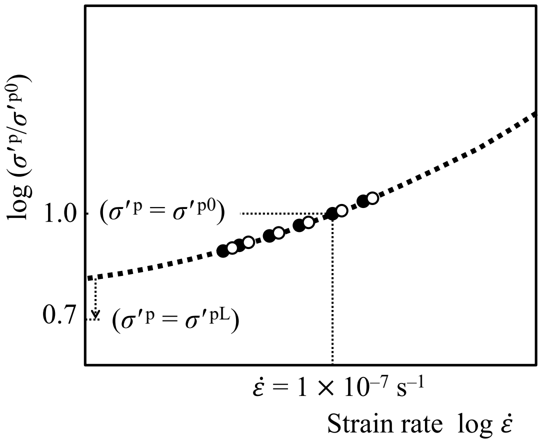

Fig. 2. Strain rate dependency of the consolidation yield stress.

Watabe et al. [9] reported $\sigma'^{\mathrm{pL}}/\sigma'^{\mathrm{p0}} = 0.7$ and proposed $c_1 = 0.935$ as an average value. Because isotache parameters, σ'^{pL}, c_1, and c_2, are mutually dependent through σ'^{p0}, determining two parameters uniquely defines the third.

Rearranging Equation (1), viscoplastic strain rate can be expressed in overstress form:

$$\dot{\varepsilon}^{\mathrm{vp}} = c_3 \left(\frac{\sigma'^{\mathrm{p}} - \sigma'^{\mathrm{pL}}}{\sigma'^{\mathrm{pL}}} \right)^{c_4} \qquad (2)$$

where $c_3 = \exp(-c_1/c_2)$, and $c_4 = 1/c_2$.

Watabe and Miyata [11] demonstrated that this expression is equivalent to the power-law creep model with overstress formulation:

$$\dot{\varepsilon}^{\mathrm{vp}} = a \left(\frac{\sigma' - \sigma'^{\mathrm{y}}}{\sigma'^{\mathrm{y}}} \right)^{b} \qquad (3)$$

where a and b correspond to c_3 and c_4, respectively, and σ'^{y} is the yield stress at current condition. Thus Equations (1)–(3) represent essentially the same constitutive framework.

In the present study, the temporal variation of viscoplastic strain is computed using a time-marching finite-difference scheme based on Equation (2), and following the calculation flow shown in Fig. 3, where, λ_ε and κ_ε in the calculation flow are the compression index and swelling index, respectively, for strain when the effective stress σ' is expressed as a natural logarithm. Ideally, excess pore water pressure dissipation and elasto-viscoplastic deformation should be solved in a fully coupled manner. However, because viscous effects are significant in compression but less influential in excess pore water pressure dissipation, an uncoupled approach is adopted for simplification.

First, the temporal variation of effective stress is computed using Terzaghi's one-dimensional consolidation theory. Subsequently, the temporal variation of strain is calculated following the calculation flow shown in Fig. 3 using the isotache law based on the obtained effective stress history.

The calculations performed in this study are based on a simplified framework in which the widely used Terzaghi's one-dimensional consolidation theory is extended by incorporating the isotache concept to represent the viscous behavior of the soil skeleton. The primary objective is to conceptually examine how layer thickness itself, as well as the effect of self-weight—which becomes increasingly significant with increasing thickness—affects consolidation settlement behavior. In this study, the dissipation of excess pore water pressure and the drainage (i.e., settlement) behavior are treated in an uncoupled manner. Therefore, particularly in the early stages of consolidation, any mismatch between the strain rate and the drainage rate is neglected. In rigorous consolidation analyses, these processes must be solved in a fully coupled manner. However, such an approach introduces various computational constraints, including the need to prescribe an appropriate initial strain rate consistent with the drainage capacity of the clay layer. The calculations in this study circumvent these complexities by adopting a simplified approach in which the isotache concept is incorporated into the conventional framework for predicting consolidation settlement.

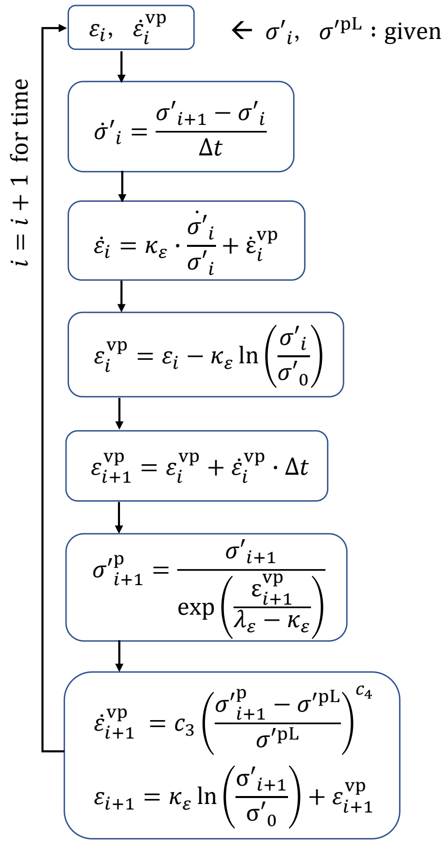

Fig. 3. Calculation flow.

3 Calculation conditions

The input parameters used in the calculations are summarized in Table 1. These values were obtained from a multi-stepwise CRS consolidation test (in which the strain rate is changed stepwise) conducted on the Pleistocene Osaka Bay marine clay Ma9, collected from seabed deposits prior to the construction of the Phase-2 island of Kansai International Airport (Yoneda and Watabe [12]). In this study, $c_3 = 2.127 \times 10^{-4}$ and $c_4 = 9.043$ were calculated using the relationships $c_3 = \exp(-c_1/c_2)$ and $c_4 = 1/c_2$.

Calculations were performed under double-drainage boundary conditions for different layer thicknesses: of 0.02 m (equivalent to a typical incremental loading oedometer test), 0.1 m, 1 m, 5 m, 10 m, 20 m (equivalent to the expected maximum in situ layer thickness), 30 m, 50 m, and 100 m. The initial effective stress, σ'^{v0}, was set to 100 kPa, representing a clay layer at a relatively shallow depth, corresponding either to the stress at the top of the clay layer or to that at the mid-depth.

Calculations were also carried out both with and without consideration of the self-weight of the clay. The applied increment of consolidation pressure was 350 kPa in all cases. The void ratio corresponding to the initial effective stress was calculated from the void ratio at an effective consolidation pressure of 1016 kPa and a compression index C_c of 1.0.

Table 1. Input parameters for calculations (based on marine clay Ma9 from the Phase-2 island of Kansai International Airport).

Parameters	Input values
Initial effective stress σ'^{v0}	100 kPa (set at the surface or mid-depth of the clay layer)
Lower limit of consolidation yield stress (preconsolidation pressure) σ'^{pL}	0.7 times the consolidation yield stress σ'^{p0} corresponding to viscoplastic strain rate of $\dot{\varepsilon}^{vp} = 1.0 \times 10^{-7}$ s^{-1} (corresponding to an elapsed time of 24 h in the oedometer test)
Incremental consolidation pressure $\Delta\sigma$	350 kPa
Parameter c_3	2.127×10^{-4}
Parameter c_4	9.043
Effective unit weight γ'	0 (without self-weight) or 7.0 kN/m^3 (with self-weight)
Void ratio e	1.42 (at an effective stress of 1016 kPa)
Compression index C_c (λ_ε)	1.0 (0.179)
Swelling index C_s (κ_ε)	0.1 (0.0179)
Coefficient of consolidation c_v	1.0×10^{-7} m^2/s (86.4 cm^2/day)

The calculation begins from a state in the normally consolidated domain (i.e., overconsolidation ratio, OCR = 1). This implies that the consolidation yield stress corresponding to a strain rate of 1.0×10^{-7} s^{-1}, which can be regarded as equivalent to the strain rate after 24 hours in practical incremental loading oedometer test, is equal to the initial effective stress before incremental loading. Because the purpose of this study is to examine the similitude law for strain, analogous to the H^2 law for dissipation of excess pore water pressure (i.e., the similitude law governing consolidation time), the coefficient of consolidation, c_v, is assumed to remain constant during consolidation, as is commonly assumed in practical analyses.

Furthermore, when applying the isotache law to calculate the dissipation of excess pore water pressure coupled with the elasto-viscoplastic deformation of the soil skeleton, if the overconsolidation ratio (OCR) is set close to 1 (i.e., normally consolidated conditions), based on incremental loading oedometer tests, the resulting strain rate becomes high, approaching 1.0×10^{-7} s^{-1}.

Under this initial condition, when the layer thickness is large, the drainage required due to compression of the soil skeleton exceeds the drainage capacity of the clay layer. As a result, excess pore water pressure is generated, the effective stress decreases, and the calculation proceeds only after the strain rate decreases to a level consistent with the drainage capacity. To avoid this unrealistic behaviour, an uncoupled analysis was conducted as a practical manner. First, the dissipation of excess pore water pressure, i.e., the temporal variation of the effective stress, was calculated using Terzaghi's one-dimensional consolidation theory, which is commonly used in practice. Then, strain was calculated using the isotache law based on the temporal variation of effective stress obtained from the consolidation analysis.

Finally, the influence of layer thickness on the time-dependent settlement behaviour, i.e., the consolidation curve expressed in terms of strain, was examined by applying the isotache law to the effective stress histories obtained. Accordingly, calculations were performed for normally consolidated clay (OCR = 1) with a constant coefficient of consolidation, c_v, consistent with the basic case assumptions of Terzaghi's one-dimensional consolidation theory, under conditions with different layer thicknesses.

In the present calculations, the coefficient of consolidation, c_v, listed in Table 1 is treated as constant. This is a simplification adopted to directly apply the framework of Terzaghi's one-dimensional consolidation theory, which is widely used in engineering practice. To more accurately represent actual behavior, particularly in cases involving thick clay layers, it is necessary to assign values of the coefficient of consolidation corresponding to the stress level. Furthermore, these values should be updated as consolidation progresses. It should be noted that, in this study, a normally consolidated state is assumed as the initial condition. As consolidation proceeds, both the hydraulic conductivity and the coefficient of volume compressibility decrease. Therefore, the change in the coefficient of consolidation—defined as the ratio of these two parameters—is expected to be relatively small.

4 Calculation results

The results of calculations for cases assuming a clay layer at shallow depth with an initial effective stress of 100 kPa are described below.

4.1 Cases without self-weight

The consolidation curves for the cases not considering the self-weight of the clay are shown in Fig. 4. The increment in the consolidation pressure of 350 kPa is relatively large compared with the initial effective stress of 100 kPa, resulting in a large total strain.

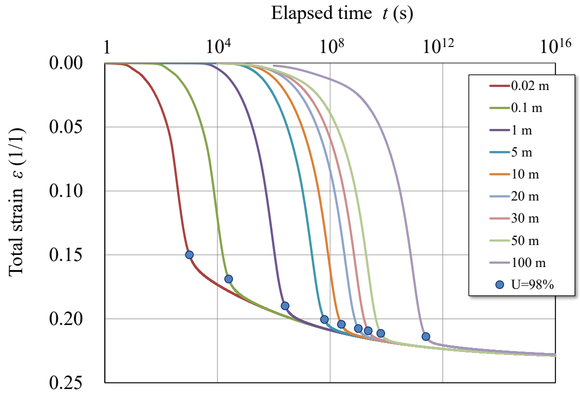

Fig. 4. Consolidation curves for cases where self-weight is not considered.

The consolidation curves converge over time to a single downward-convex curve. However, the strain at the end of primary consolidation, indicated by circle markers in the figure corresponding to 98% dissipation of excess pore water pressure, increases with increasing layer thickness. This indicates that the effect of layer thickness on the consolidation curve expressed in strain supports Hypothesis B when the initial stress distribution due to the self-weight of the clay is not considered (effective unit weight $\gamma' = 0$). This means that the consolidation curve calculated based on the isotache law without considering self-weight is consistent with Hypothesis B.

4.2 Cases where self-weight is considered with reference to the effective stress at the top of the clay layer

Consolidation curves for the cases considering the self-weight of the clay, with the initial effective stress at the top of the clay layer set to 100 kPa (initial void ratio of 2.43), are shown in Fig. 5.

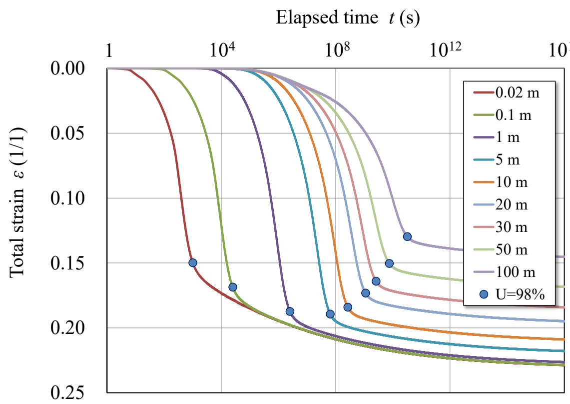

Fig. 5. Consolidation curves for cases where self-weight is considered with reference to the effective stress at the top of the clay layer.

Because the effective unit weight γ' is 7.0 kN/m^3, the initial effective stress at the bottom of the clay layer is 107 kPa (initial void ratio of 2.40) for a layer thickness of 1 m, indicating only a small difference between the top and bottom of the layer. However, for a layer thickness of 5 m, it increases to 135 kPa (initial void ratio of 2.30), and for a layer thickness of 10 m, it increases to 170 kPa (initial void ratio of 2.20). As the layer thickness increases, the initial effective stress at the bottom becomes significantly larger, accompanied by a smaller initial void ratio. In extreme cases, for a layer thickness of 50 m, the initial effective stress at the bottom reaches 450 kPa (initial void ratio of 1.77), and for a layer thickness of 100 m, it increases to 800 kPa (initial void ratio of 1.52).

As the initial void ratio decreases, the amount of strain generated also decreases. In other words, as the layer thickness increases, consolidation progresses in the deeper portion due to its self-weight prior to incremental loading. Consequently, the average initial void ratio of the entire layer decreases, resulting in smaller strain under the consolidation pressure increment of 350 kPa.

4.3 Cases where self-weight is considered with reference to the effective stress at the mid-depth of the layer

Consolidation curves obtained by assuming the initial effective stress of 100 kPa (initial void ratio of 2.43) at the mid-depth of the clay layer and considering the self-weight of the clay are shown in Fig. 6.

Because the effective unit weight γ' is 7.0 kN/m^3, the initial effective stress at the top and bottom of the clay layer are 96.5 kPa and 103.5 kPa (initial void ratio of 2.44 and 2.41, respectively) for a layer thickness of 1 m, indicating only a small difference. For a layer thickness of 5 m, the difference becomes more pronounced, reaching 82.5 kPa and 117.5 kPa (initial void ratio of 2.51 and 2.36, respectively), and for a layer thickness of 10 m, the values become 65 kPa and 135 kPa (initial void ratio of 2.61 and 2.30, respectively).

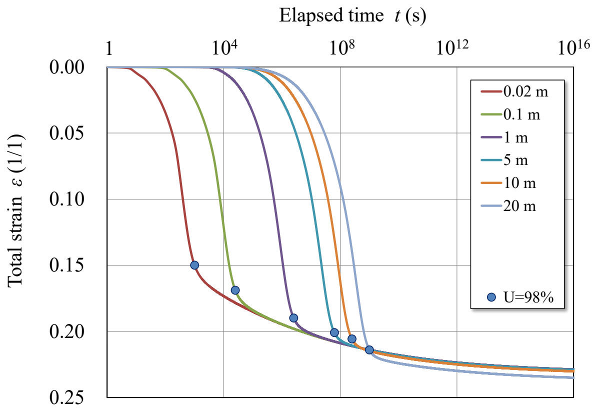

Fig. 6. Consolidation curves for cases where self-weight is considered with reference to the effective stress at the mid-depth of the layer.

As the layer thickness increases, the difference in the initial effective stress between the top and bottom becomes significantly larger, and the difference in initial void ratio also increases. In other words, as the layer thickness increases, consolidation has progressed in the deeper portion due to the higher effective stress caused by self-weight, whereas the shallower portion remains less consolidated due to lower effective stress.

As a result, these opposing effects in the deeper and shallower portions largely cancel each other out, and the average initial void ratio of the entire layer does not change significantly. This means that the strain due to elasto-plastic deformation caused by the consolidation pressure increment of 350 kPa is not strongly influenced by layer thickness. However, as the layer thickness increases, the time required for consolidation increases in proportion to the square of the layer thickness, and creep strain due to viscous effects increases even during primary consolidation, resulting in behaviour consistent with Hypothesis B.

For a layer thickness of 20 m, the initial effective stress at the top of the clay layer decreases significantly to 30 kPa, showing a slight deviation from Hypothesis B. If the layer thickness increases further, the initial effective stress at the top becomes negative, which is physically unrealistic.

4.4 Discussion

The above results suggest that the consolidation curves in Fig. 4, which support Hypothesis B when self-weight is not considered, changes significantly in Fig. 5 when self-weight is considered with the reference depth set at the top of the clay layer. As the layer thickness increases, the consolidation curves gradually shift upward from the convergence curve corresponding to Hypothesis B.

When comparing the consolidation curves for thin layers (0.02 m and 0.1 m) with those for thick layers (30 m and 50 m), the strains at the end of primary consolidation become nearly identical, and the secondary consolidation behaviour is expressed as approximately parallel curves. Although creep occurs during primary consolidation, the overall calculation results appear to support Hypothesis A.

These results suggest that when the clay layer is thin (1 m or less), the effect of the self-weight is relatively small, and the consolidation curve expressed in strain supports Hypothesis B. However, when comparing thin layers used in laboratory tests with thick layers in in-situ ground conditions, the behaviour gradually shifts from Hypothesis B toward Hypothesis A as the layer thickness increases. Furthermore, when the layer thickness becomes unrealistically large, such as 100 m, the strain at the end of primary consolidation becomes even smaller, exceeding the range expected under Hypothesis A.

However, even when self-weight is considered, if the reference depth for the initial effective stress is set at the mid-depth of the clay layer, the overall effect of self-weight remains relatively small. This is because relatively large compression occurs in the shallower portion above the reference depth, while relatively small compression occurs in the deeper portion below the reference depth. These opposing effects cancel each other out, resulting in consolidation behaviour that continues to support Hypothesis B even for moderately thick layers.

Based on the above findings, adopting the top of the clay layer as the reference level for stress definition can be regarded as a rational extension of laboratory consolidation testing when extrapolated to field conditions. However, in practical calculations, it is more appropriate to represent the clay layer by its mid-depth; in such cases, the results tend to be consistent with Hypothesis B.

5 Summary

The effect of layer thickness on the consolidation curve expressed in terms of strain supports Hypothesis B when the self-weight of the clay is not considered (i.e., the effective unit weight is assumed to be $\gamma' = 0$). However, when a clay layer at a relatively shallow depth, where the effect of the self-weight is significant, is considered, Hypothesis A may appear to be more favourable than Hypothesis B.

Even for a relatively shallow clay layer where the effect of self-weight is significant, if the reference depth for defining the initial effective stress is taken as the mid-depth of the clay layer, the influence of the small initial effective stress (high initial void ratio) in the shallower portion and that of the large initial effective stress (low initial void ratio) in the deeper portion tend to cancel each other out. As a result, Hypothesis B tends to be more favourable even when the self-weight of the clay is taken into account.

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